Optimal Oblivious Path Selection on the Mesh

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Outline

- Introduction
- Related Work and Our Contributions
- Results on Mesh Networks
- Future Work

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Oblivious Routing Algorithms

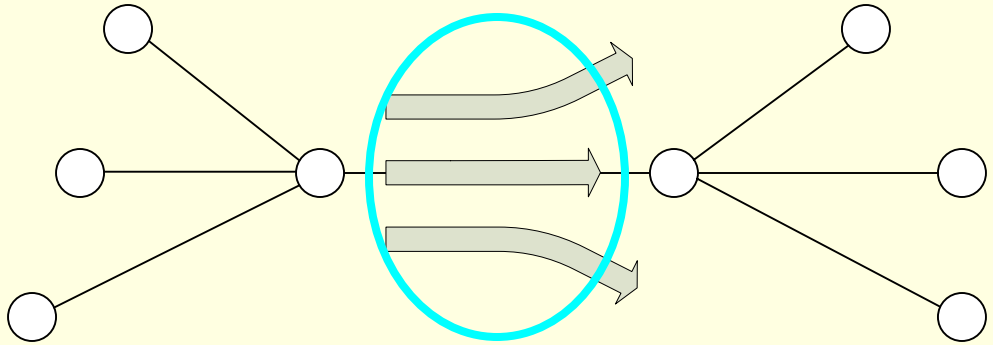
- Routing algorithms provide paths for packets in the network.
- A routing algorithm is **oblivious** if the path of every packet is specified independently of the paths of the other packets.

Benefits of Oblivious Routing

- Distributed algorithm
- Capable of solving online routing problems, where packets continuously arrive in the network.

Congestion C

- Congestion C: the maximum number of paths that use any edge in the network

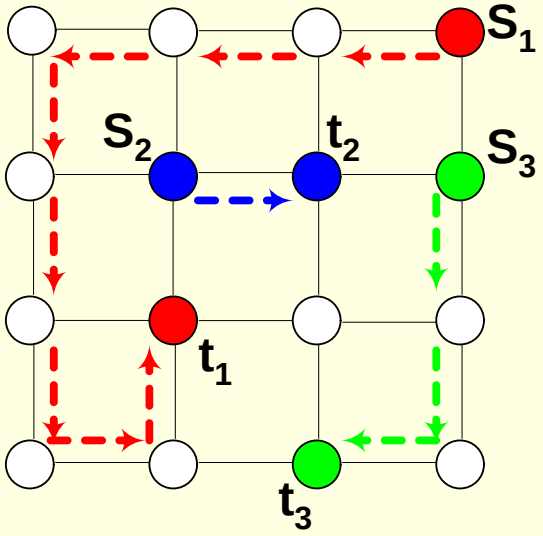


The total number of paths that use this edge

Congestion C takes the maximum over all edges.

Dilation D

- Dilation D: the maximum length of any path

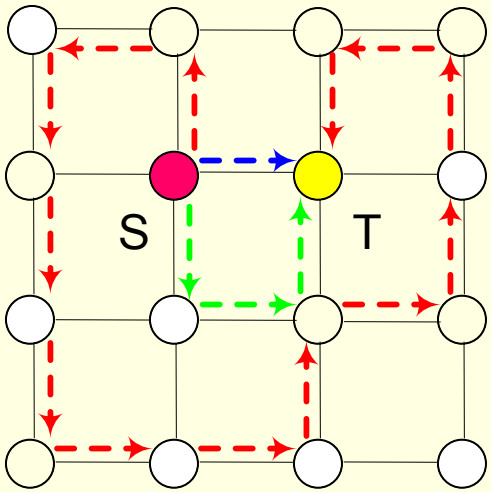


The length of the red path is 8 hops

Dilation D takes the maximum over all paths.

Stretch

- *dist(s,t):* the length of the shortest path from *s* to *t.*
- $stretch(p_i) = |p_i|/dist(s_i, t_i)$.
- $stretch(P) = max_i\ stretch(p_i)$.



dist(s,t) =1

stretch(p_i) = 1

stretch(p_i) = 3

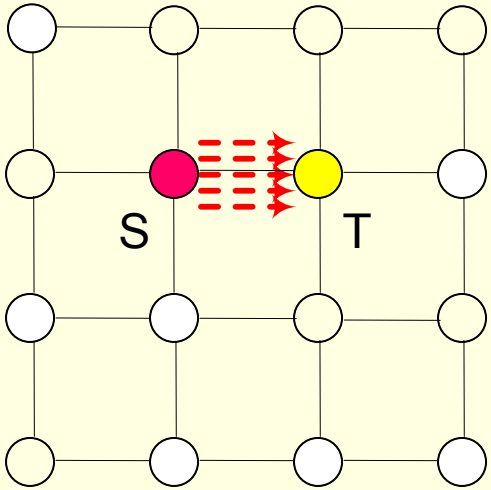
stretch(p_i) = 13

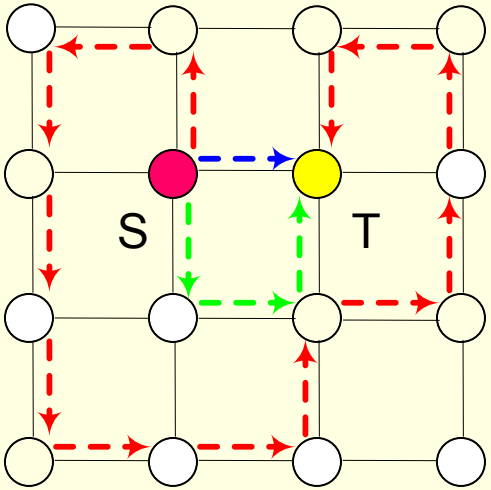
*stretch(**P**) = 13*

Motivations

- A trivial lower bound on transfer time: $\Omega(C+D)$
- So far, the oblivious algorithms focused on minimizing the congestion while ignoring the dilation
- An open question is whether C and D can be controlled simultaneously.

Motivations





Good Stretch but Bad Congestion!

Good Congestion but Bad Stretch!

We want to find such set of paths that minimize C+D!

Related Work

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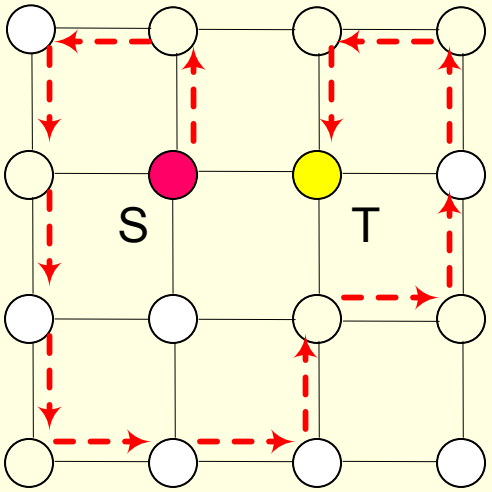
Related Work

- Maggs, et al. in *[FOCS1997]* presented an oblivious algorithm with optimal congestion $O(C^* \log n)$ on Mesh.

  C*: Optimal Congestion for any routing algorithm

- Problems: The stretch factor is unbounded.



Related Work

- Y.Azar et al. in *[STOC 2003]*

  Marcin Bienkowski et al. in *[SPAA 2003]*

  Chris Harrelson et al. in *[SPAA 2003]*

  Harald Racke et al. in *[FOCS 2002]* gave progressively better oblivious algorithms with near optimal congestion on general networks

- Problems: Unbounded stretch

- Our algorithm: Constant stretch without sacrificing on the congestion

Related Work

- A.Srinivasan et al. in *[STOC 1997]* presented near optimal algorithm but the algorithm is *offline and non-oblivious*

- Problems: Require knowledge of the traffic distribution and generally do not scale well

Our Contributions

- For 2-dimensional Mesh, our path selection algorithm has congestion $O(C^* \log n)$, and stretch $\Theta(1)$.
- For d-dimensional Mesh, our path selection algorithm has congestion $O(dC^* \log n)$, and stretch $O(d^2)$.
- Number of random bits required per packet is within $O(d)$ of the optimal.

Our Contributions

- First oblivious algorithm to control C+D simultaneously!

	Congestion	Stretch
Maggs' algorithm	O(dC*logn)	O(n)
Our algorithm	O(dC*logn)	$O(d^2)$

- Randomization: Number of random bits required per packet is within $O(d)$ of the optimal.

Oblivious Routing Algorithm on Mesh

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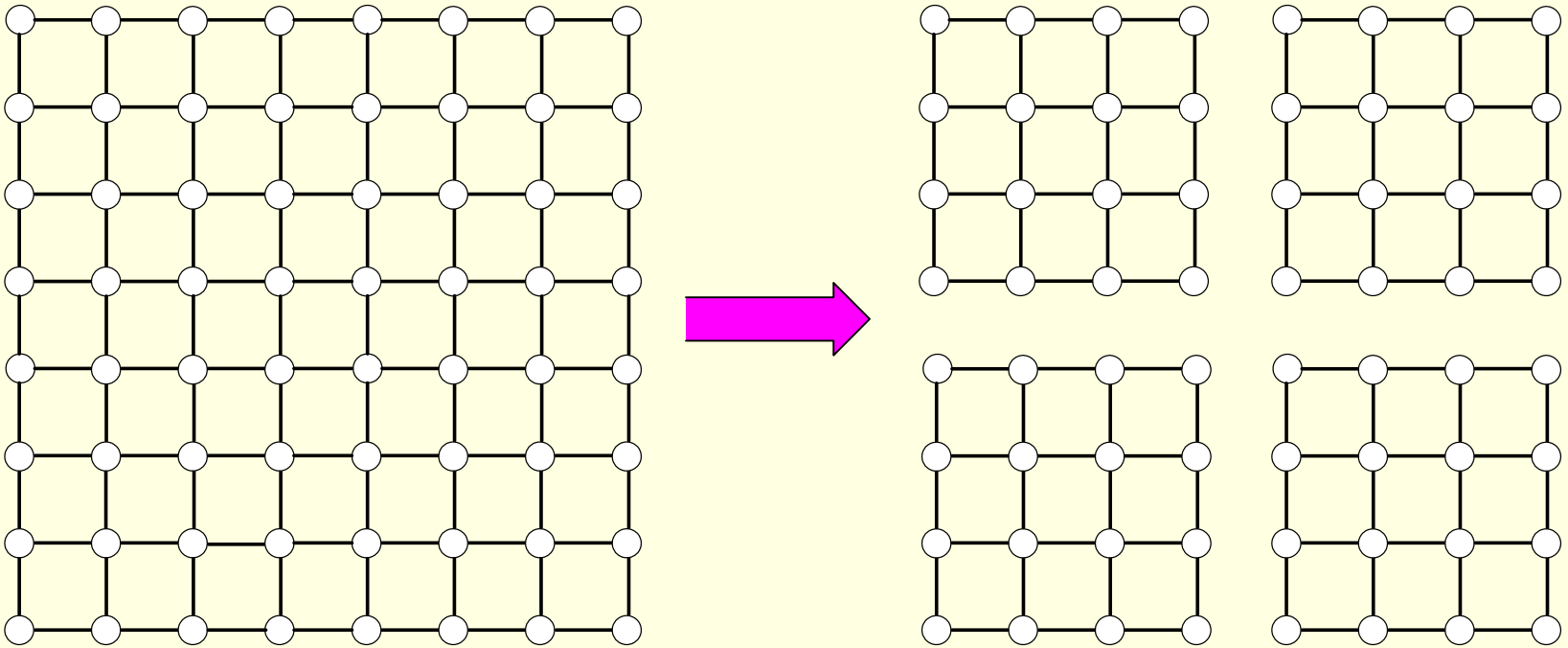
Type I Mesh Decomposition



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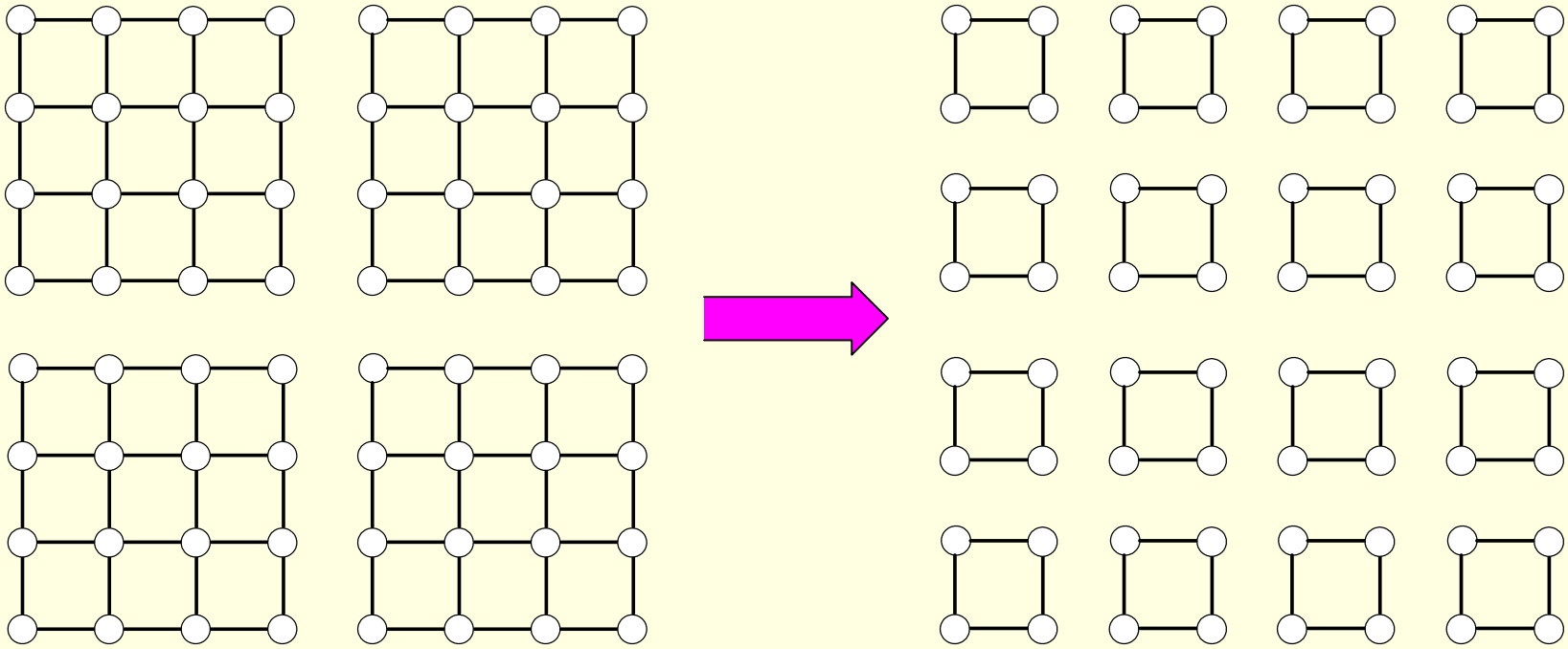

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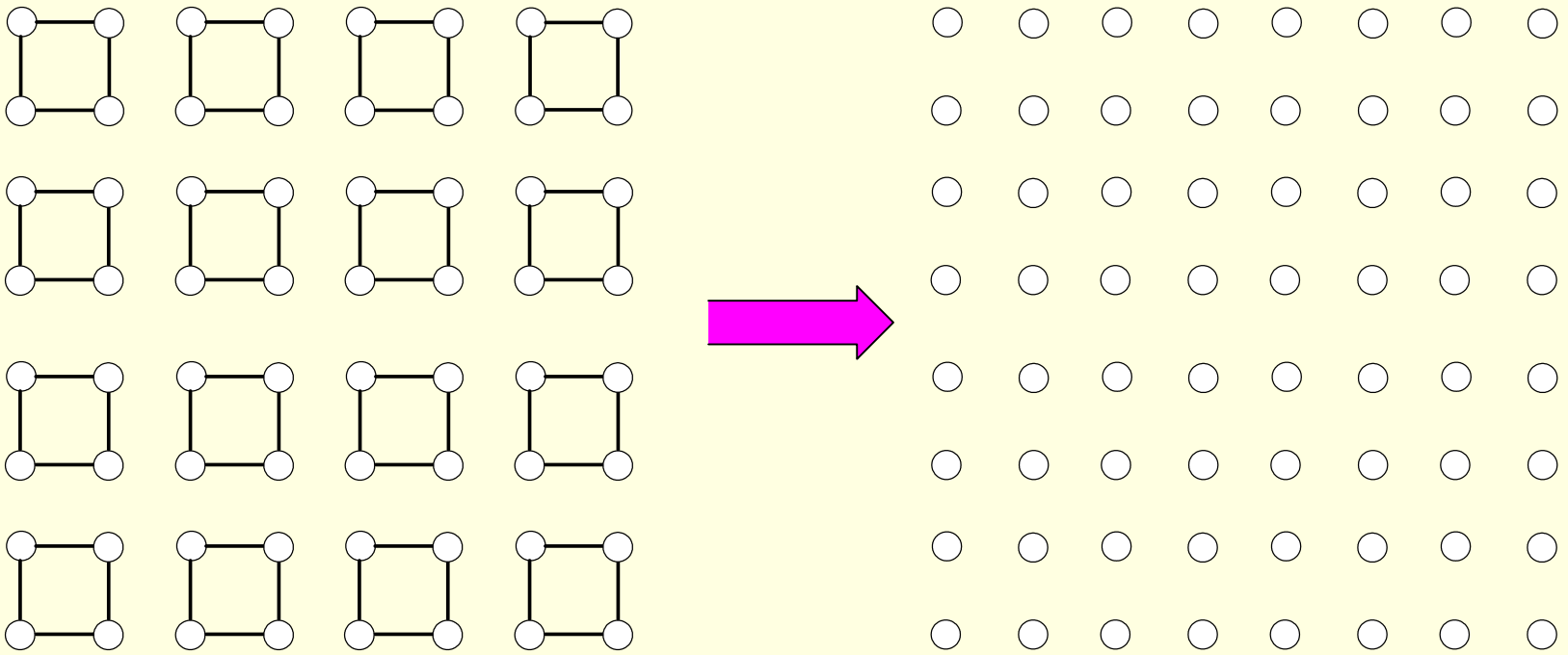
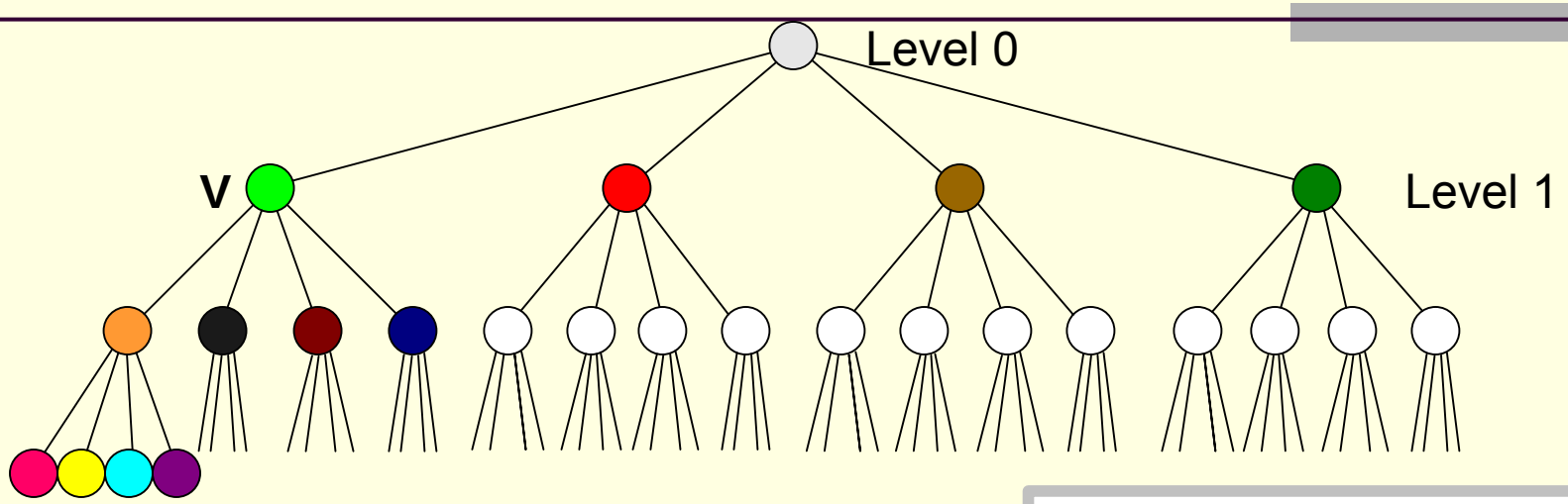
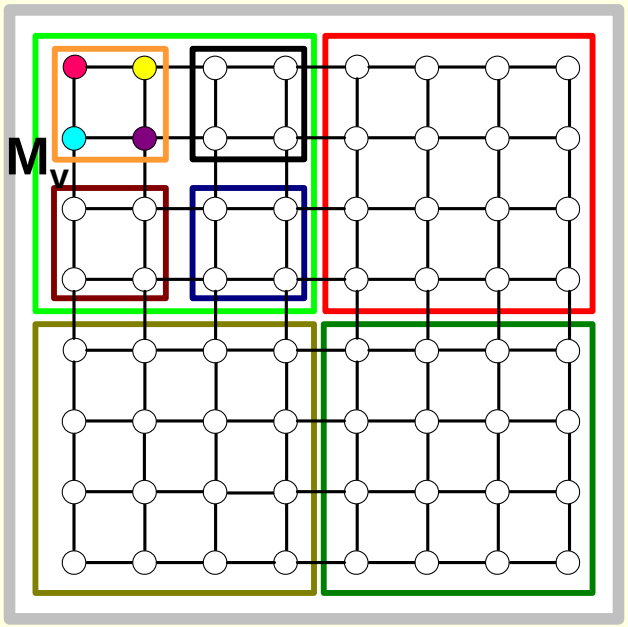
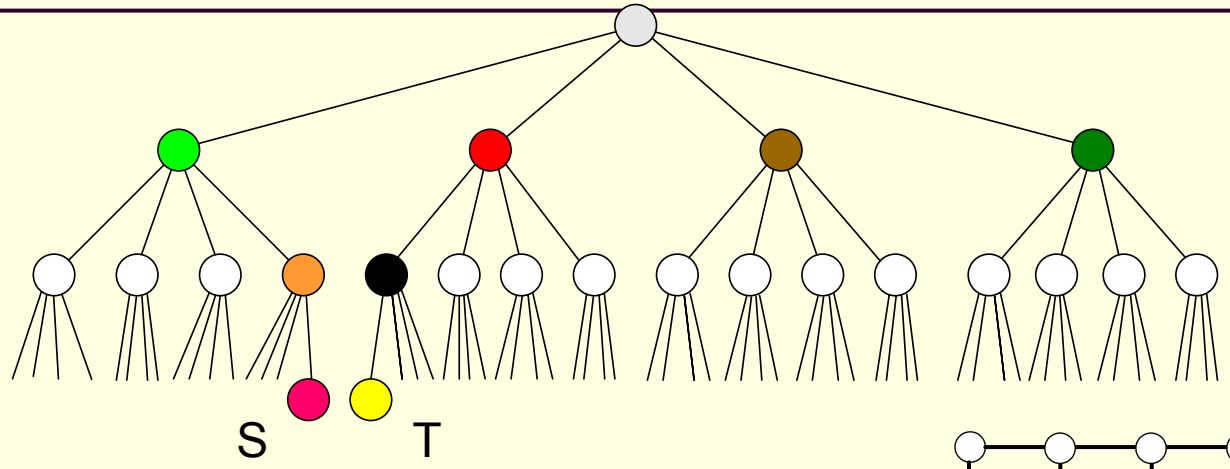


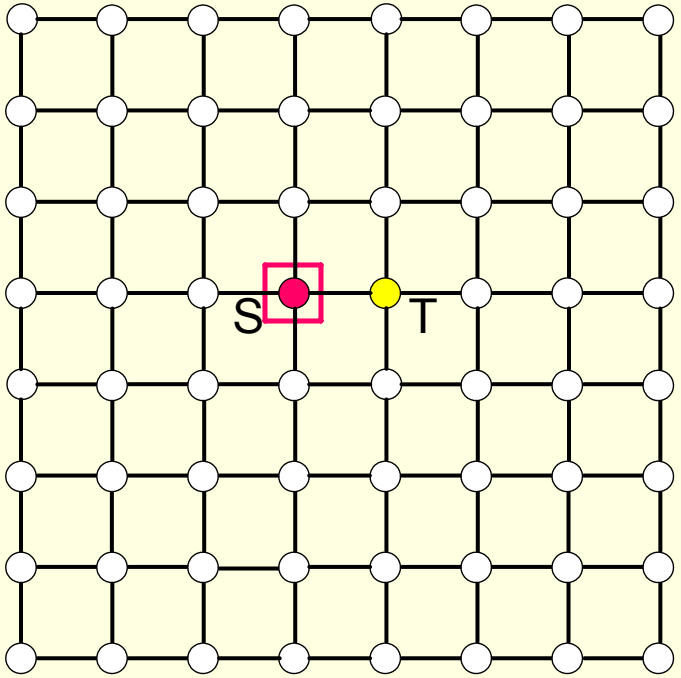
Access Graph for Type I Mesh



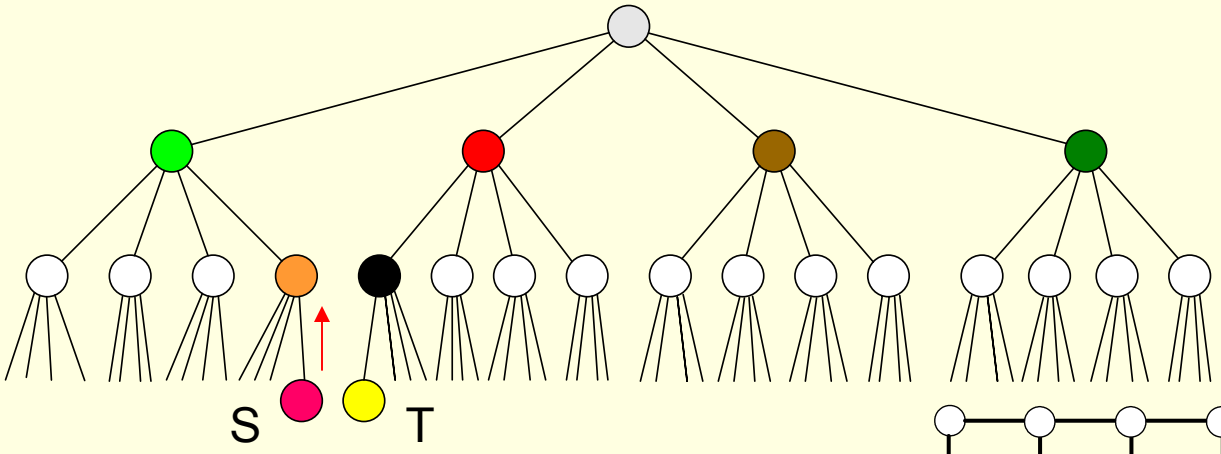


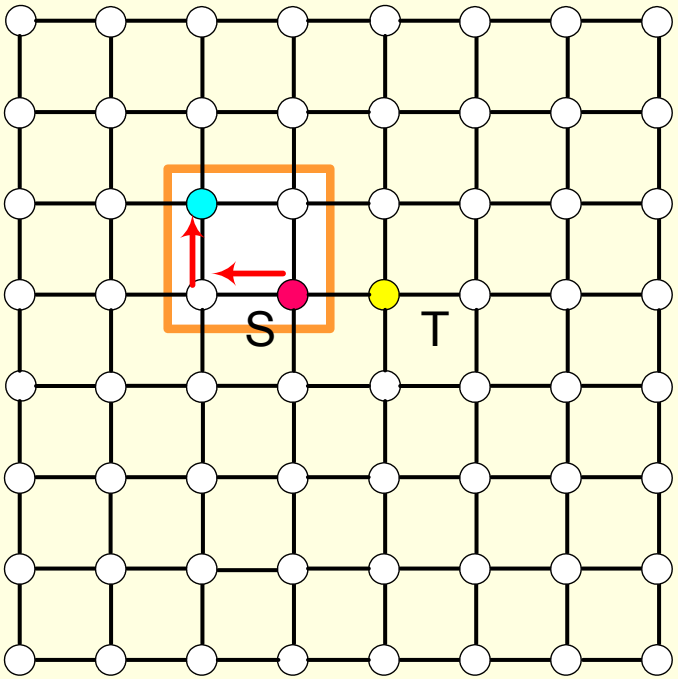
Routing for Type I Mesh Decomposition





Routing for Type I Mesh Partition





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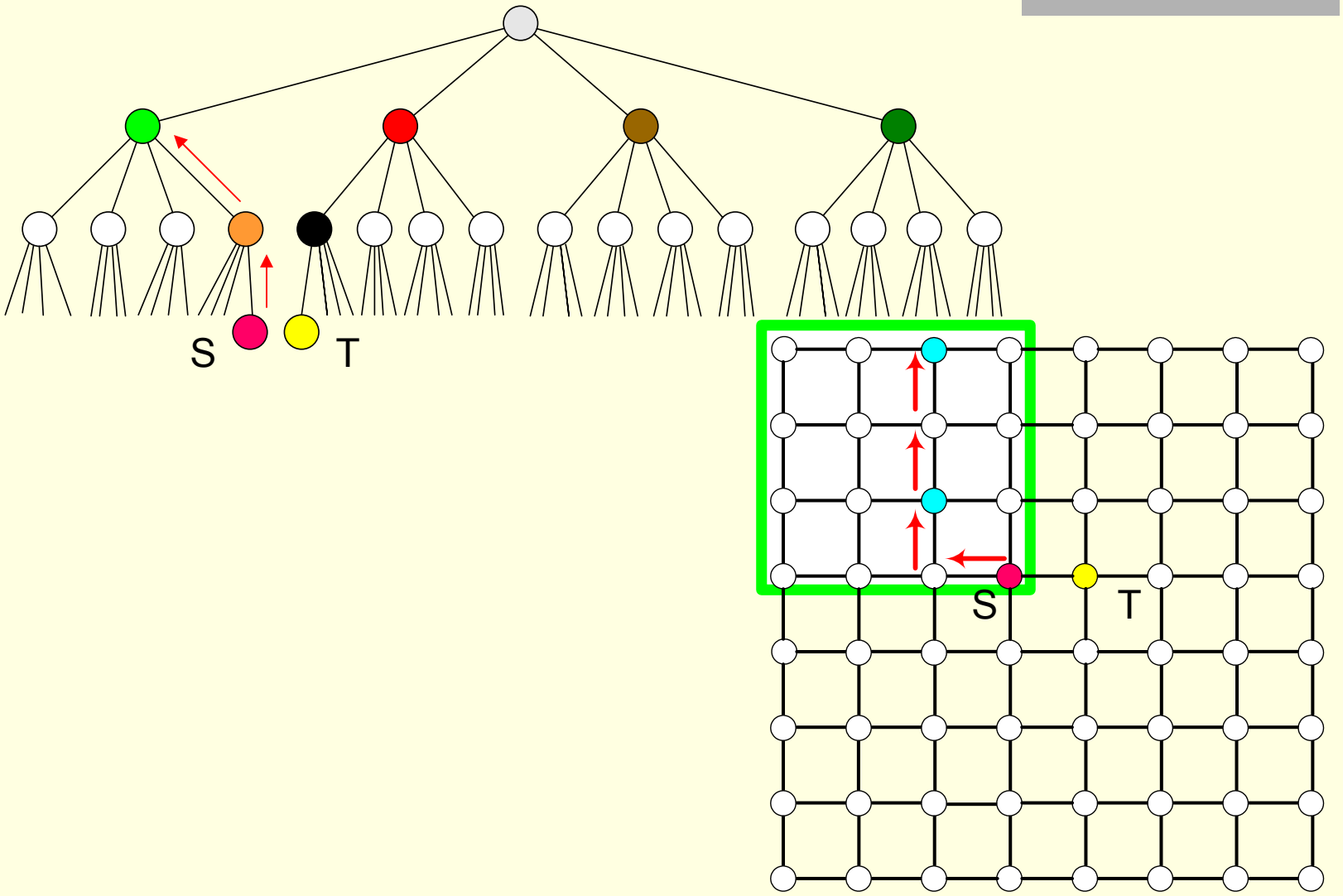

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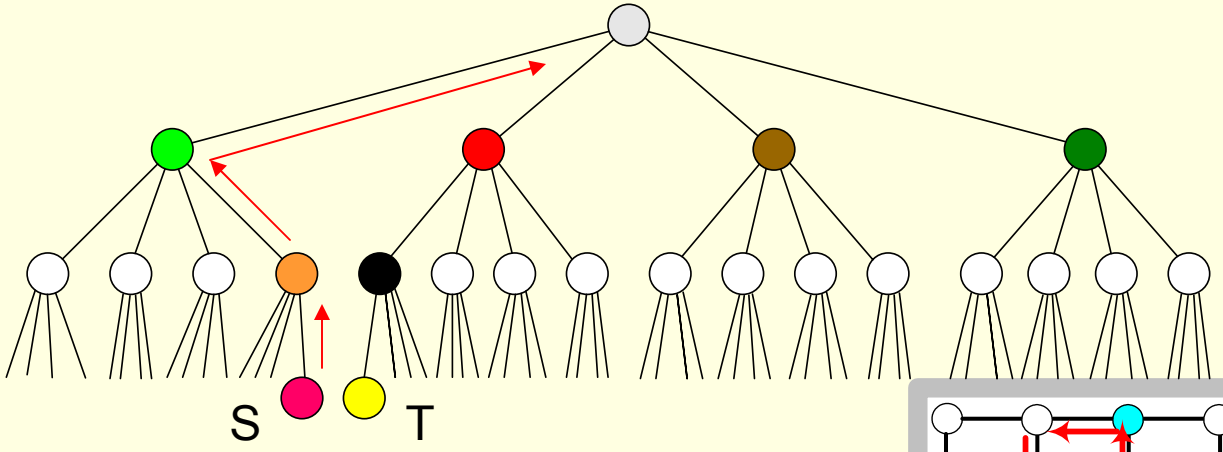

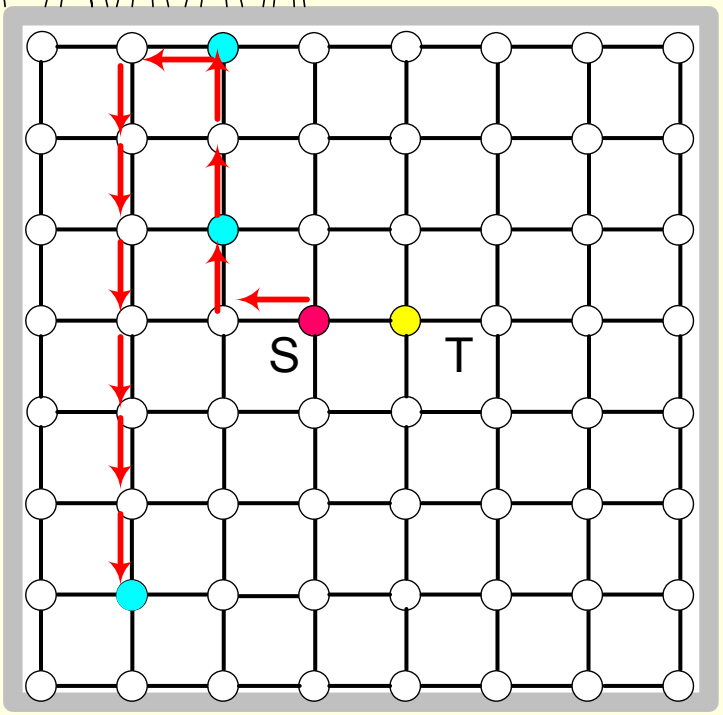

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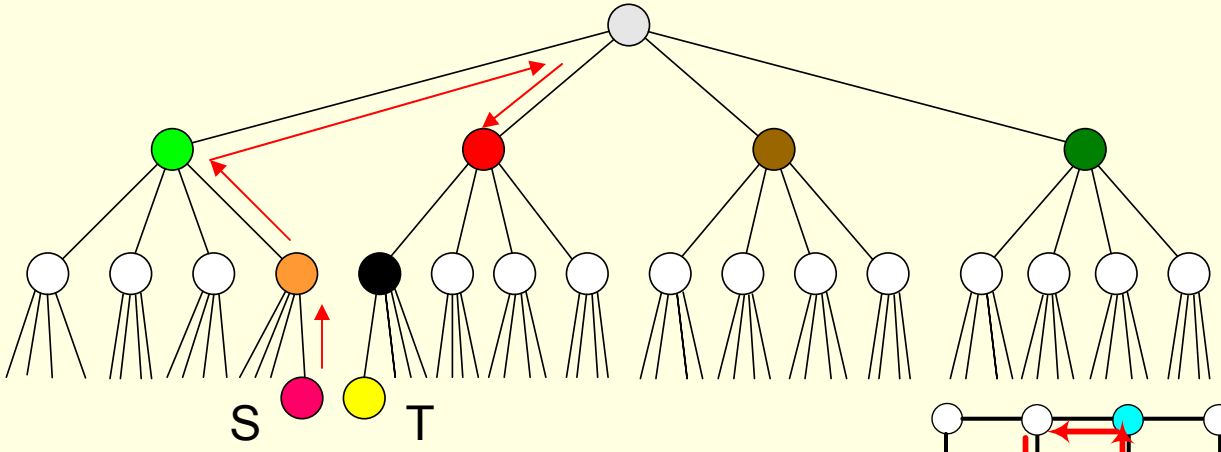

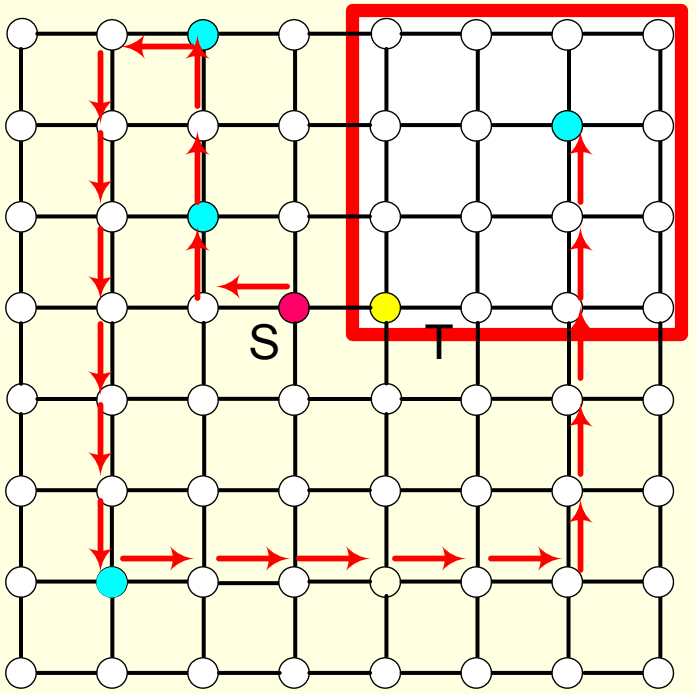

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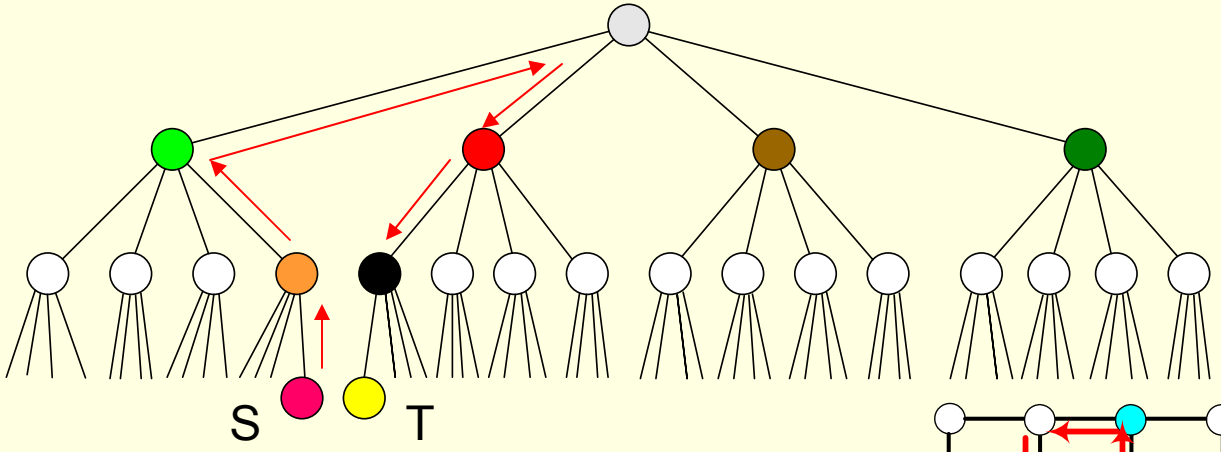

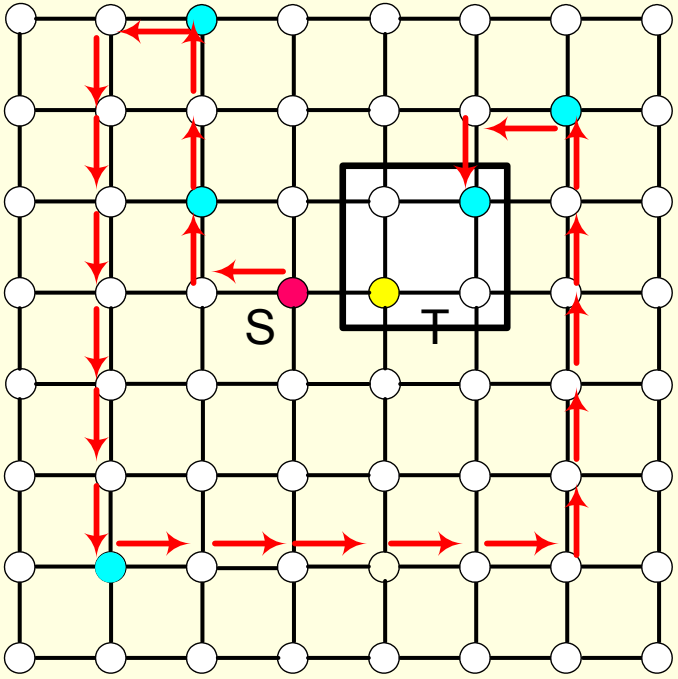

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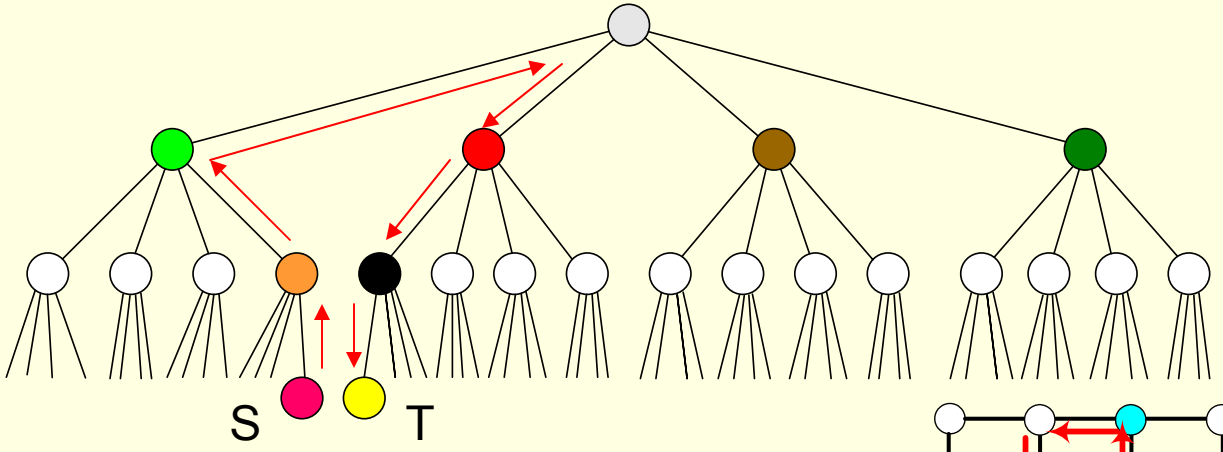
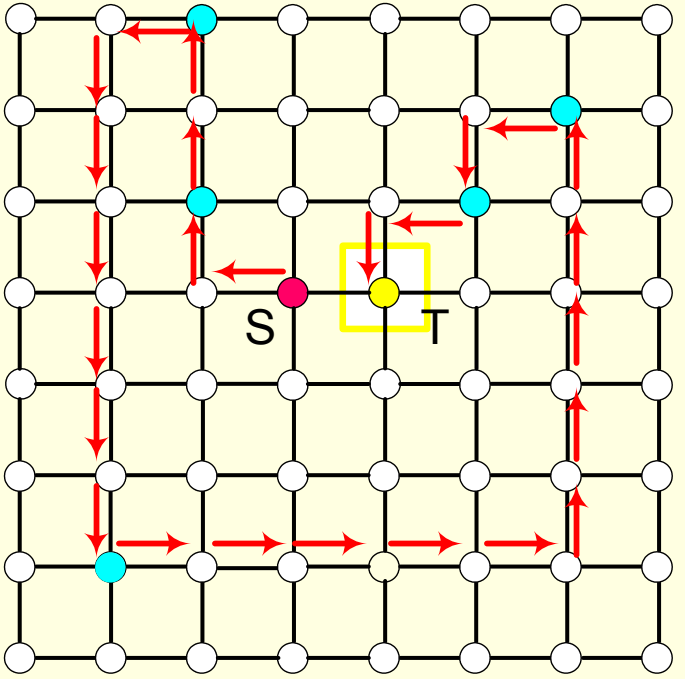
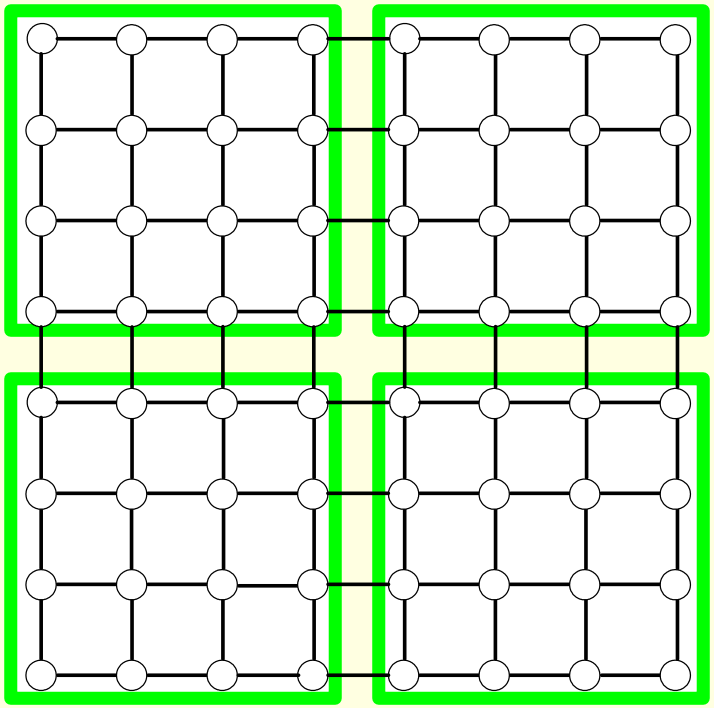


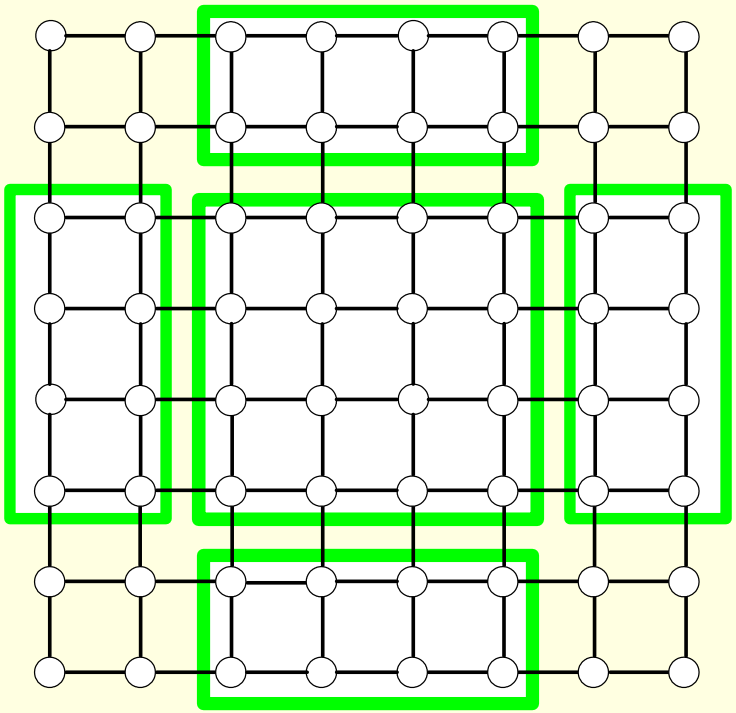


Even for neighbor nodes, it may take a path with length O(n)!
Stretch = O(n)!
n: # of nodes in the network

Type II Mesh Decomposition

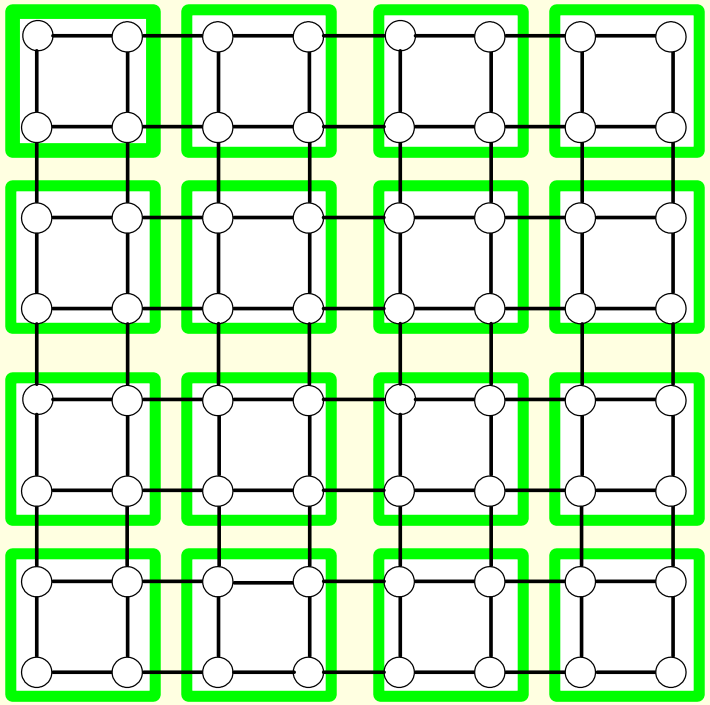


Level 1, Type 1

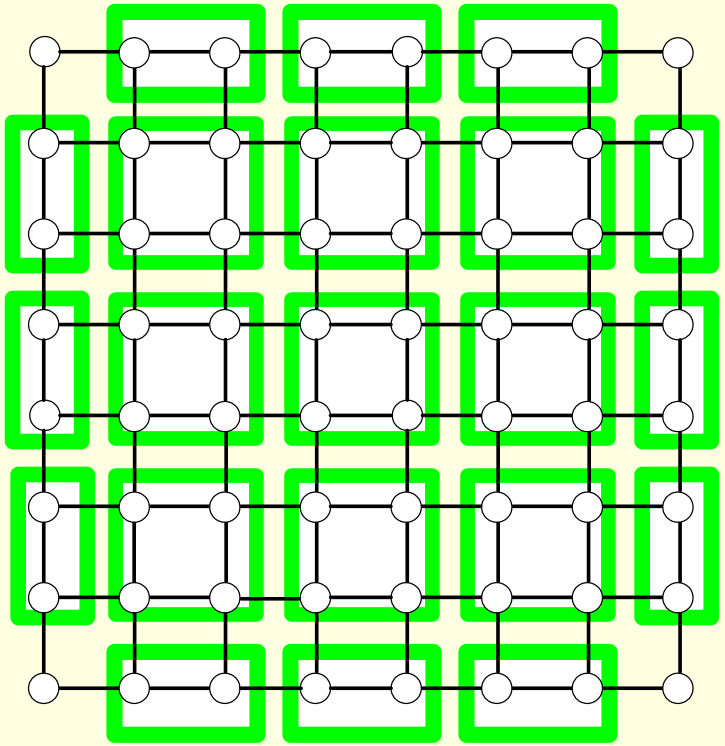


Level 1, Type 2

Type II Mesh Decomposition

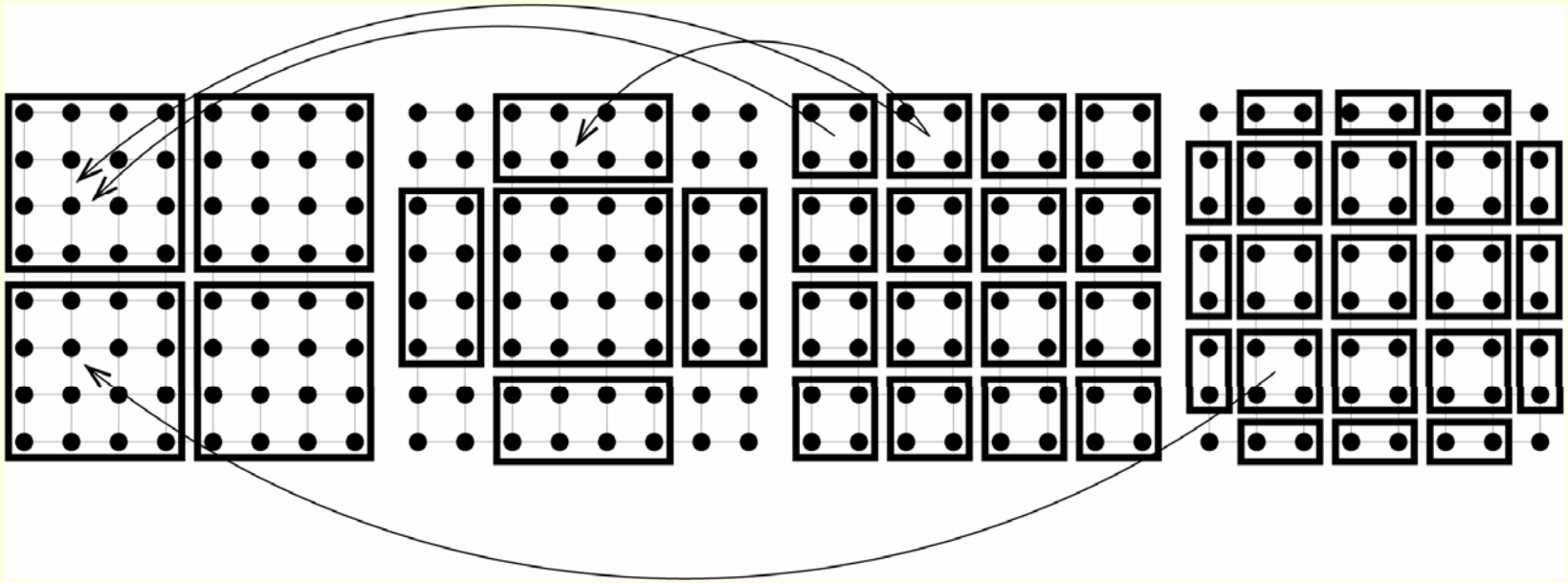


Level 2, Type 1



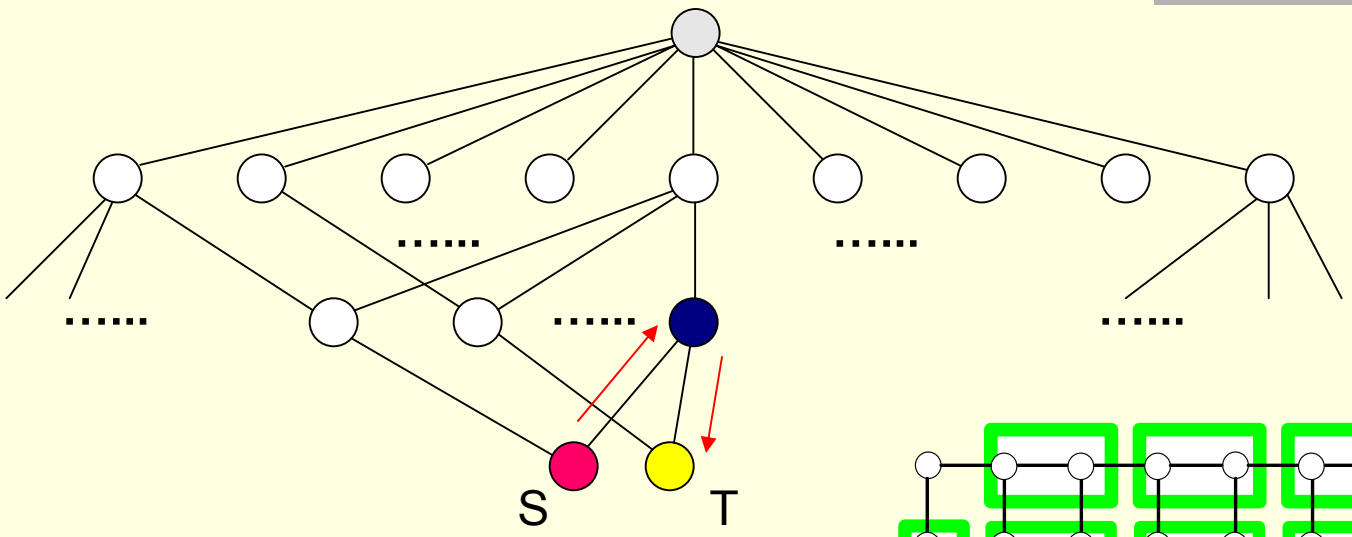
Level 2, Type 2

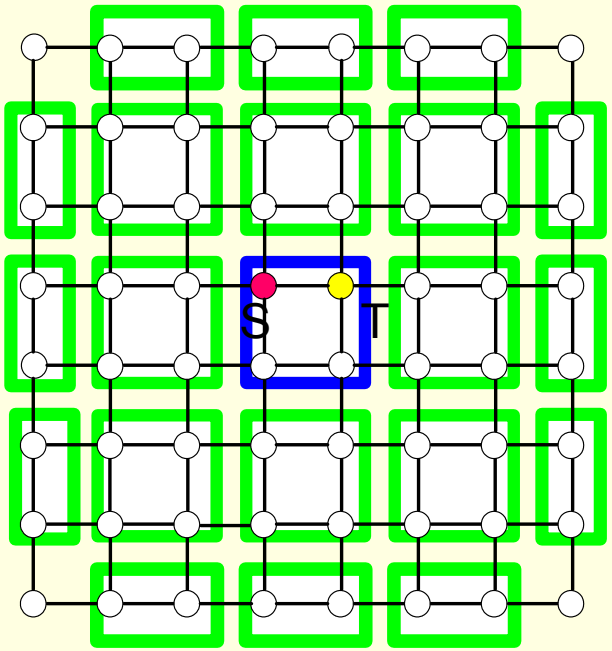
Type II Mesh Decomposition



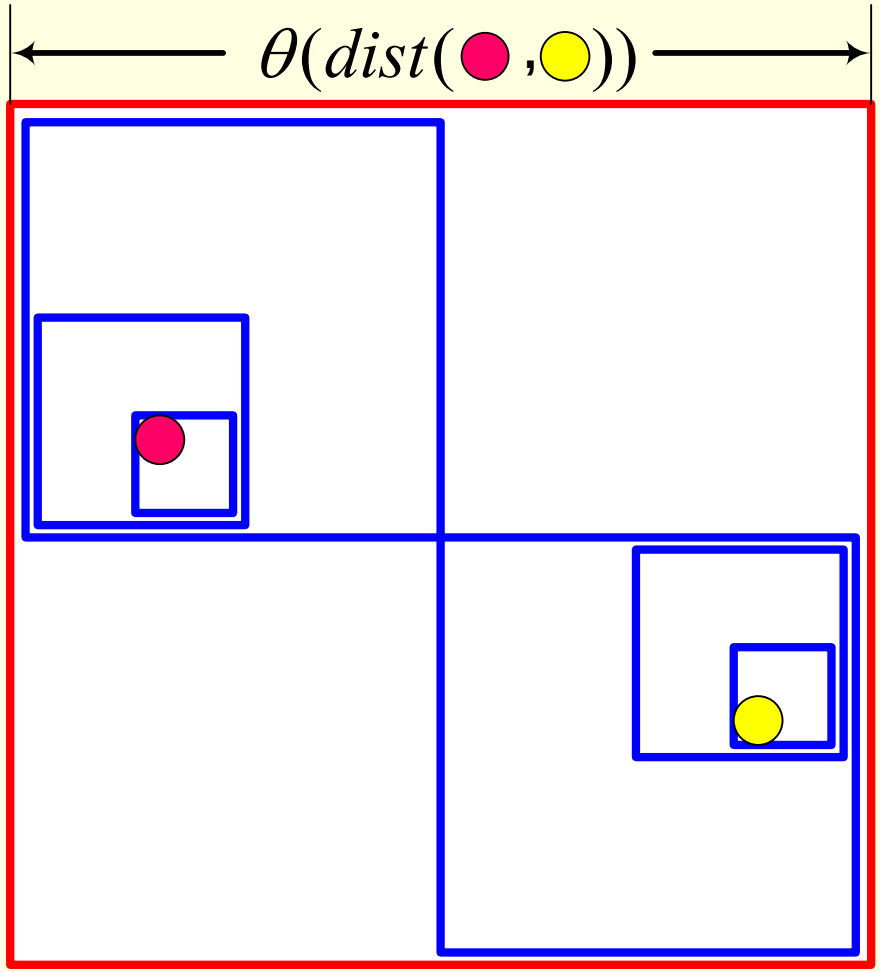
- Level 1 type 1. Level 1, type 2. Level 2, type 1. Level 2, type 2.

An Routing Example





Analysis on Stretch

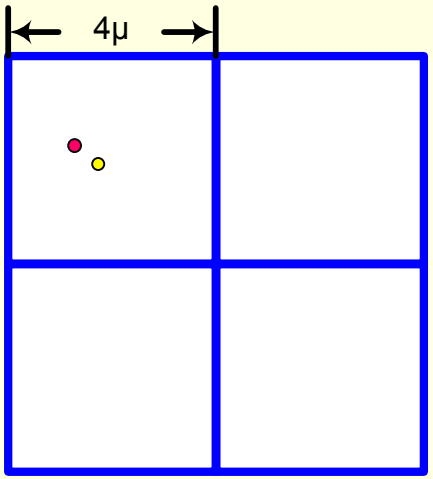


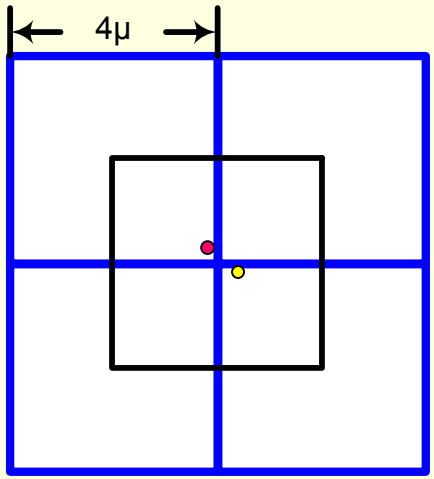
Bridge Submesh: Type I or Type II.

Only 1 Type II submesh is ever needed

Analysis on Stretch

$$\mu = 2^{\lceil \log dist(s,t) \rceil} \geq dist(s,t)$$





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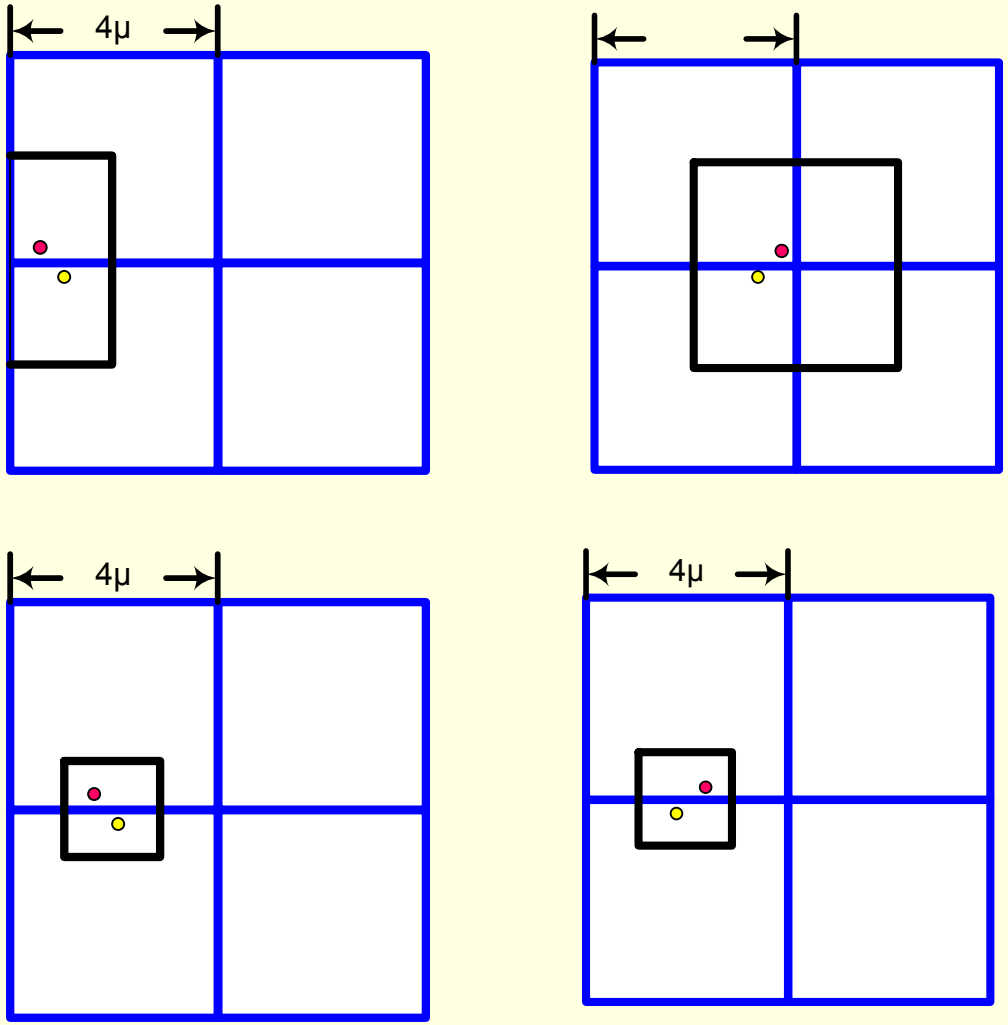


In all cases, ● , ○ are contained in a submesh of side length 4 μ , and each follows type I submeshes only until reach the bridge.

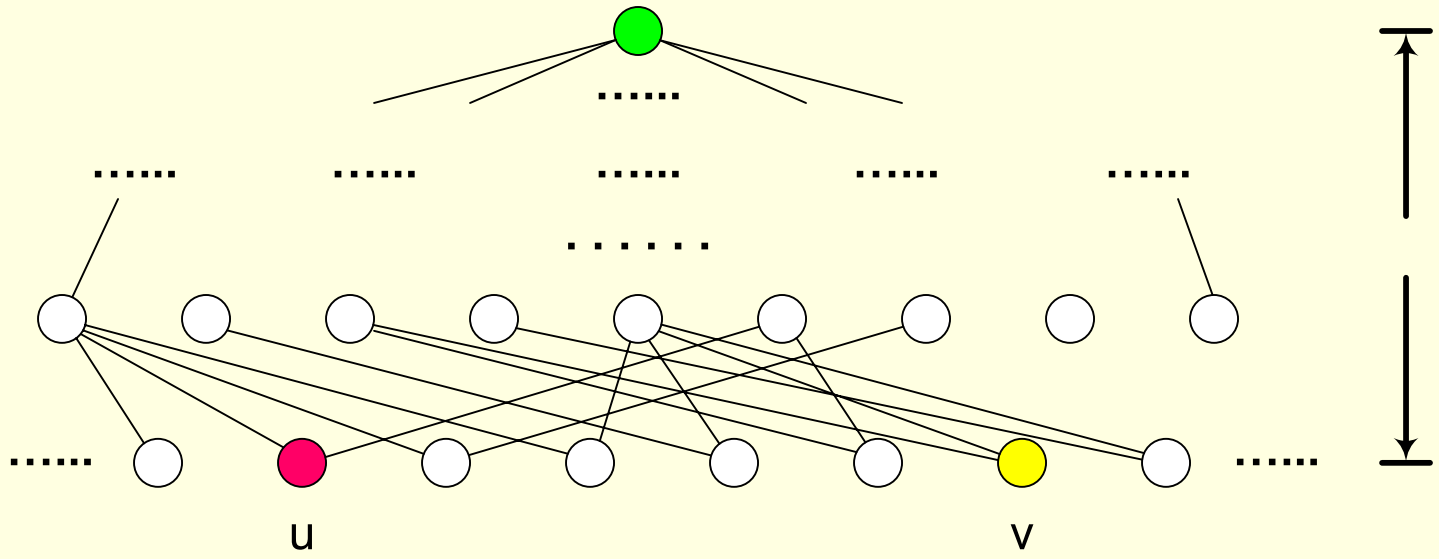
4μ

Theorem For any two distinct nodes *s* and *t* of the mesh, *stretch(p(s, t))* $\leq$ *64*.

Analysis on Stretch

- **Lemma** The deepest common ancestor of two leaves u and v has height at most

  $\lceil \log dist(g(u), g(v)) \rceil + 2$.



Analysis on Edge Congestion

- Subpath uses edge e with probability at most $2/m_l$
- P': set of paths from M_1 to M_2 or vice-versa

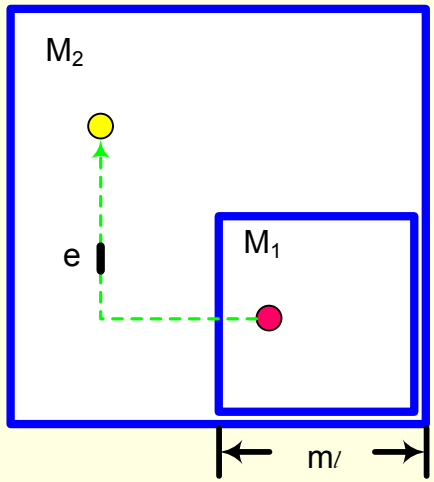
  C'(e): congestion caused by P'
- $E[C'(e)] \le 2|P'|/m_l$

  $B \ge |P'|/out(M_1)$

  $out(M_1) \le 4m_l$

  $C^* \ge B$

  $\Rightarrow E[C'(e)] \le 8C^*.$



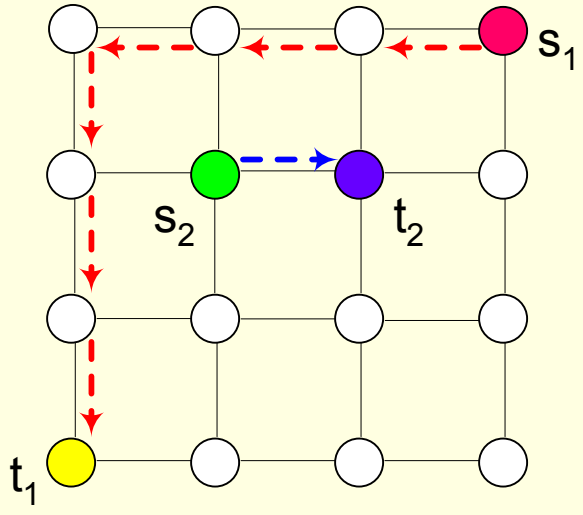
Analysis on Edge Congestion

- **Theorem** The deepest common ancestor of two leaves u and v has height at most

  $\lceil \log dist(g(u), g(v)) \rceil + 2 < \log D^* + 3.$

- $D^* = max_i\ dist(s_i, t_i)$

$dist(s_2,t_2) = 1$
$D^* = 6$



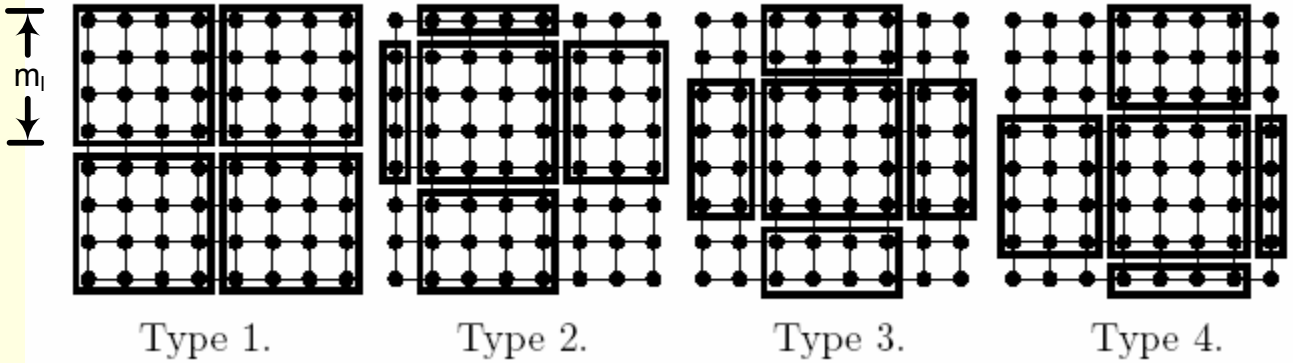
Analysis on Edge Congestion

- **Lemma** $E[C(e)] \leq 16C^*(\log D^* + 3)$.
- **Theorem** $C = O(C^* \log n)$ with high probability.

Analysis on d-dimensional mesh

- The 2-dimensional decomposition can be directly generalized to a d-dimensional mesh but the stretch becomes $O(2^d)$.
- An another decomposition is used to control congestion in $O(C^* \log n)$ and stretch $O(d^2)$.

d-dimensional Mesh Decomposition



3−dimensional Mesh decomposition. Only 2 of the 3 dimensions are depicted

O(d) types on each level

Set $\lambda = max\{1, m_l/2^{\lceil \log(d+1) \rceil}\} = O(m_l/d)$

We shift the *type-1* submeshes by *(j-1)* λ nodes in each dimension to get the *type-j* submeshes, for *j>1*.

Analysis on d-dimensional Mesh

- **Theorem** (Stretch for d dimensions) For any two distinct nodes *s* and *t* of the mesh, $stretch(p(s, t)) = O(d^2)$.

- **Theorem** (Congestion for d Dimensions) $C = O(dC^* \log n)$ with high probability.

Random Choices

- Randomization is unvoidable for oblivious algorithms which obtain near optimal congestion.
- A is a *k-choices* algorithm if for every source-destination pairs (s,t), A chooses the resulting path from *k* possible different paths from s to t.
- A *k-choice* algorithm requires *log k* bits of randomization per packet to select any path

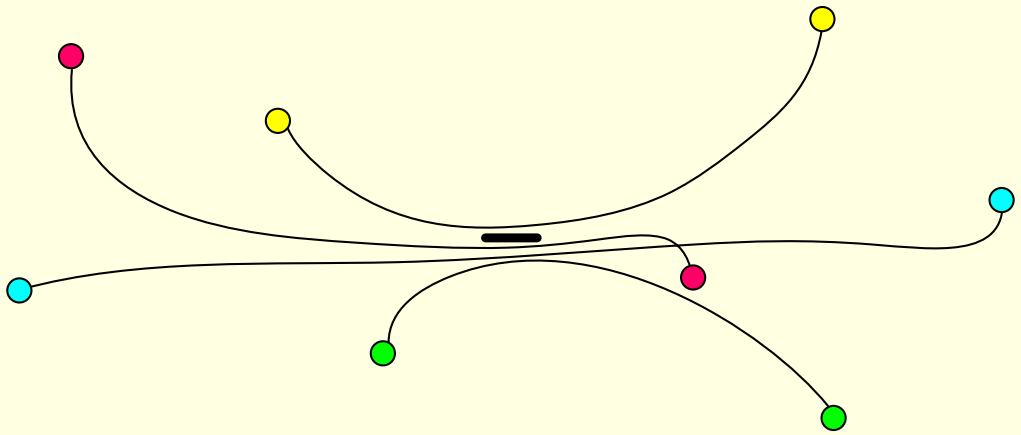
Construction of a Routing Problem

- Consider a *k-choice* algorithm A.
- Each node is the source of one packet and the destination of one packet (permutation).
- The distance from all pairs are $l = 2^s$.
- Each packet uses the path with max probability among the *k* possible paths provided by algorithm A.
- Average # of paths crossing an edge e is at least

$$\frac{m^d l}{dm^d} = \frac{l}{d}$$

Construction of a Routing Problem

- Π_A denote a set consisting of $\frac{l}{d}$ packets crossing e.



- **Lemma** For any *k-choice* algorithm A and corresponding routing problem Π_A with $D^* = l$, the expected congestion Y is at least $\frac{l}{dk}$.

Lower Bound on Randomization

- Lemma Let C_H denote the expected congestion of our algorithm for Π_A , then

$$C_H = O((\frac{l}{d})^{\frac{1}{d}} \cdot \log n).$$

- Let C_A denote the expected congestion of algo A

$C_A \le C_H$ and $\frac{l}{dk} \le C_A \Rightarrow$ algorithm A requires

$$k = \Omega((\frac{l}{d})^{1-\frac{1}{d}} \cdot \frac{1}{\log n})$$ path choices.

Lower Bound on Randomization

- Equivalently, $\Omega((1-\frac{1}{d})\log\frac{l}{d}-\log\log n)$ random bits are required per packet.

  $D^* = l$

- **Lemma** There is a routing problem with $D^* = \Omega(\log n)$ for which any algorithm A with $C_A = O(C_H)$ requires $\Omega((1-\frac{1}{d})\log\frac{D^*}{d})$ random bits per packet.

Upper Bound on Randomization

- **Lemma** For any routing algorithm, algorithm H requires $O(d \log(dD^*))$ random bits.

- **Theorem** The number of random bits used by algorithm *H* is within *O(d)* of optimal.

Conclusions and Future Work

- Introduction
- Related Work
- Results on Mesh Networks
- Conclusions and Future Work

Conclusions

- We have shown that for Mesh, congestion and stretch can be controlled simultaneously!

	Congestion	Stretch
Maggs' algorithm	O(dC*logn)	O(n)
Our algorithm	O(dC*logn)	$O(d^2)$

Future Work

- Develop similar algorithm for other specific networks.
- Develop oblivious algorithms that minimize C+D for general networks.

THANK
YOU!