



# Semantic Analysis

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Read: Scott, Chapter 4.1-4.3

# Announcements

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- HW 1 grades are up
- Quiz 1,2,3 grades up
  - We will release answers in review lecture
- Rainbow grades
  - Please check if your grade shows up correctly
- Exam 1 a week from today --- Oct 11<sup>th</sup>
  - Links to practice problems on Submittity forum
- HW3 is posted
  - Due in 10 days

# Lecture Outline

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- Quiz 4
- Attribute grammars
  - Attributes and rules
  - Synthesized and inherited attributes
  - S-attributed grammars
  - L-attributed grammars
- Attribute evaluation

# Attribute Grammars: Foundation for Static Semantic Analysis

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- **Attribute Grammars:** generalization of Context-Free Grammars
  - Associate meaning with parse trees
  - Attributes
    - Each grammar symbol has one or more values called **attributes** associated with it. Each parse tree node has its own **instances** of those attributes; attribute value carries the “meaning” of the parse tree rooted at node
  - Semantic rules
    - Each grammar production has associated **rule**, which may refer to and compute the values of attributes

# Example: Attribute Grammar to Compute Value of Expression (denote grammar by AG1)

$$S \rightarrow E \quad E \rightarrow E + T \mid T \quad T \rightarrow T * F \mid F \quad F \rightarrow \text{num}$$

Production	Semantic Rule
$S \rightarrow E$	$\text{print}(E.val)$
$E \rightarrow E_1 + T$	$E.val := E_1.val + T.val$
$E \rightarrow T$	$E.val := T.val$
• $T \rightarrow T_1 * F$	$T.val := T_1.val * F.val$
• $T \rightarrow F$	$T.val := F.val$
• $F \rightarrow \text{num}$	• $F.val := \text{num}.val$

$val$ : Attributes

# Another Grammar

$E$  stands for *expr*

$T$  stands for *term*

$TT$  stands for *term\_tail*

- Now, the right-recursive LL(1) grammar:

$$E \rightarrow T TT$$
$$TT \rightarrow - T TT$$
$$TT \rightarrow \epsilon$$
$$T \rightarrow \text{num}$$

- Goal: construct an attribute grammar that computes the value of an expression

- Values must be computed “normally”, i.e.,

5-3-2 must be evaluated as (5-3)-2, not as

5-(3-2)  
4 ~~X~~ No

# Question

- What happens if we wrote a “bottom-up attribute flow” grammar?

$$E \rightarrow TTT$$

$$E.val = T.val - TT.val$$

$$TT \rightarrow - TTT_1$$

$$TT.val = T.val - TT_1.val$$

$$TT \rightarrow \epsilon$$

$$TT.val = 0$$

$$T \rightarrow \text{num}$$

$$T.val = \text{num.val}$$

A hack:

$$t_1 - t_2 - t_3 - \dots - t_n = b_{1+} (t_2 + t_3 + \dots + t_n)$$

$$E \rightarrow TTT$$

$$E.val = T.val - TT.val$$

$$\rightarrow TT \rightarrow - TTT_1$$

$$TT.val = T.val + TT_1.val$$

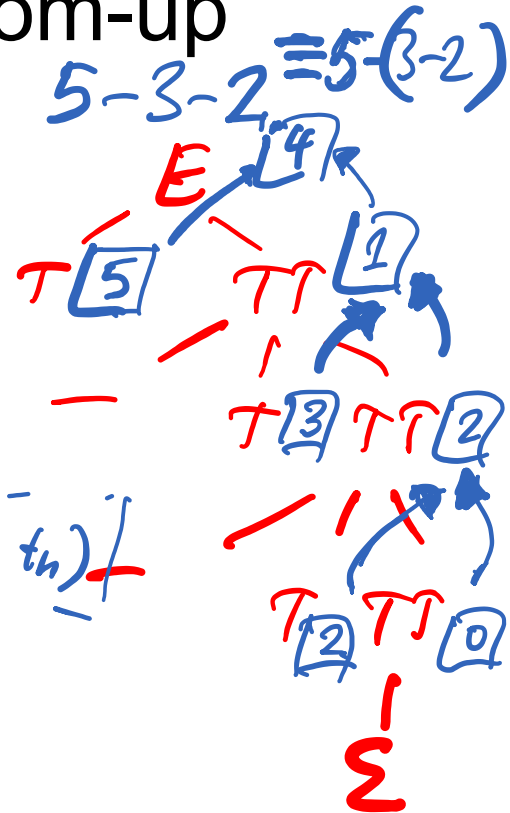
$$TT \rightarrow \epsilon$$

$$TT.val = 0$$

$$T \rightarrow \text{num}$$

$$T.val = \text{num.val}$$

Unfortunately, this won't work if we add  $TT \rightarrow + TTT_1$



# Attribute Grammar to Compute Value of Expressions (denote by AG3)

$$E \rightarrow T TT \quad TT \rightarrow -T TT \mid +T TT \mid \varepsilon \quad T \rightarrow \text{num}$$

Production	Semantic Rules
$E \rightarrow T TT$	(1) $TT.sub := T.val$ (2) $E.val := TT.val$
$TT \rightarrow -T TT_1$	(1) $TT_1.sub := TT.sub - T.val$ (2) $TT.val := TT_1.val$
$TT \rightarrow +T TT_1$	(1) $TT_1.sub := TT.sub + T.val$ (2) $TT.val := TT_1.val$
$TT \rightarrow \varepsilon$	(1) $TT.val := TT.sub$
$T \rightarrow \text{num}$	(1) $T.val := \text{num.val}$ (provided by scanner)

Handwritten example for  $E \rightarrow T TT$  with  $T=5$ ,  $TT_1=3$ ,  $TT_2=2$ :

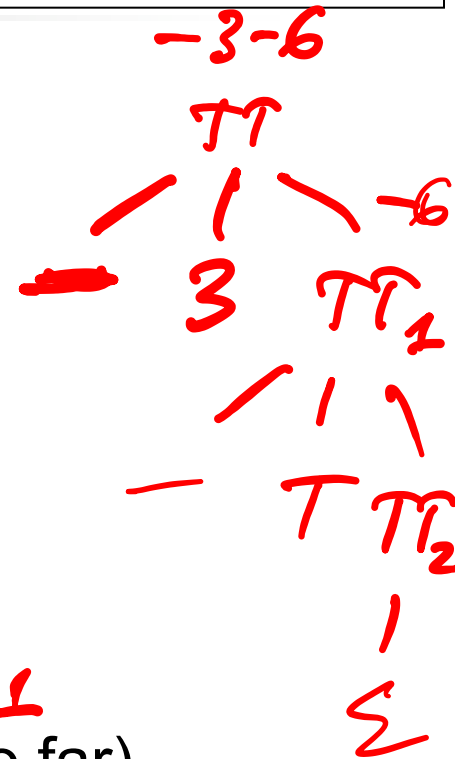
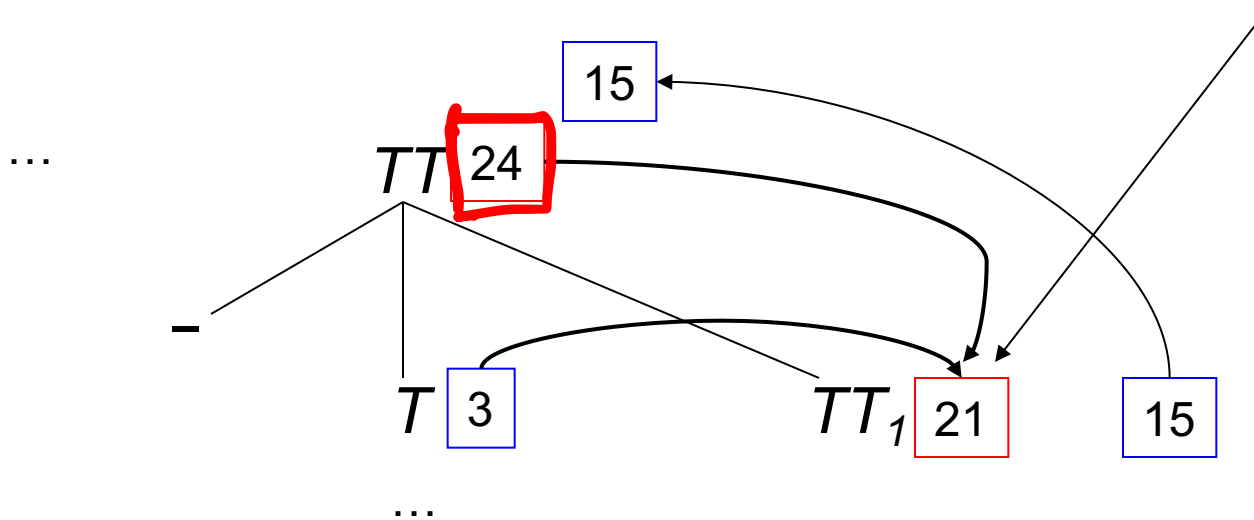
- $TT.sub = 5$  (value of  $TT_1$ )
- $TT_1.sub = 2$  (value of  $TT_2$ )
- Final value:  $(5-3)$

Attributes flow from parent to node, and from “siblings” to node!



# Attribute Flow

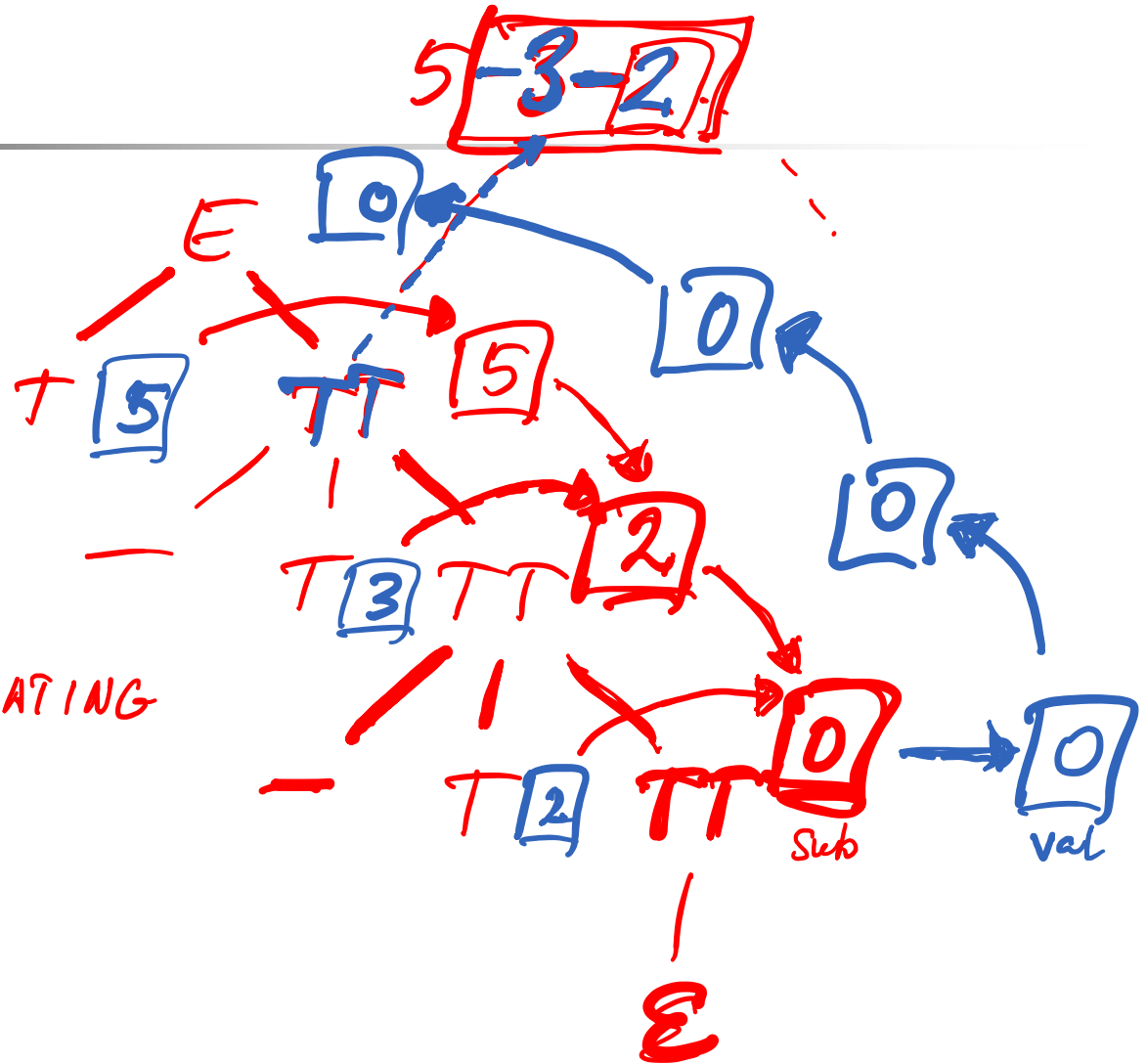
Attribute  $TT_1.sub$ : computed based on parent  $TT$  and sibling  $T$ :  $TT.sub - T.val$



E.g.,  $25 - 1 - 3 - 6$   $\rightarrow TT$   
 $\rightarrow TT_2.sub = 21$   
 $TT$  holds **subtotal** 24 (for  $25 - 1$ , computed so far)  
 $T$  holds **value** 3 (i.e., the value of next term)  
 $TT_1$  gets **subtotal** 21 (for  $25 - 1 - 3$ )

Passed down the tree of  $TT_1$  to next  $TT$  on chain  
 Eventually, we hit  $TT \rightarrow \epsilon$  and **value** gets **subtotal** 15  
 Value 15 is passed back up

# Example



# Attribute Flow

- Attribute *.val* carries the total value
- Attribute *.sub* is the subtotal carried from left



- Rules for nonterminals  $E$ ,  $T$  do not perform computation
  - No need for *.sub* attribute
- *T.val* attribute flows to the right
  - In  $E \rightarrow T TT$ : *val* of  $T$  is passed to sibling  $TT$
  - In  $TT \rightarrow -T TT_1$ : *val* of  $T$  is passed to sibling  $TT_1$

# Attribute Flow

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- Rules for nonterminal  $TT$  do perform computation
  - $TT$  needs to carry subtotal in *.sub*
    - E.g., in  $TT \rightarrow - T TT_1$  the subtotal of  $TT_1$  is computed by subtracting the value of  $T$  from the subtotal of  $TT$
- *TT.val* attribute flows up
  - In  $E \rightarrow T TT$  : *val* of  $TT$  is passed to parent  $E$
  - In  $TT \rightarrow -T TT_1$  : *val* of  $TT_1$  is passed to parent  $TT$

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  - L-attributed grammars
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# Synthesized and Inherited Attributes

## ■ Synthesized attributes



- Attribute value computed from attributes of **descendants** in parse tree or attributes of self

- E.g., attributes val in AG1, val in AG3

- E.g., attributes nptr in AG2 (*Constructs the AST*)

## ■ Inherited attributes

- Attribute value computed from attributes of **parent** in tree, or attributes of **siblings** in tree

- E.g., attributes *sub* in AG3

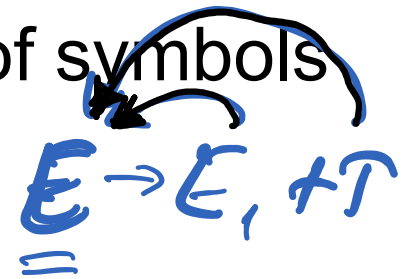


- In order to compute value “normally” we needed to pass *sub* down the tree (*sub* is inherited attribute).

# S-attributed Grammars

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- An attribute grammar for which all attributes are **synthesized** is said to be **S-attributed**
  - “Arguments” of rules are attributes of symbols from the production right-hand-side
    - I.e., attributes of children in parse tree
  - “Result” is placed in attribute of the symbol on the left-hand-side of the production
    - I.e., computes attribute of parent in parse tree
  - I.e., attribute values depend only on descendants in tree. They do not depend on parents or siblings in tree!



# Questions

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- Can you give examples of S-attributed grammars?
  - Answer: AG1 and AG2
- How can we evaluate S-attributed grammars?
  - I.e., can we evaluate the attributes during a bottom-up parse?
  - Answer: Yes



# L-attributed Grammar

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- An attribute grammar is **L-attributed** if each inherited attribute of  $X_j$  on the right-hand-side of  $A \rightarrow X_1 X_2 \dots X_{j-1} X_j \dots X_n$  depends only on
  - (1) the attributes of symbols to the left of  $X_j$ :  $X_1, X_2, \dots, X_{j-1}$
  - (2) the inherited attributes of  $A$

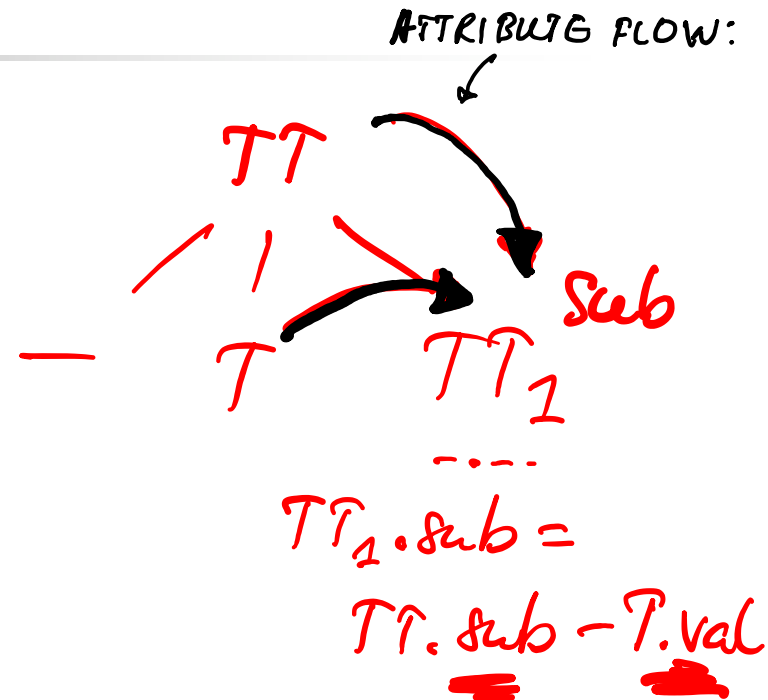
# Questions

- Can you give examples of L-attributed grammars?

- Answer: AG3

- How can we evaluate L-attributed grammars?

- Answer: in a top-down (recursive descent) parse



# Attribute Evaluation

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- S-attributed grammars
  - A very special case of attribute grammars
  - Most important case in practice
  - Can be evaluated on-the-fly during a bottom-up (LR) parse
- L-attributed grammars
  - A proper superset of S-attributed grammars
    - Each S-attributed grammar is also L-attributed because restriction applies only to inherited attributes
  - Can be evaluated on-the-fly during a top-down (LL) parse

# Bottom-up Parsing

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- Also called LR parsing
- LR parsers work with LR(k) grammars
  - ♥ ■ L stands for “left-to-right” scan of input
  - ➔ ■ R stands for “rightmost” derivation
  - ■ k stands for “need k tokens of lookahead”
- We are interested in LR(0) and LR(1) and variants in between *LALR(1)*
- LR parsing is better than LL parsing!
  - Accepts larger class of languages
  - Just as efficient!

# Main Idea

- Stack ← Input
  - Stack: holds the part of the input seen so far
    - A string of both terminals and nonterminals
  - Input: holds the remaining part of the input
    - A string of terminals
  - Parser performs two actions
    - **Reduce**: parser pops a “suitable” production right-hand-side off top of stack, and pushes production’s left-hand-side on the stack
    - **Shift**: parser pushes next terminal from the input on top of the stack
- Handwritten notes:* **math** (under 'Parser'), **OTHER TWO ACTIONS: ERROR, ACCEPT** (at the bottom)
- 
- ```
graph TD; E1[E] --- E2[E]; E1 --- P1[+]; E1 --- T1[T]; E2 == E; P1 == +; T1 == T; S[S]
```

# Example

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- Recall the grammar

- $expr \rightarrow expr + term \mid term$
- $term \rightarrow term * num \mid num$

- This is not LL(1) because it is left recursive
- LR parsers can handle left recursion!

- Consider string

**num + num \* num**

num + num\*num

Stack

Input

Action

EMPTY  
STACK

num+num\*num

shift num

expr  
|  
term  
|  
num

num

+num\*num

reduce by *term* → num

term

+num\*num

reduce by *expr* → term

expr

+num\*num

shift +

expr+

num\*num

shift num

expr+num

\*num

reduce by *term* → num

$expr \rightarrow expr + term \mid term$   
 $term \rightarrow term * num \mid num$

num + num\*num

~~expr \* term~~

Stack                      Input    Action

*expr*+term                \*num    shift \*   

*expr*+term\*                num    shift num   

*expr*+term\*num                reduce by *term* → *term* \* num   

*expr*+term                      reduce by *expr* → *expr*+term   

*expr*                              EMPTY  
INPUT                            ACCEPT, SUCCESS

*expr* → *expr* + *term* | *term*  
*term* → *term* \* num | num



# num + num\*num

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Sequence of reductions performed by parser

↑  
**num+num\*num**  
*term*+**num\*num**  
*expr*+**num\*num**  
*expr*+*term*\***num**  
*expr*+*term*  
*expr*

- A rightmost derivation in reverse
- The stack (e.g., *expr*) concatenated with remaining input (e.g., **+num\*num**) gives a sentential form (*expr*+**num\*num**) in the rightmost derivation

$expr \rightarrow expr + term \mid term$   
 $term \rightarrow term * num \mid num$

# Evaluation $5 + 3 * 2$

| Stack                                                                                                       | Input              | Action                                                               |
|-------------------------------------------------------------------------------------------------------------|--------------------|----------------------------------------------------------------------|
|                                                                                                             | <b>num+num*num</b> | shift <b>num</b>                                                     |
| <u>num</u>                                                                                                  | <b>+num*num</b>    | • reduce by <i>term</i> → <b>num</b><br><i>term.val := num.val</i>   |
| <u>term</u>                | <b>+num*num</b>    | • reduce by <i>expr</i> → <i>term</i><br><i>expr.val := term.val</i> |
| <u>expr</u>                | <b>+num*num</b>    | shift <b>+</b>                                                       |
| <u>expr+</u>              | <b>num*num</b>     | shift <b>num</b>                                                     |
| <u>expr+</u>  <u>num</u> | <b>*num</b>        | reduce by <i>term</i> → <b>num</b>                                   |

# Evaluation $5 + 3 * 2$

Stack

Input Action

  
 $expr + term$

$* num$  shift  $*$

$expr + term *$

$num$  shift  $num$

$expr + term * num$

reduce by  $term \rightarrow term_1 * num$

  
 $expr + term$

$term.val := term_1.val * num.val$

  
 $expr$

reduce by  $expr \rightarrow expr + term$

ACCEPT, SUCCESS

# Question

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- An attribute grammar is **L-attributed** if each inherited attribute of  $X_j$  on the right-hand-side of  $A \rightarrow X_1 X_2 \dots X_{j-1} X_j \dots X_n$  depends only on
  - (1) the attributes of symbols to the left of  $X_j$ :  $X_1, X_2, \dots, X_{j-1}$
  - (2) the inherited attributes of  $A$
- Why the restriction on siblings and kinds of attributes of parent? Why not allow dependence on siblings to the right of  $X_j$ , e.g.,  $X_{j+1}$ , etc.?

# (Top-down) Recursive Descent

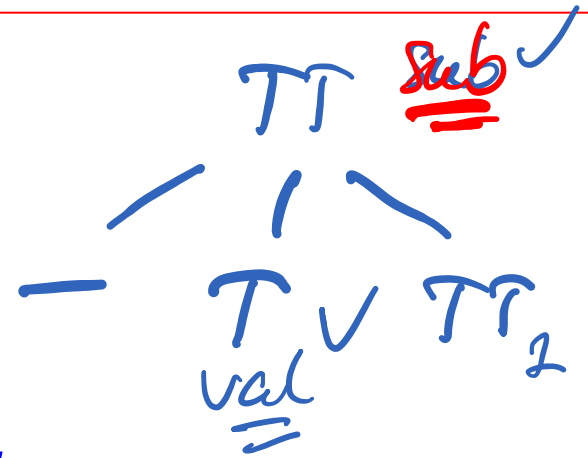
$S \rightarrow E \$\$$   
 $E \rightarrow T TT$        $TT \rightarrow - T TT \mid + T TT \mid \epsilon$        $T \rightarrow \text{num}$

num S()

case lookahead() of  
   num:  $val = E()$ ; match(\$\$); return  $val$   
   otherwise PARSE\_ERROR

num E()

case lookahead() of  
   num:  $sub = T()$ ;  $val = TT(sub)$ ; return  $val$   
   otherwise PARSE\_ERROR



num TT(num sub)

case lookahead() of  
   - : match( '-' );  $Tval = T()$ ;  $val = TT(sub - Tval)$ ; return  $val$   
   + : match( '+' );  $Tval = T()$ ;  $val = TT(sub - Tval)$ ; return  $val$   
   \$\$ :  $val = sub$ ; return  $val$   
   otherwise: PARSE\_ERROR

*sub is fully evaluated.*  
*Tval is fully evaluated.*

# The End

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