



Exam 2 Topics

Topics

- Scheme (Lectures 12 and 13, plus chapters)
 - S-expression syntax
 - Lists and recursion
 - Shallow and deep recursion
 - Equality
 - Higher-order functions
 - **map, foldl, and foldr**
 - Programming with **map, foldl, and foldr**
 - Tail recursion

Topics

- Scheme (Lecture 14, plus chapters)
 - Binding with **let**, **let***, **letrec**
 - Scoping in Scheme
 - Closures and closure bindings

Topics

- Scoping, revisited (Lecture 14, plus chapters)
 - Static scoping
 - Reference environment
 - Functions as third-class values vs.
 - Functions as first-class values
 - Dynamic scoping
 - With shallow binding
 - With deep binding

Topics

(define (fun x) S-expr)
(define fun (lambda (x) S-expr))

- Lambda calculus (Lectures 15, 16, and 17)
 - Syntax and semantics
 - Free and bound variables
 - Substitution
 - Rules of the Lambda calculus: Alpha-conversion and Beta-reduction
 - Normal forms
 - Reduction strategies: Normal order and Applicative order
 - Fix-point combinator and recursion

Quiz 5

Question 1. (2pts) Scheme's scoping discipline is

Select one:

- static scoping
- dynamic scoping

Question 2. (2pts) Scheme's typing discipline is

Select one:

(cdr e)

- static typing
- dynamic typing

Quiz 5

Questions 3 and 4 refer to the following Scheme function:

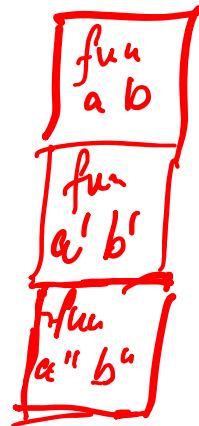
```
(define (fun a b)
  (cond ((= a b) a)
        ((> a b) (fun (- a b) b))
        (else (fun a (- b a)))))
```

Question 3. (2pts) What does fun compute? Note: you may assume that the arguments are positive integers.

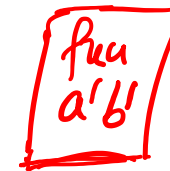
gcd

Question 4. (2pts) fun is tail-recursive.

Standard:



optimised:



A single frame on stack.

Select one:

true

false

Quiz 5

Question 5. (2pts) Function `atomcount`, defined below, attempts to count the number of atoms nested in a list. Recall predicate `atom?` that we wrote in class — it returns true if given an object that is *not a pair* (i.e., an object we cannot take the `car` or the `cdr` of).

```
(define (atomcount lis)
  (cond ((atom? lis) 1)
        ((null? lis) 0)
        (else (+ (atomcount (car lis)) (atomcount (cdr lis))))))
(atomcount '(1 (2 (3)))) yields
```

6

Scoping with First-Class Functions

- Functions as **first-class values**
- Static scoping is more involved
 - Function value may outlive static referencing environment!
 - Therefore, we need “immortal” closure bindings

Scoping with First-Class Functions

- Dynamic scoping is more involved
 - Shallow binding vs. deep binding
- **Dynamic scoping with shallow binding**
 - Reference environment for function/routine is not created until the function is called
 - As a result, all non-local references are resolved using the most-recent-frame-on-stack rule

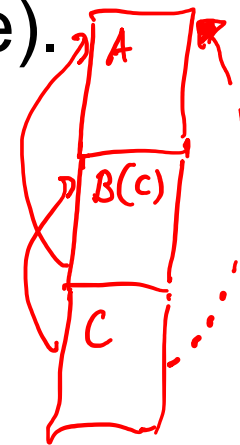
Scoping with First-Class Functions

- **Dynamic scoping with deep binding**
 - When a function/routine is passed as an argument, the code that passes the function/routine has a particular reference environment (the current one!) in mind. It passes this reference environment along with the function value (it passes a closure).

A
B(C)

C = { function value
A's ref. env.

C is a closure. Includes
function value + A's reference
environment.



resolution of
non-local refs
in C uses link
to A.

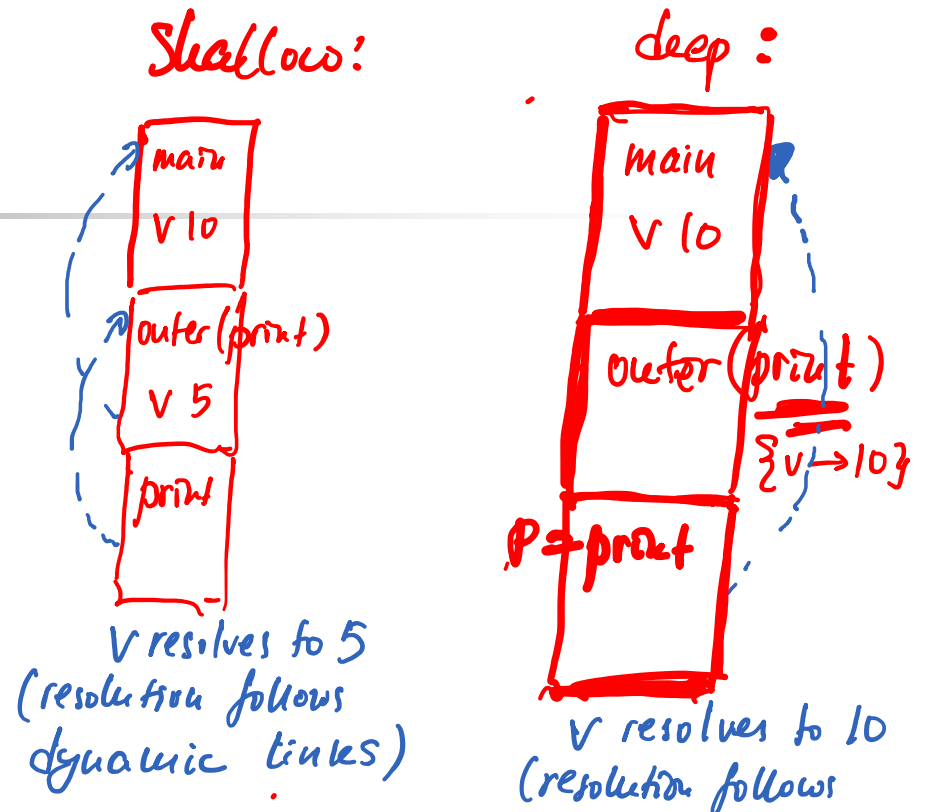
Example

```
v : integer := 10
people : database
print_routine (p : person)
  if p.age > v
    write_person(p)
```

```
other_routine (db : database, P : procedure) link to main
```

```
  v : integer := 5
  foreach record r in db
    P(r)
```

```
other_routine(people, print_routine) /* call in main */
```



Quiz 6

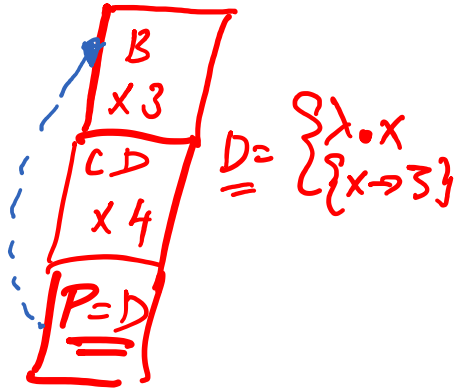
Questions 1-3 below refer to the following Scheme function:

```
(define A
  (lambda ()
    (let* ((x 2)
           • (C (lambda (P) (let ((x 4)) (P) )))
           • (D (lambda () x))
           • (B (lambda () (let ((x 3)) (C D))))))
      (B))))
```

shallow



deep



Question 1. (1pts) What gets printed when we call the function in the interpreter?

2

Question 2. (2pts) What would get printed if Scheme used *dynamic scoping with shallow binding*?

4

Question 3. (2pts) What would get printed if Scheme used *dynamic scoping with deep binding*?

3

Quiz 6

Question 4. (2pts) Willy Wazoo wrote a function f. What does it do?

```
(define (f lis)
  (foldl (lambda (x y) (if (and (number? x) (> x 5)) (cons x y) y)) lis '()))
```

Handwritten annotations:
- "partial result" with an arrow pointing to the lambda function.
- "next element" with an arrow pointing to the lambda function.
- "y" under the first 'y' in the lambda function.
- "y" under the second 'y' in the lambda function.
- "y x x" under the lambda function's body.
- "(append x (cons y '()))" written below the lambda function.

Select one:

- selects all numerical elements in lis greater than 5
- selects all numerical elements in lis less or equal than 5
- neither; Willy's function has a bug

Quiz 6

Question 5. (1pt) Consider the lambda term $(\lambda v.v) ((\lambda v.v) (\lambda x. (\lambda x.x) z))$. There are this many reducible expressions in this term:

3

Question 6. (1pt) Is lambda term $x \overbrace{((\lambda y.y) z)}^{E_1 \quad E_2}$ in Weak Head Normal Form (WHNF)?

YES

$\lambda x. (\lambda y.y) z x$ is in WHNF

$\lambda x. (\lambda y.y) z x$ is not in HNF

Question 7. (1pt) Is term $x ((\lambda y.y) z)$ in Normal Form (NF)?

No

Unification (simplified)

- **Unify**: tries to unify terms τ_1 and τ_2 and binds variables to values if $\tau_1 = \tau_2$ if unification is successful

def **Unify**(τ_1, τ_2) =

This is the **occurs check!**

case (τ_1, τ_2)

(τ_1, \mathbf{V}_2) -> **success** with binding [τ_1/\mathbf{V}_2] if \mathbf{V}_2 does not **occur** in τ_1 ;
fail otherwise

(\mathbf{V}_1, τ_2) -> **success** with binding [τ_2/\mathbf{V}_1] if \mathbf{V}_1 does not **occur** in τ_2 ;
fail otherwise

($\mathbf{c}_1, \mathbf{c}_2$) -> **success** if (eq? \mathbf{c}_1 \mathbf{c}_2); **fail** otherwise

($\mathbf{p}(\tau_{11}, \tau_{12}), \mathbf{p}(\tau_{21}, \tau_{22}))$) -> **Unify**(τ_{11}, τ_{21}); **Unify**(τ_{12}, τ_{22})

otherwise = **fail**

Questions?

Abbrev. $\lambda x. \lambda y. \lambda z. x \ z \ (y \ z)$

$$S = \lambda x y z. x \ z \ (y \ z) \quad I = \lambda x. x$$

$$S \ I \ I \ I = ((S \ I) \ I) \ I \quad \text{Applicative order}$$

$$\underline{(\lambda x. \lambda y. \lambda z. x \ z \ (y \ z)) \ I \ I \ I} \rightarrow_{\beta}$$

$$(\lambda y. \lambda z. \underline{I \ z} \ (y \ z)) \ I \ I =$$

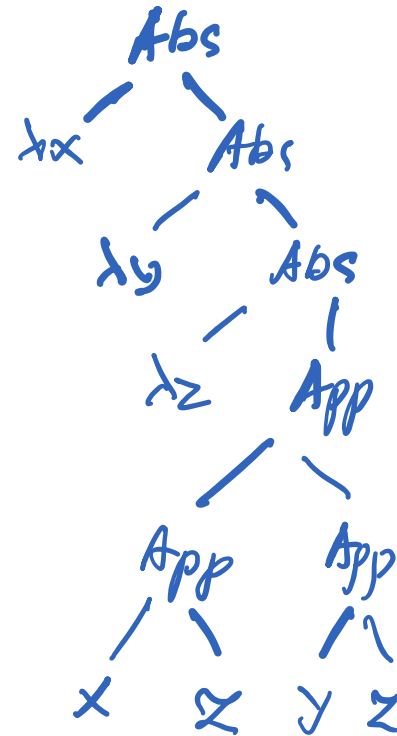
$$(\lambda y. \lambda z. (\lambda x. x) \ z \ (y \ z)) \ I \ I \rightarrow_{\beta}$$

$$\underline{(\lambda y. \lambda z. z \ (y \ z)) \ I \ I} \rightarrow_{\beta}$$

$$(\lambda z. z \ (\underline{I \ z})) \ I =$$

$$(\lambda z. z \ ((\lambda x. x) \ z)) \ I \rightarrow_{\beta}$$

$$\underline{(\lambda z. z \ z) \ I} \rightarrow_{\beta} \ I \ I = \underline{(\lambda x. x) \ I} \rightarrow_{\beta} \ I = \underline{\lambda x. x}$$



The End

Normal order:

$$\underline{(\lambda x. \lambda y. \lambda z. x z (y z))} \underline{I} \underline{I} \underline{I} \rightarrow$$

$$\underline{(\lambda y. \lambda z. I z (y z))} \underline{I} \underline{I} \rightarrow$$

$$\underline{(\lambda z. I z (I z))} \underline{I} \rightarrow$$

$$\underline{II} \text{ (II)} =$$

$$(\lambda x. x) I (II) \rightarrow$$

$$\underline{I (II)} =$$

$$\underline{(\lambda x. x) (II)} \rightarrow$$

$$\underline{II} = (\lambda x. x) I \rightarrow \boxed{I}$$

