Logic Programming and Prolog

Finish reading: Scott, Chapter 12

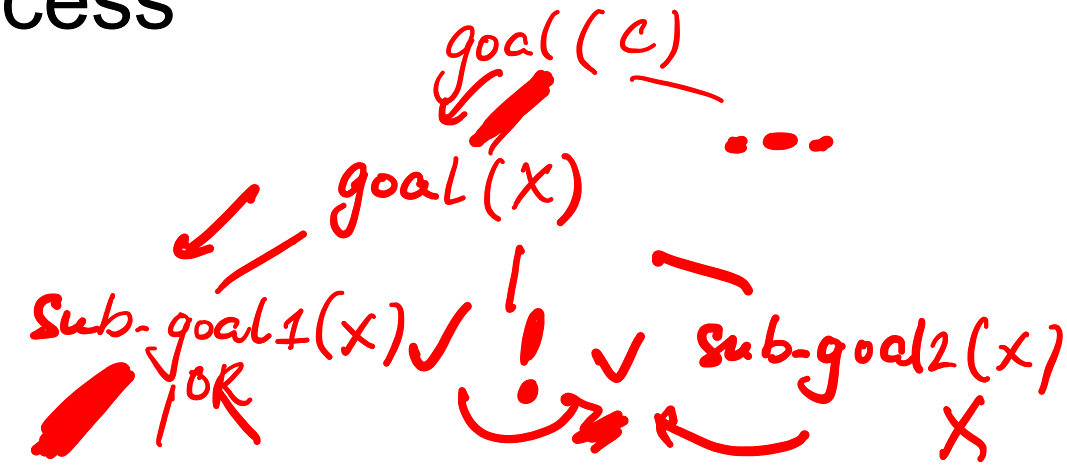
Lecture Outline

- Quiz 3
- Prolog
  - Imperative control flow !
  - Negation by failure not(x)
  - Generate and test paradigm

Imperative Control Flow

- Programmer has explicit control on backtracking process

`cut (!)`



- **`!`** is a subgoal
- As a goal it succeeds, but with a <u>side effect</u>:
  - Commits interpreter to all bindings made since unifying left-hand side of current rule with parent goal

Cut (!) Example

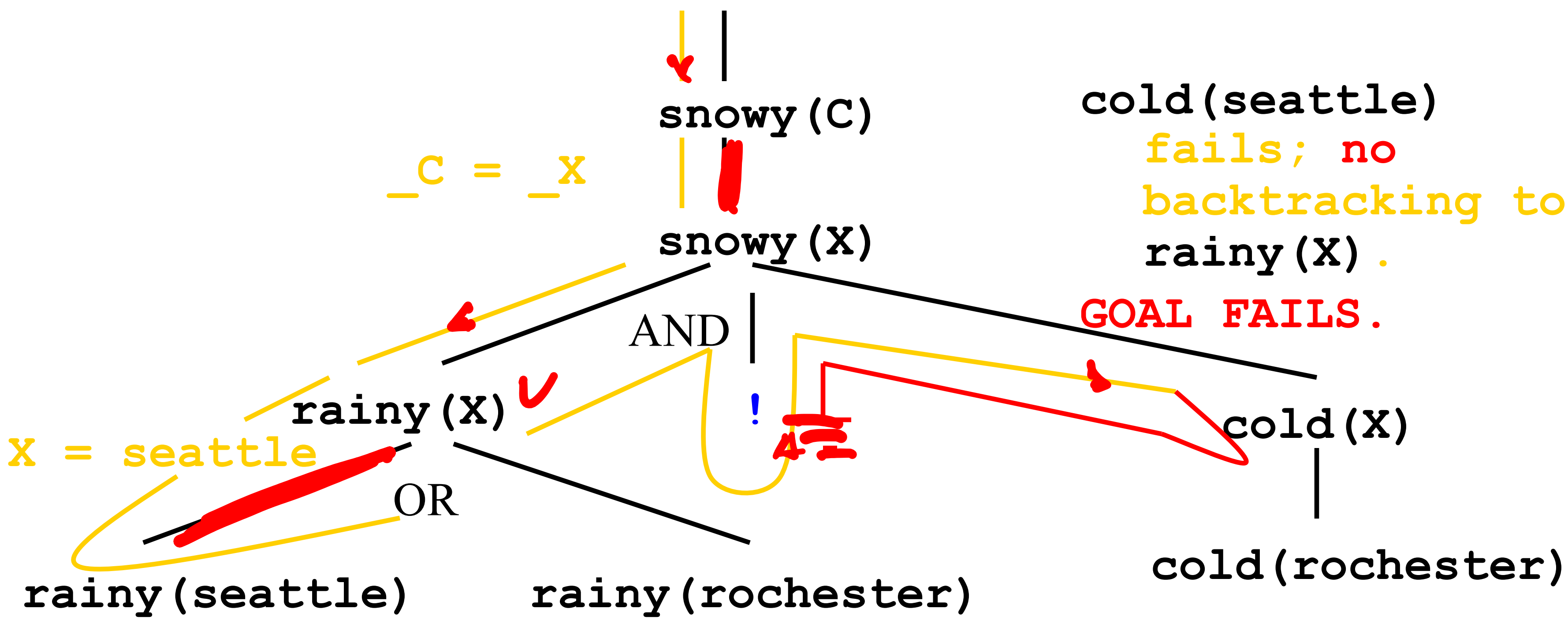
```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), !, cold(X).

?- snowy(C).
```

?- false.

Cut (!) Example

```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), !, cold(X).
```



Cut (!) Example 2

```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), !, cold(X).
snowy(troy).

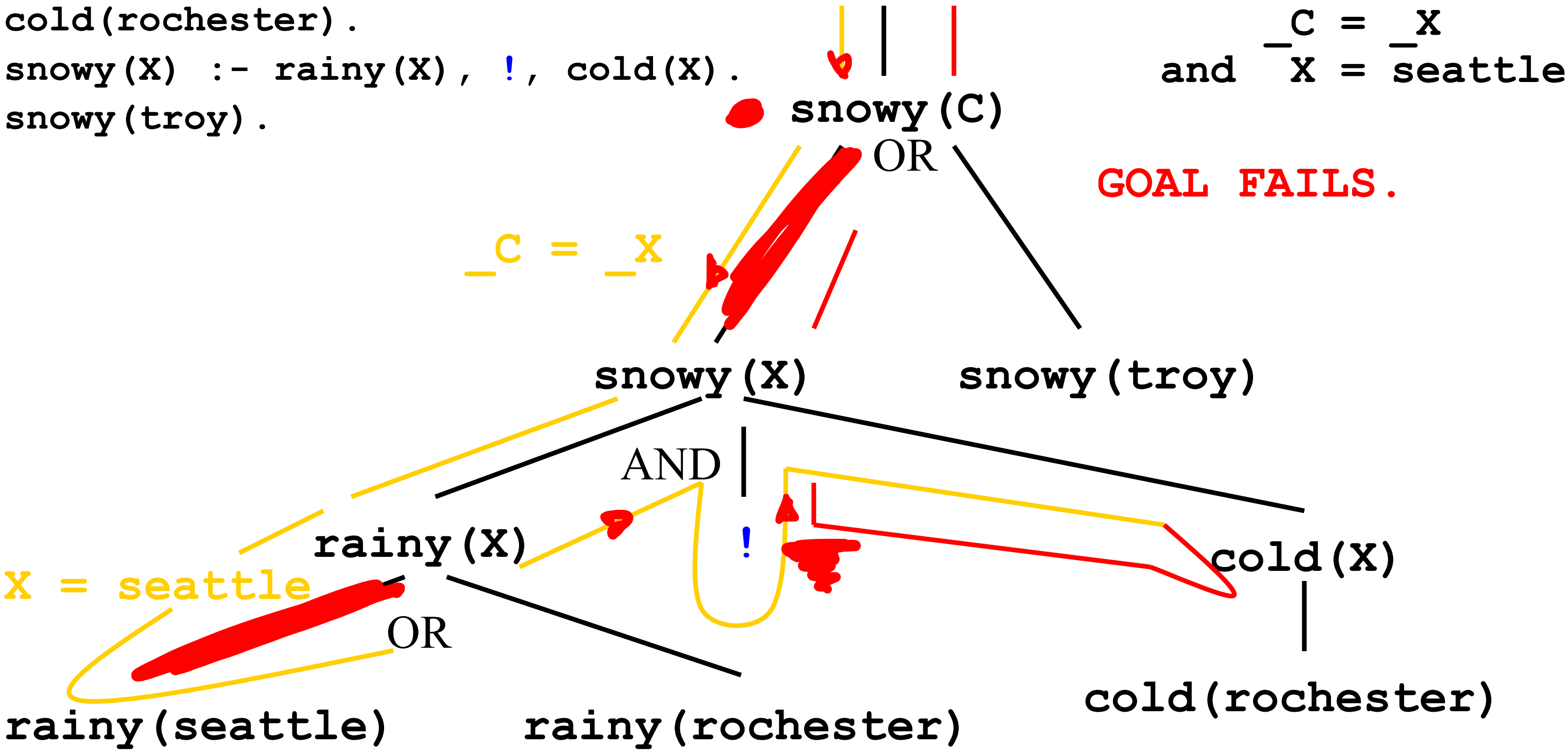
?- snowy(C).
```

Cut (!) Example 2

```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), !, cold(X).
snowy(troy).
```

2 committed OR bindings:

_C = _X
and _X = seattle



How about query ?- snowy(troy)?

?- true.

Cut (!) Example 3

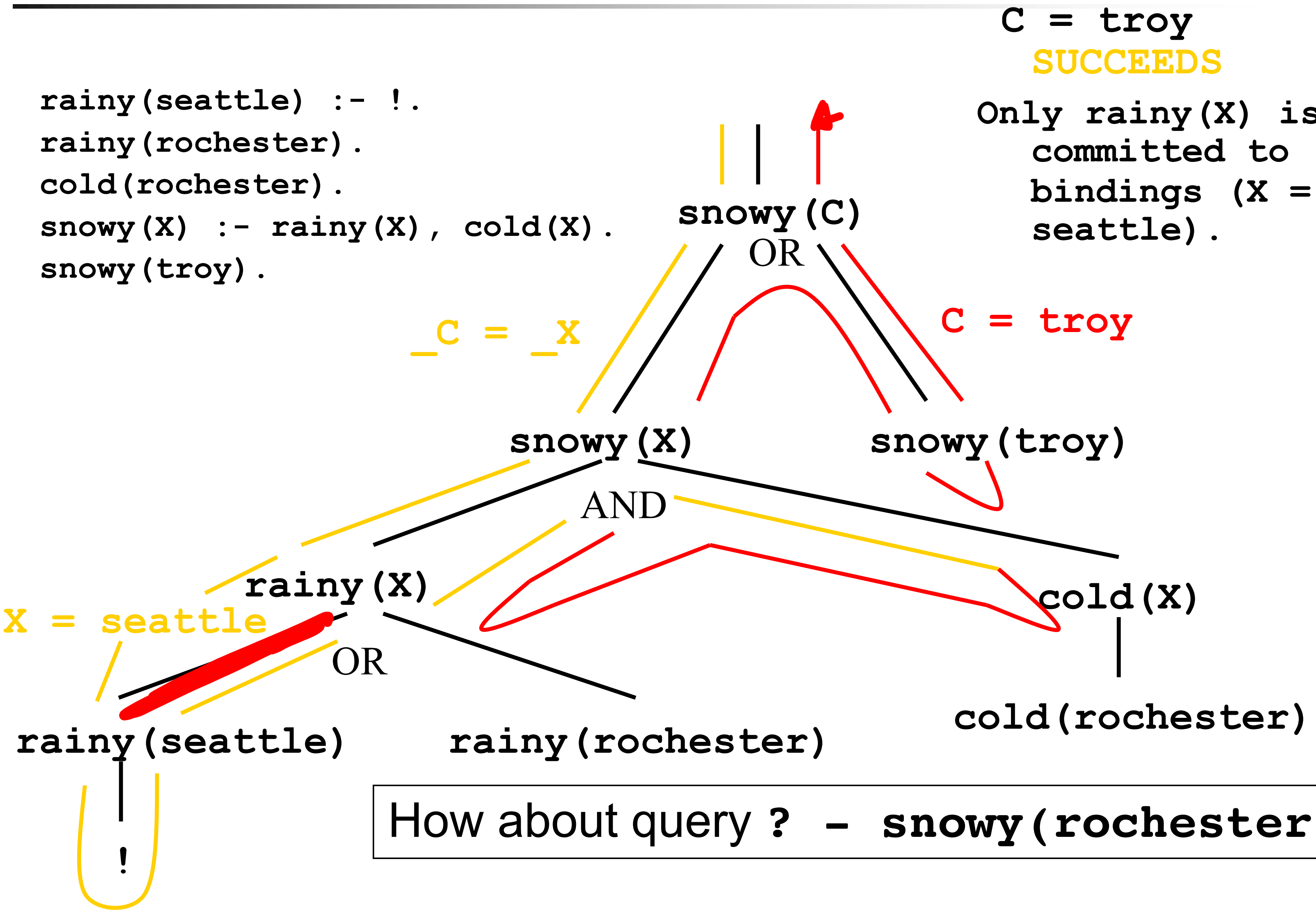
```
rainy(seattle) :- !.
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), cold(X).
snowy(troy).

?- snowy(C).
```

?- C= troy.

Cut (!) Example 3

```
rainy(seattle) :- !.
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), cold(X).
snowy(troy).
```



How about query **?- snowy(rochester)?**

Cut (!) Example 4

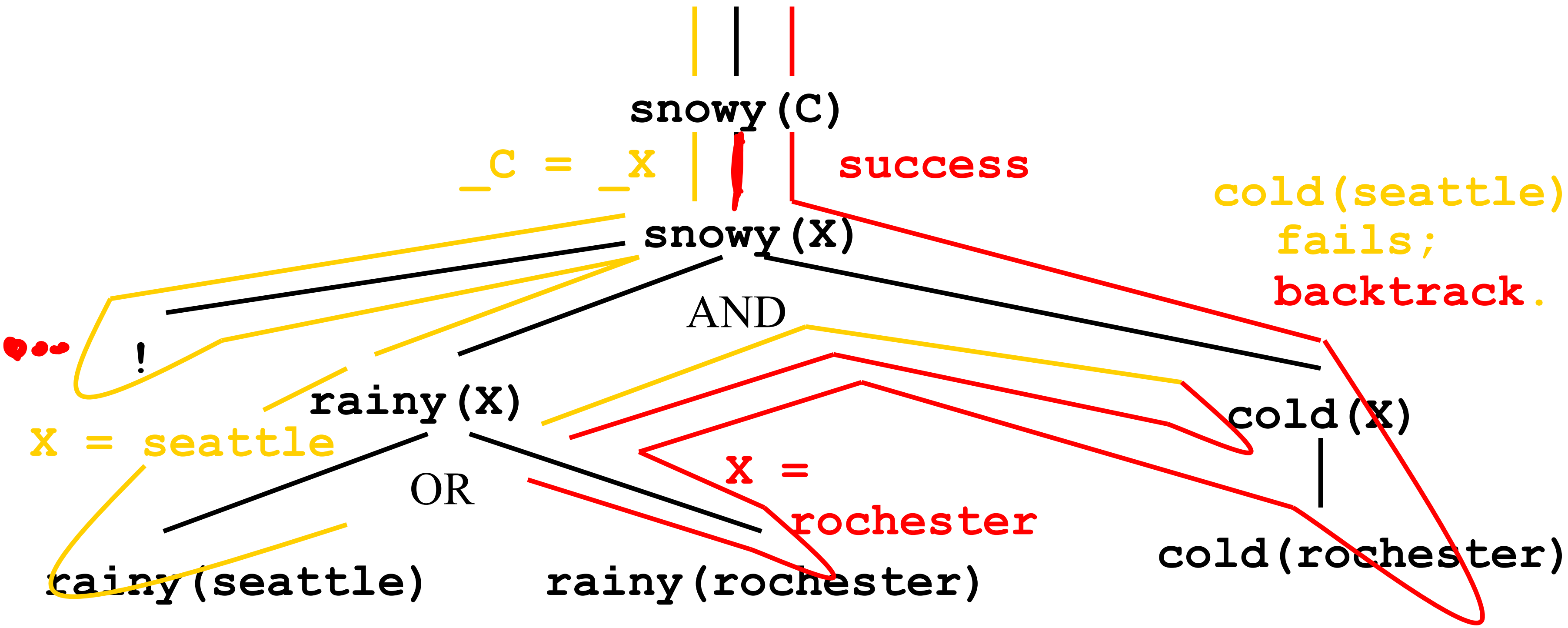
```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- !, rainy(X), cold(X).
```

```
?- snowy(C).
```

?- C=rochester.

Cut (!) Example 4

```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- !, rainy(X), cold(X).
```



Cut (!) Example 5

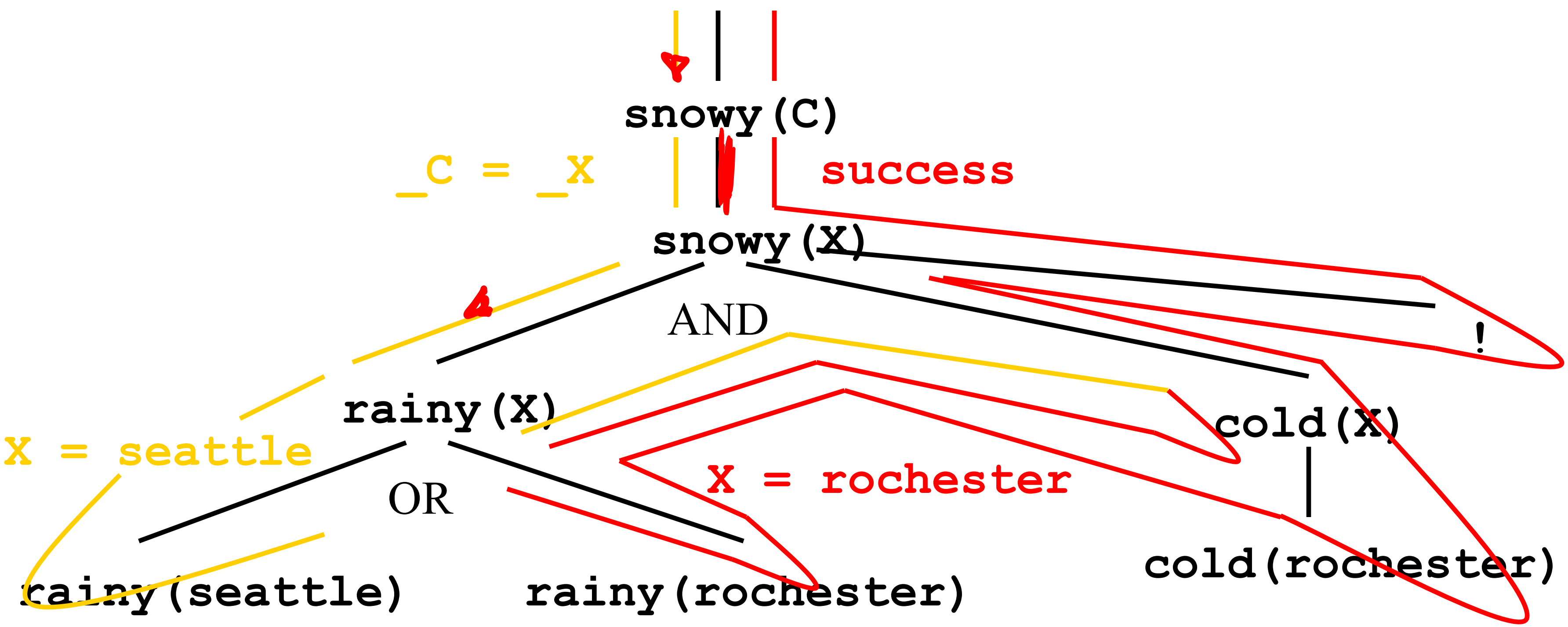
```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), cold(X), !.
```

```
?- snowy(C).
```

?- C = rochester.

Cut (!) Example 5

```
rainy(seattle).
rainy(rochester).
cold(rochester).
snowy(X) :- rainy(X), cold(X), !.
```



Negation by Failure: `not(X)`, `\+(X)`

- **`not(C)`** succeeds when **`C`** fails
  - Called negation by failure, defined:

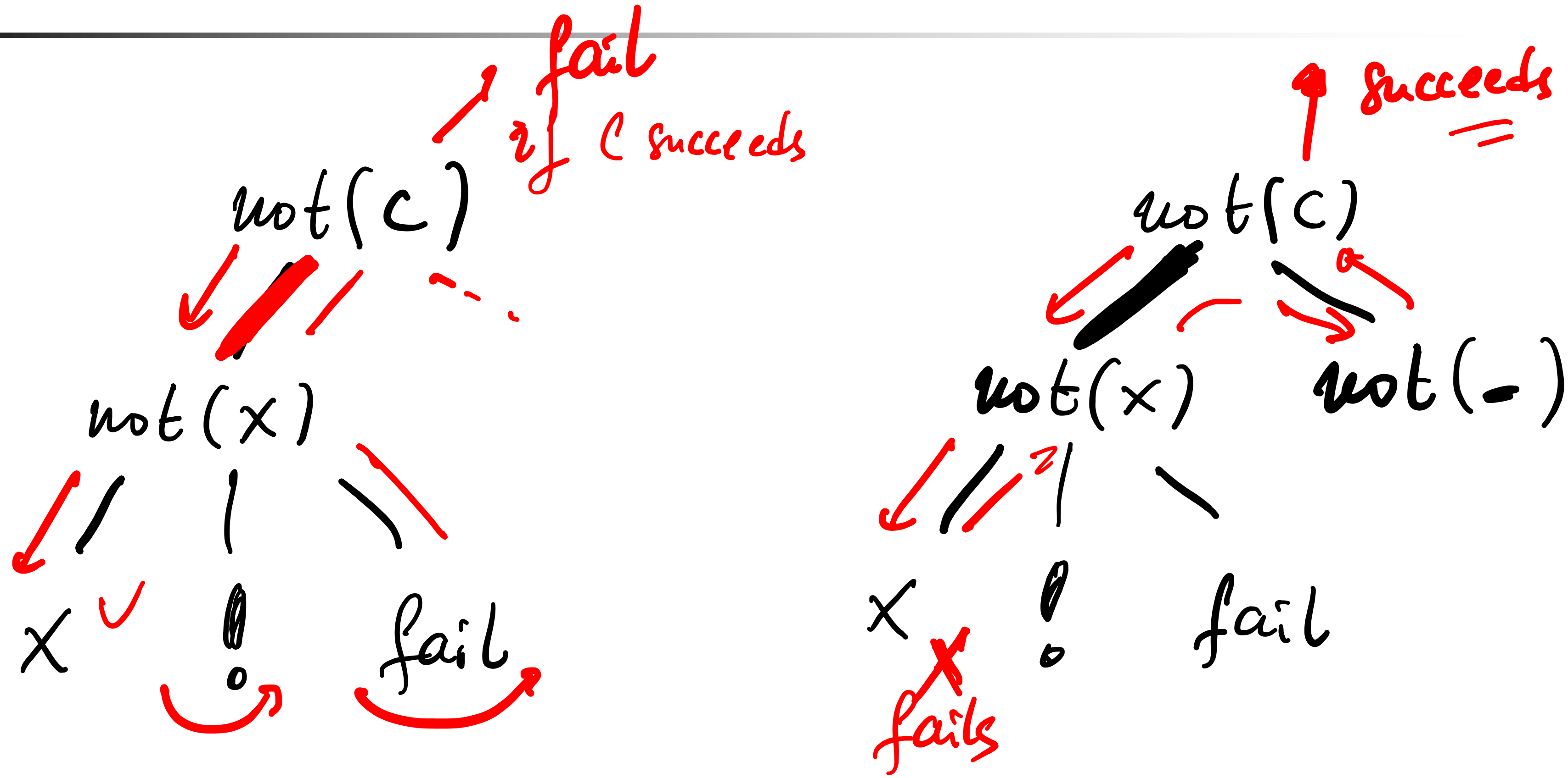
```
not(X) :- X,!,fail.
not(_).
```

don't care

- Not the same as negation in logic **¬X**!

- In Prolog, we can assert that something is true, but we cannot assert that something is false

likes(eve, pie). ~~¬ Likes(eve, pie).~~


fail
if C succeeds
not(C)
not(X)
X
!
fail
succeeds
not(C)
not(X)
not(_)
X
!
fail
fails

Exercise

```
takes(jane, his).
takes(jane, cs).
takes(ajit, art).
takes(ajit, cs).
classmates(X,Y) :- takes(X,Z),takes(Y,Z),
                                        not(X == Y).

?- classmates(jane,Y).
```

What are the bindings of **Y**?

```
?- Y= jane ; // Y=jane , Z= his
?= Y= jane ; // Z= cs
?= Y= ajit ; // Z= cs
```

Exercise

- ```
  p(X) :- q(X), not(r(X)).
  r(X) :- w(X), not(s(X)).
  q(a). q(b). q(c).
  s(a). s(c).
  w(a). w(b).
  ```

  q(a) ✓ r(a) true for p(a).
  r(a) is false w(a) ✓ s(a) false

- Evaluate:
  - `?- p(a).` true
  - `?- p(b).` false
  - `?- p(c).` true
  ```

Lecture Outline

- Prolog
  - Imperative control flow
  - Negation by failure
  - Generate and test paradigm

Generate and Test Paradigm

- Search in space

- Prolog rules to generate potential solutions
- Prolog rules to test potential solutions for desired properties

- Easy prototyping of search

```
solve(P) :- generate(P), test(P).
```

A Classical Example: n Queens

- Given an n by n chessboard, place each of n queens on the board so that no queen can attack another in one move
  - Queens can move either vertically,
  - horizontally, or
  - diagonally.
- A classical generate and test problem

n Queens

```
my_not(X):- X, !, fail.   %same as not
my_not(_).
in(H,[H|_]).              %same as member
in(H,[_|T]):- in(H,T).
```

→ result

nums(L,H,R).

```
nums(H,H,[H]).
```

result of toplevel call

L+1, H, R

```
nums(L,H,[L|R]):- L<H, N is L+1, nums(N,H,R).
%%%nums generates a list of integers between two other
   numbers, L,H by putting the first number at the front
   of the list returned by a recursive call with a number
   1 greater than the first.  It works when L and H are
   bound to integers. It stops when it gets to the higher
   number
```

nums(1,4,R).

```
queen_no(4).
%%%The number of queens/size of board - use 4
```

R = [1,2,3,4].

n Queens (ii)

generation rules

```
ranks(L):- queen_no(N), nums(1,N,L).
files(L):- queen_no(N), nums(1,N,L).
```

%%%ranks and files generate the x and y axes of the chess board. Both are lists of numbers up to the number of queens; that is, ranks(L) binds L to the list [1,2,3,…,#queens].

ranks(L).
L=[1,2,3,4]

```
rank(R):- ranks(L), in(R,L).
```

%%% R is a rank on the board; selects a particular rank R from the list of all ranks L.

?- rank(R).
?- 1;
?- 2;
...

```
file(F):- files(L), in(F,L).
```

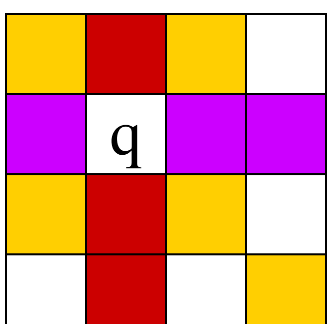
%%% F is a file on the board; selects a particular file F from the list of all files L.

n Queens (iii)

test rules

attacks((R1,F1),(R2,F2))

```
%%% Squares on the board are (rank,file) coordinates.
   attacks decides if a queen on the square at rank R1,
   file F1 attacks the square at rank R2, file F2 or
   vice versa. A queen attacks every square on the same
   rank, the same file, or the same diagonal.
attacks((R,_),(R,_)).
attacks((_,F),(_,F)). %a Prolog tuple
attacks((R1,F1),(R2,F2)):-
                         diagonal((R1,F1),(R2,F2)).
%%%can decompose a Prolog tuple by unification
   (X,Y)=(1,2) results in X=1,Y=2; tuples have fixed
   size and there is not head-tail type construct for
   tuples
```



same rank (X coordinate)

same file (Y coordinate)

same diagonal

What is safe placement for next queen on board?

n Queens (iv)

```
%%% Two squares are on the same diagonal if the slope of
  the line between them is 1 or -1.  Since / is used, real
  number values for 1 and -1 are needed.

diagonal((X,Y),(X,Y)). %degenerate case
diagonal((X1,Y1),(X2,Y2)):-N is Y2-Y1,D is X2-X1,
  Q is N/D, Q is 1 . %diagonal needs bound
  arguments!
diagonal((X1,Y1),(X2,Y2)):-N is Y2-Y1,D is X2-X1,
  Q is N/D, Q is -1 .
%%%because of use of “is”, diagonal is NOT invertible.
```

n Queens (v)

```
%%% This solution works by generating every list of
  squares, such that the length of the list is the same
  as the number of queens, and then checks every list
  generated to see if it represents a valid placement of
  queens to solve the N queens problem;
  assume list length function
```

4

[_X1,_X2,_X3,_X4]

```
queens(P):- queen_no(N), length(P,N),
  placement(P), ok_place(P).
```

"generate" code given first

"test" code follows

n Queens (vi)

placement(P) P=[_X1,_X2,_X3,_X4]
P is of fixed length

```
%%%placement can be used as a generator.  If placement
   is called with a free variable, it will construct
   every possible list of squares on a chess board.
   The first predicate will allow it to establish the
   empty list as a list of squares on the board.  The
   second predicate will allow it to add any (R,F) pair
   onto the front of a list of squares if R is a rank
   of the board and F is a file of the board.
   placement first generates all 1 element lists, then
   all 2 element lists, etc.  Switching the order of
   predicates in the second clause will cause it to try
   varying the length of the list before it varies the
   squares added to the list
placement([]).
placement([(R,F)|P]):- placement(P), rank(R), file(F).
```

_X1 _X2,_X3,_X4

n Queens (vii)

ok_place(P)

```
%%%these two routines check the placement of the next
   queen
```

fully instantiated

```
%%%Checks a list of squares to see that no queen on
   any of them would attack any other. does by checking
   that position j doesn't conflict with positions
   (j+1),(j+2) etc.
ok_place([]).
ok_place([(R,F)|P]):- no_attacks((R,F),P),ok_place(P).
%%% Checks that a queen at square (R,F) doesn't attack
   any square (rank,file pair) in list L; uses attacks
   predicate defined previously
no_attacks(_,[]).
no_attacks((R,F),[(R2,F2)|P]):-
   my_not(attacks((R,F),(R2,F2))), no_attacks((R,F),P).
```

Homework Solution Structure

- Typical Prolog homework: search in space (e.g., paths in a maze, paths in graph, parsing sequences, various puzzles)
- Typical solution: % base case

```
search(F,Partial,Total) :-
   final(F), … % get Total from Partial
search(C,Partial,Total) :-
   generate(C,N), % generate next position
   valid(N),…    % test if N is a valid position
   augment(Partial,New_partial),
   % augment Partial solution with N, typically
       we would need not(member(N,Partial)) too.
   search(N,New_partial,Total).  % recurse
```

A Harder Exercise

- Remember grammar from one of the quizzes…

1. $S \rightarrow \mathbf{a}S\mathbf{b}S$
2. $S \rightarrow \mathbf{b}S\mathbf{a}S$
3. $S \rightarrow \varepsilon$

- Write a **top-down depth-first** parser in Prolog:

`?- parse([a,b,a,b],R).`

`R = [1, 2, 3, 3, 3] ;` // seq. of productions

`R = [1, 3, 1, 3, 3] ;` // different seq

`false.` // no more seqs

- Hint: break list into constituent parts

The End

```
search(P, Final) :- queens_no(N), length(P,N),
                    Final = P.
% search(P, P) :- queens_no(N), length(P,N).

search(P, Final) :-
    rank(R), file(F), no_attacks((R,F), P),
    search([(R,F)|P], Final).

solve(P) :- search([], P).
```