HW 7 - Sample Solution

8.1 Assume TSP(G, b) returns false if no tour of length b or less exists G.

Sum up all the distances of G.

TSP_opt calls TSP(G,b) as a binary search primitive.

```
TSP_opt (G)
    let S be the sum of all distances
    return BinarySearchTour(G, 0, S)
BinarySearchTour (G, l, u)
    b = (l+u)/2
    if TSP(G,b) ≠ false
        return BinarySearchTour (G, l, b)
    else return BinarySearchTour (G, b, u)
```

8.4

a. ~~Given~~ clique3 can be checked in polynomial time.

b. Reduction is in the wrong reduction.

We must reduce known NP-complete Problem clique to clique 3.

c. C is a vertex iff V - C is an independent set in G.

d. Largest clique can be of size 4.

Take all possible subsets of 4, to test whether there is clique of size k.

8.10

a. We can view this as a generalization of clique problem. Let (G, k) be a clique instance. Construct a subgraph H which is clique of size k.

b. This is a generalization of Rudrata path. Given a graph with n, vertices, let $g = n-1$.

9.4. Keep inserting the smallest degree vertex in the independent set. Delete that vertex. Repeat the step till no more vertices is left.

Let. Largest independent set size is x.

For each vertex we would have picked one of the $d+1$ vertices. Size will be atmost $\frac{x}{d+1}$

9.6. Find the MST of the designated nodes.

Let that cost be X.

If the optimal Steiner tree has cost y.

We can do an eulerian traversal of the optimal Steiner and keeping only the designated vertices.

$$2y \geq X$$

$$\therefore X \leq 2y.$$

7.1 The optimal solution is on the upper right corner of the convex feasible region (5, 2) and has the value $5x + 3y = 31$.

7.2. Let MN the quantity between Mexico and NewYork

MC between Mexico & California

KN between Kansas & NewYork

KC between Kansas & California

min $4MN + MC + 2KN + 3KC$

$MN + KN = 10$ $MN + KN \leq 10$ $MN + KN \geq 10$

$MC + KC = 13$ $MC + KC \leq 13$ $MC + KC \geq 13$

$MN + MC = 8$ $MN + MC \leq 8$ $MN + MC \geq 8$

$KN + KC = 15$ $KN + KC \leq 15$ $KN + KC \geq 15$

$MN, MC, KN, KC \geq 0$

7.3 Let q_i be the quantity in cubic meters of material i

max $1000 q_1 + 1200 q_2 + 12000 q_3$

$2q_1 + q_2 + 3q_3 \leq 100$

$q_1 + q_2 + q_3 \leq 60$

$q_1 \leq 40 \quad q_2 \leq 30 \quad q_3 \leq 20$

$q_1, q_2, q_3 \geq 0$

7.4. Let R be regular Diff bear

S' be Strong Diff bear

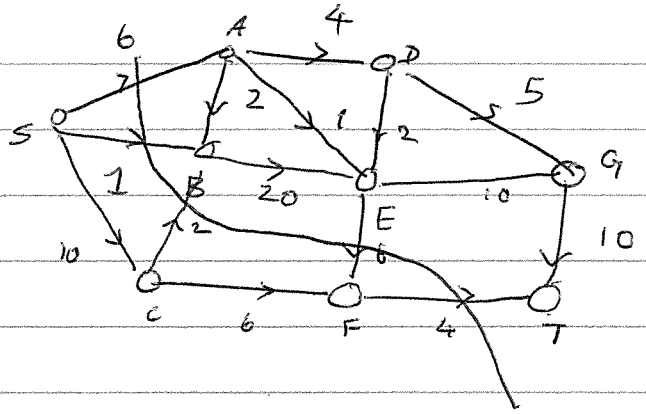
$$\max \; R + 1.5S$$

$$S \le 2R$$

$$R + S \le 3000$$

$$R, S \ge 0$$

7.10



S A D G T	4
S A B E G T	2
S B E G T	1
S C F T	4
S C B E G T	2
	13

cut $\{S, C, F\}$ and $\{A, B, D, E, G, T\}$