ICCOPT II
MOPTA 07

COMPARATIVE STUDY OF METHODS FOR MODELING CONTACT AND FRICTION IN PHYSICAL SIMULATION

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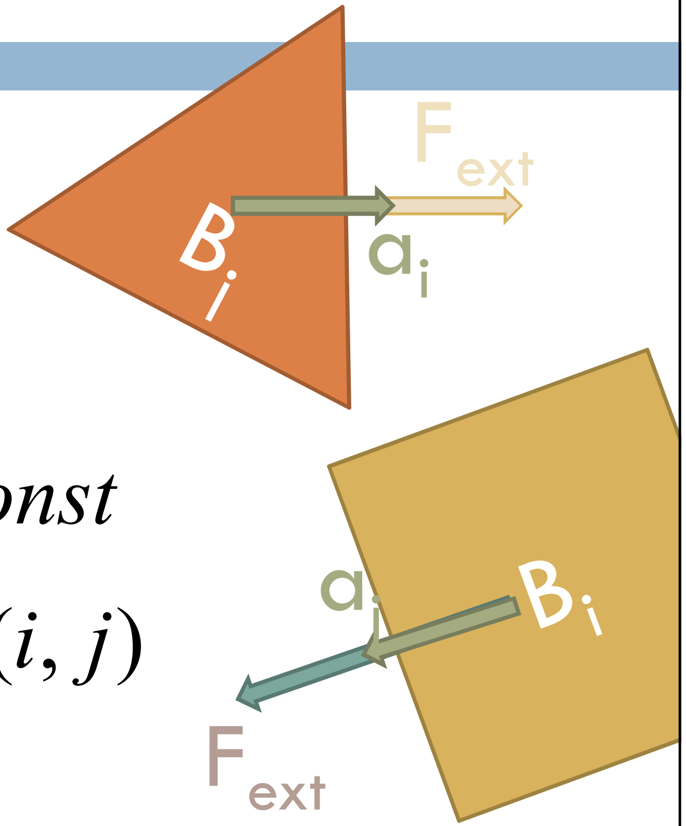
Introduction

- Motivation
  - Computer games
  - Machine design
- Set of methods originated from (David Stewart, Jeff Trinkle) and (Mihai Anitescu, Florian Potra, Cremer) [1996-2003], (Chakraborty, Steven Berard, Jeff Trinkle, Srinivas Akella) [2006-2007].

Outline

- **Introduction**
- **Our dynamics simulation formulations**
  - **Continuous formulation**
  - Stewart Trinkle formulation
  - Egan Trinkle formulation
  - Chakraborty Berard formulation
- Summary

A popular physical model



- Newton Euler
- Rigid body and non-penetration assumption:

$$\forall (p,q) \in B_i : \|p-q\| = const$$

$$Int(B_i) \cap Int(B_j) = \varnothing, \forall (i,j)$$

- Coulomb friction:

$$\|f_{friction}\| \le \mu \|f_{normal}\|$$

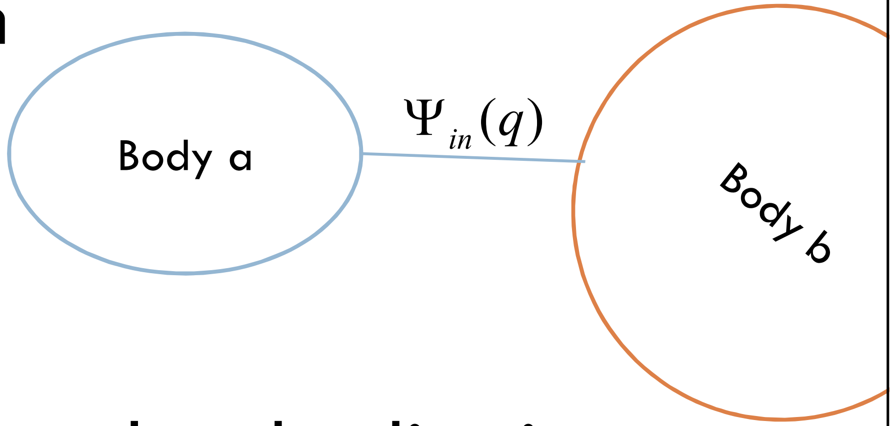
Continuous formulation

- A system of n body $\{B_1, B_2, \ldots, B_n\}$ with configuration vector $q = \{q_1, q_2, \ldots, q_n\}$.
- Velocity vector: $v = [v_1\; v_2 \ldots v_n]^T$
- Inertia matrix: M (stack of n inertia matrix)
- Newton-Euler equation:

$$M(q)\dot{v} = \underset{\nearrow\;\text{External force}}{F_{applied}} + F_{coriolis} + \overbrace{\underset{\nwarrow\;\text{Forces to prevent penetration}}{F_{contact}} + \underset{\swarrow\;\text{Forces to prevent sliding}}{F_{friction}}}^{\text{Constraint forces}}.$$

Kinematic map: $\dot{q} = Gv$

Rigid body assumption



I)Signed distance function

$$\Psi_{in}(q) \geq 0$$

II)Lagrange multiplier

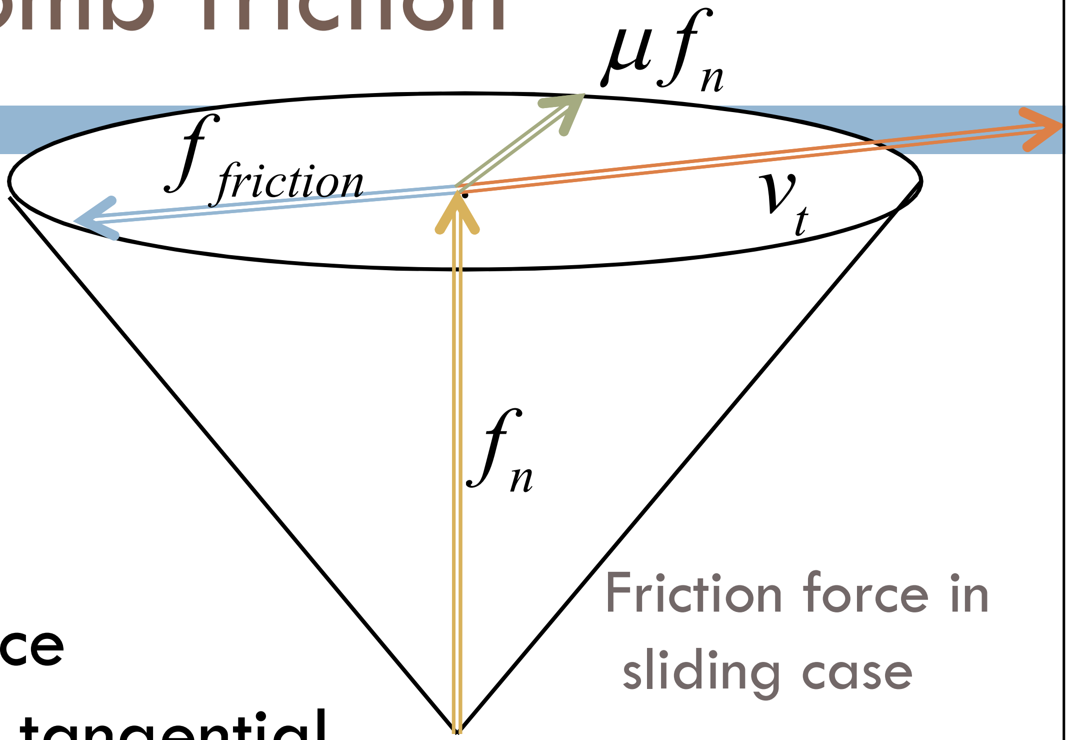
for contact force: $\lambda_{in} \geq 0$

III)Normal force only exists when bodies in contact: $\lambda_{in}^{T} \bullet \Psi_{in}(q) = 0$.

These 3 conditions are called Signorini condition and can be formulated as:

$$\boxed{0 \leq \lambda_{in} \perp \Psi_{in}(q) \geq 0}$$

Coulomb friction



Friction force in sliding case

- ☐ Friction cone (f_n, μ)
- ☐ Friction force must line on the disc defined by (f_n, μ) .
- ☐ Friction force must face opposite direction of tangential velocity (sliding).
- ☐ f_n : normal contact force, v_t : tangential velocity,
  f_c : friction force.

Papers by (David Stewart, Jeff Trinkle) (Mihai Anitescu, Florian Potra, Cremer)[1996-2003]

Time stepping method

- Time stepping method: introduce time step *h*.
- Only calculate system state values at time $t_l = l * h, l = 0, 1, 2, \ldots$ and $X^{(l)}$ = value of X at time l.
- Approximate: $\boxed{\dot{v} \approx \frac{v^{(l+1)} - v^{(l)}}{h}, \dot{q} \approx \frac{q^{(l+1)} - q^{(l)}}{h}.}$
- Using velocity-impulse instead of acceleration-force. (Impulse $p = f * h.$)

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Stewart Trinkle formulation

- Newton-Euler equation:

$$\boxed{M\cdot(v^{(l+1)}-v^{l})=W_n p_n^{(l+1)}+W_f p_f^{(l+1)}+p_{ext}}$$

W_n, W_f : Normal and friction “wrenches” and

$$W_n^T=\frac{\partial\Psi_n}{\partial q}G, W_f^T=\frac{\partial\Psi_f}{\partial q}G.$$

p_n, p_f, p_{ext} : normal, friction, external impulse $(p_n=h\lambda_n, p_f=h\lambda_f, p_{ext}=h\lambda_{ext})$

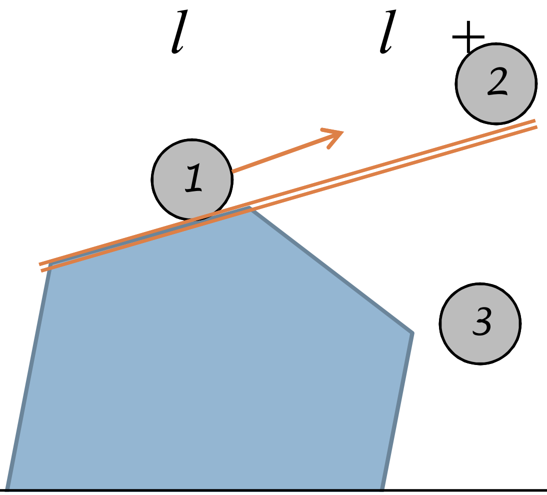
Stewart Trinkle formulation

- Linearized distance function:

$$\Psi_{in}^{(l+1)} \approx \Psi_{in}^{(l)} + \frac{\partial \Psi_{in}^{(l)}}{\partial q}\left(q^{(l+1)} - q^{(l)}\right) + \frac{\partial \Psi_{in}^{(l)}}{\partial t} h$$

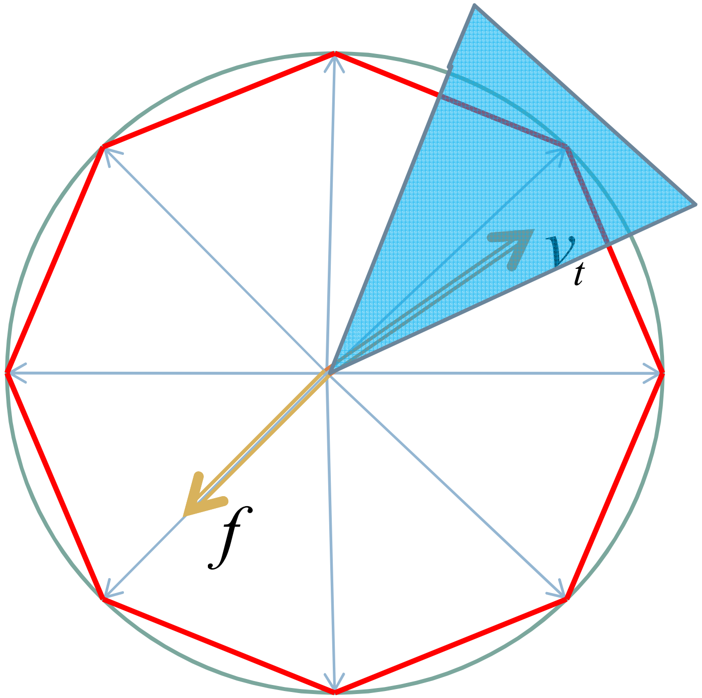
$$= \Psi_{in}^{(l)} + W_{in}^{T} \nu^{(l+1)} h + \frac{\partial \Psi_{in}^{(l)}}{\partial t} h \geq 0.$$

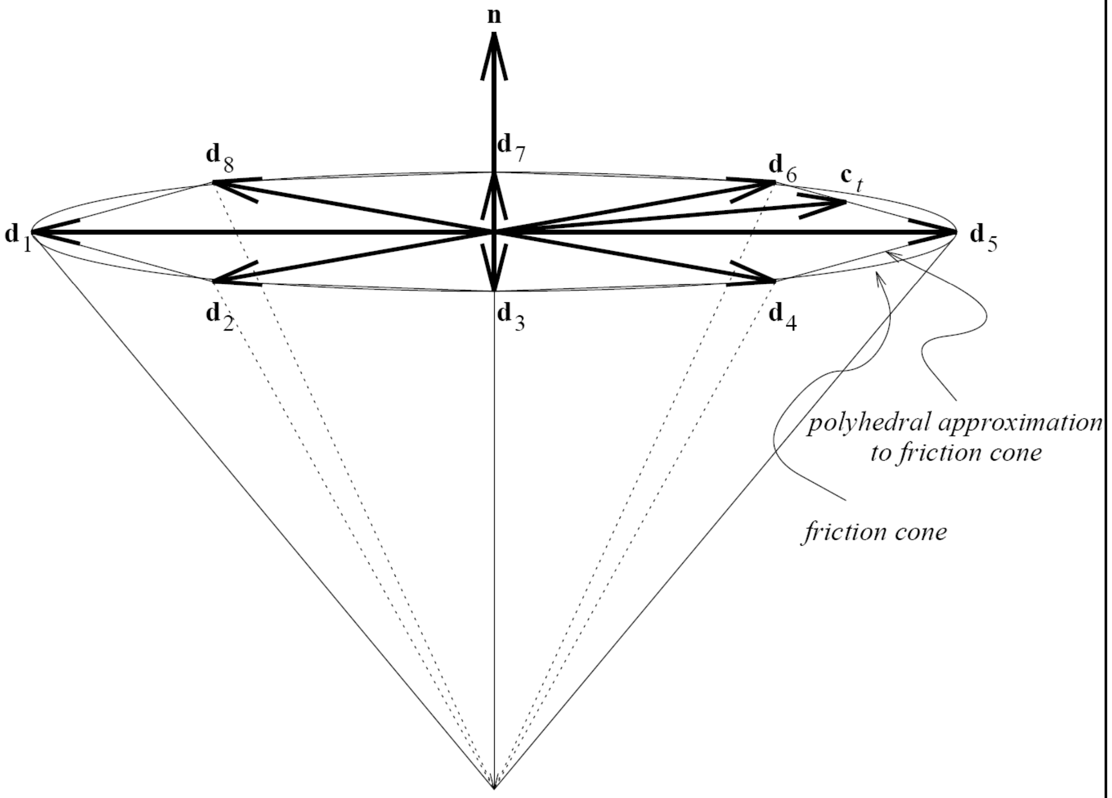
- Use information at time *l* to build constraints that should be satisfied at time *l*+1.



Stewart Trinkle formulation

- Polygonal approximation of friction cone.





Stewart Trinkle formulation

$$\begin{bmatrix} 0 \\ \rho_{\mathrm{n}}^{\ell+1} \\ \rho_{\mathrm{f}}^{\ell+1} \\ s^{\ell+1} \end{bmatrix} = \begin{array}{c} \begin{bmatrix} -M & W_{\mathrm{n}} & W_{\mathrm{f}} & 0 \\ W_{\mathrm{n}}^{\mathrm{T}} & 0 & 0 & \boxed{0} \\ W_{\mathrm{f}}^{\mathrm{T}} & 0 & 0 & E \\ 0 & \boxed{U} & -E^{\mathrm{T}} & 0 \end{bmatrix} \\ \begin{array}{cccc} 6n & n_c & n_c n_d & n_c \end{array} \end{array} \begin{bmatrix} \nu^{\ell+1} \\ p_{\mathrm{n}}^{\ell+1} \\ p_{\mathrm{f}}^{\ell+1} \\ \sigma^{\ell+1} \end{bmatrix} + \begin{bmatrix} M\nu^{\ell} + hf \\ \frac{\psi_{\mathrm{n}}^{\ell}}{h} + \frac{\partial \psi_{\mathrm{n}}^{\ell}}{\partial t} \\ \frac{\partial \psi_{\mathrm{f}}^{\ell}}{\partial t} \\ 0 \end{bmatrix}$$

n : *number of bodies.*

nc : *number of contacts*

nd: *number friction directions*

$$0 \le \begin{bmatrix} \rho_{\mathrm{n}}^{\ell+1} \\ \rho_{\mathrm{f}}^{\ell+1} \\ s^{\ell+1} \end{bmatrix} \perp \begin{bmatrix} p_{\mathrm{n}}^{\ell+1} \\ p_{\mathrm{f}}^{\ell+1} \\ \sigma^{\ell+1} \end{bmatrix} \ge 0$$

$n_c\,(n_d + 2)$ complimentarity conditions

Papers by (David Stewart, Jeff Trinkle) (Mihai Anitescu, Florian Potra, Cremer)[1996-2003]

Stewart Trinkle formulation

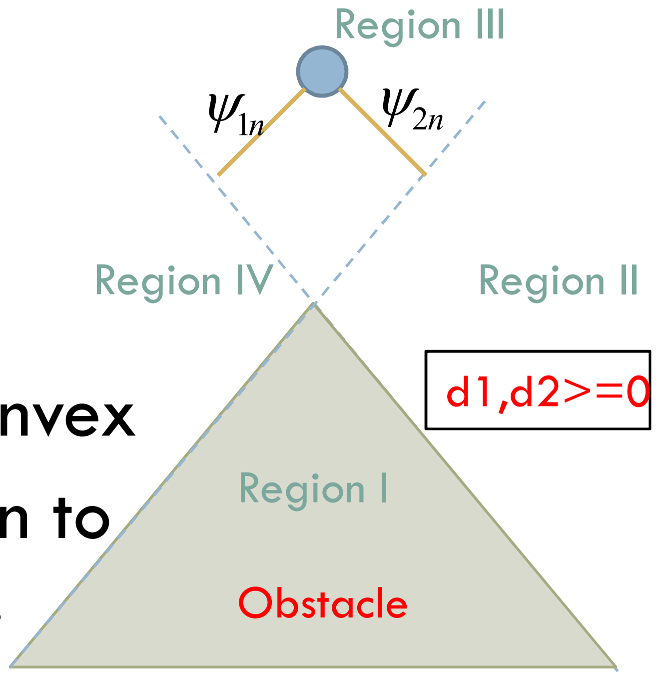
- ☐ Show triangle convex demo
- ☐ A quick look at ST non penetration constraint:

$$\boxed{\begin{aligned} 0 \le \psi_{1n} \perp p_1 \ge 0 \\ 0 \le \psi_{2n} \perp p_2 \ge 0 \end{aligned}}$$

(p_1, p_2):Normal impulses to keep ψ_{1n}, ψ_{2n} in feasible set.

=> implicit assumption locally convex

- ☐ Need to use topology information to deal with this implicit assumption.



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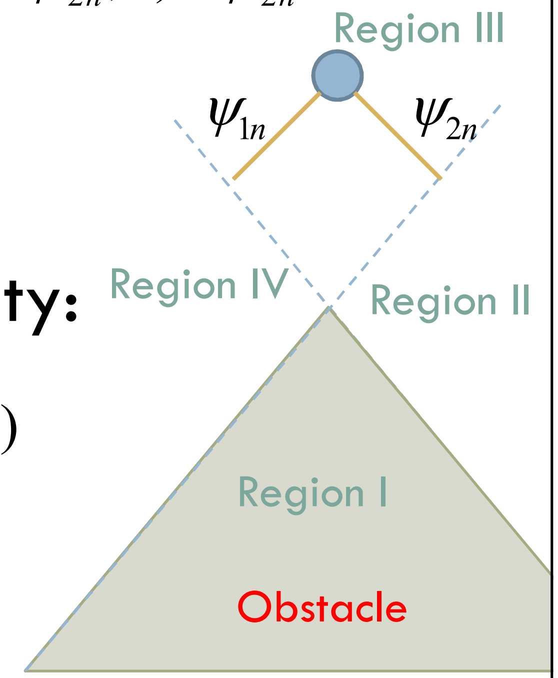
Egan Trinkle formulation

- Non penetration constraint: $(0 \le \psi_{1n} \vee \psi_{2n} \ge 0)$
- $(0 \le \psi_{1n} \vee \psi_{2n} \ge 0) \Leftrightarrow \max(\psi_{1n}, \psi_{2n}) \ge 0 \Leftrightarrow \max(\psi_{1n} - \psi_{2n}, 0) + \psi_{2n} \ge 0$
- Use $c = \max(a, 0) \Leftrightarrow 0 \le c - a \perp c \ge 0$
- The formulation of non-penetration constraint with this locally non-convexity:



$$0 \le c - \psi_{1n} + \psi_{2n} \perp c \ge 0 \rightarrow c = \max(\psi_{1n} - \psi_{2n}, 0)$$

$$\left.\begin{array}{l} 0 \le c + \psi_{2n} \perp p_1 \ge 0 \\ 0 \le c + \psi_{2n} \perp p_2 \ge 0 \end{array}\right\}$$

$(0 \le \psi_{1n}) or (\psi_{2n} \ge 0)$

Only apply normal impulses when $\psi_{1n}, \psi_{2n} \le 0$

Egan Trinkle formulation

- Show Egan Trinkle triangle convex demo
- Show Stewart Trinkle ramp demo
- Show Egan Trinkle ramp demo
- Show Stewart Trinkle hole fitting
- Show Egan Trinkle hole fitting

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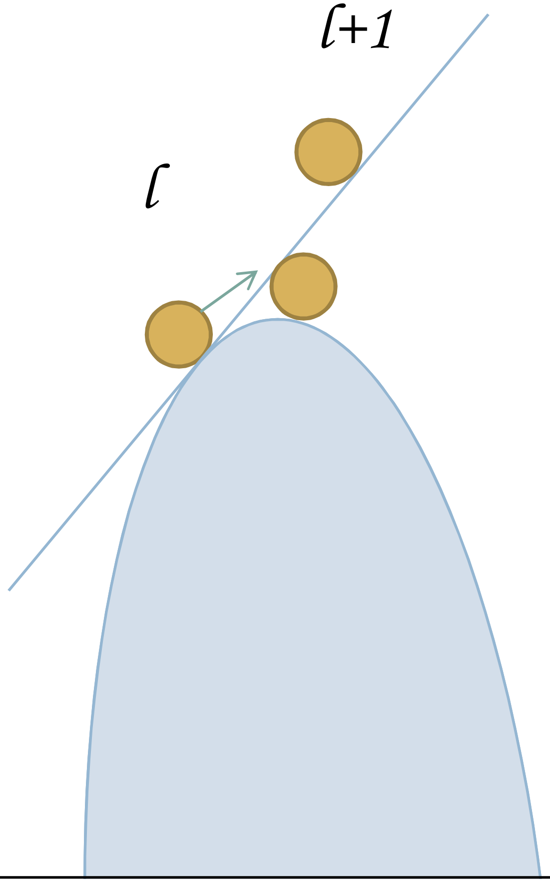
Chakraborty Berard formulation

- Newton Euler
- Rigid body assumption: non linear and implicit
- Friction: non linear



Chakraborty Berard formulation

$$0 = -\mathbf{M}\boldsymbol{\nu}^{\ell+1} + \mathbf{M}\boldsymbol{\nu}^{\ell} + \mathbf{W}_{\mathrm{n}}^{\boxed{\ell+1}}\mathbf{p}_{\mathrm{n}}^{\ell+1} + \mathbf{W}_{\mathrm{t}}^{\boxed{\ell+1}}\mathbf{p}_{\mathrm{t}}^{\ell+1}$$

$$+ \mathbf{W}_{\mathrm{o}}^{\boxed{\ell+1}}\mathbf{p}_{\mathrm{o}}^{\ell+1} + \mathbf{W}_{\mathrm{r}}^{\boxed{\ell+1}}\mathbf{p}_{\mathrm{r}}^{\ell+1} + \mathbf{p}_{\mathrm{app}} + \mathbf{p}_{\mathrm{vp}}$$

$$0 = (\mathbf{a}_1^{\ell+1} - \mathbf{a}_2^{\ell+1}) + \|\mathbf{a}_1^{\ell+1} - \mathbf{a}_2^{\ell+1}\| \frac{\nabla f(\mathbf{a}_1^{\ell+1})}{\|\nabla f(\mathbf{a}_1^{\ell+1})\|}$$

$$0 = \frac{\nabla f(\mathbf{a}_1^{\ell+1})}{\|\nabla f(\mathbf{a}_1^{\ell+1})\|} + \frac{\nabla g(\mathbf{a}_2^{\ell+1})}{\|\nabla g(\mathbf{a}_2^{\ell+1})\|}$$

$$0 = f(\mathbf{a}_1^{\ell+1})$$

$$0 = g(\mathbf{a}_2^{\ell+1})$$

Implicit and non-linear distance function

$$\boxed{0 \leq p_{\mathrm{n}}^{\ell+1} \perp f(\mathbf{a_2}^{\ell+1}) \geq 0}$$

The mathematical model is a Mixed Nonlinear Complementarity Problem.

Chakraborty Berard formulation

- A disk can roll to infinity on a flat surface even with friction in CB and stop with ST and ET: movie.

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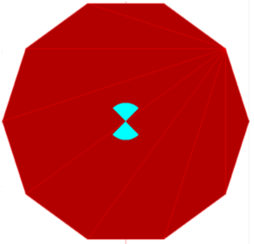
Problem’s size comparison

Scenario	Stewart Trinkle (LCP size)	Egan Trinkle (LCP size)	Chakraborty Berard (NCP size)
Sphere(10) rolling on floor	15	15	9
Sphere(50) rolling on floor	67	67	9
Sphere(100) rolling on floor	123	123	9
Sphere(1000) rolling on floor	1263	1263	9

10



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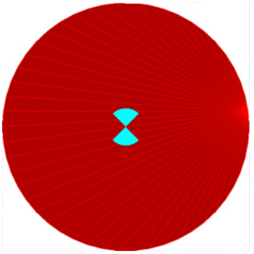

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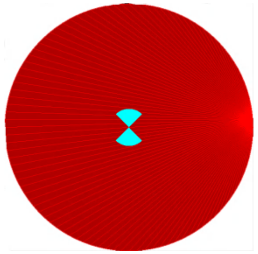

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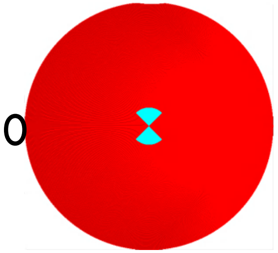

Stewart Trinkle	Egan Trinkle (1)	CB
6*n + nc*(nd+2)	6*n + nc*(nd+2)+ nk-1	9*n+4*nc

n: *number of bodies*, nc: *number of contacts*

nd: *number friction direction*, nk: *number of Egan Trinkle’s contacts*

Summary

- We can use complementary condition to model contact and friction in many ways.
- Which formulation to use in simulation is really depend on application.
- We are trying to create more formulations having a set of features that may be useful for more types of applications.

Question

- PATH is a general solver so it cannot exploit some information that we have (structure of matrix, physical interpretation,…)
- How to solve the dynamics simulation LCP/NCP efficiently?

$$\begin{bmatrix}0\\ \rho_{\mathrm{n}}^{\ell+1}\\ \rho_{\mathrm{f}}^{\ell+1}\\ s^{\ell+1}\end{bmatrix} = \begin{array}{c}\begin{bmatrix}-M & W_{\mathrm{n}} & W_{\mathrm{f}} & 0\\ W_{\mathrm{n}}^{\mathrm{T}} & 0 & 0 & 0\\ W_{\mathrm{f}}^{\mathrm{T}} & 0 & 0 & E\\ 0 & U & -E^{\mathrm{T}} & 0\end{bmatrix}\\ \begin{array}{cccc}6n & n_c & n_c n_d & n_c\end{array}\end{array} \begin{bmatrix}\nu^{\ell+1}\\ p_{\mathrm{n}}^{\ell+1}\\ p_{\mathrm{f}}^{\ell+1}\\ \sigma^{\ell+1}\end{bmatrix} + \begin{bmatrix}M\nu^{\ell}+hf\\ \frac{\psi_{\mathrm{n}}^{\ell}}{h}+\frac{\partial\psi_{\mathrm{n}}^{\ell}}{\partial t}\\ \frac{\partial\psi_{\mathrm{f}}^{\ell}}{\partial t}\\ 0\end{bmatrix}$$

$$0 \le \begin{bmatrix}\rho_{\mathrm{n}}^{\ell+1}\\ \rho_{\mathrm{f}}^{\ell+1}\\ s^{\ell+1}\end{bmatrix} \perp \begin{bmatrix}p_{\mathrm{n}}^{\ell+1}\\ p_{\mathrm{f}}^{\ell+1}\\ \sigma^{\ell+1}\end{bmatrix} \ge 0$$

$n_c\,(n_d+2)$ complimentarity conditions

THANK YOU!