Lab 21 (2004-12-02)

1. **Substitutions**

   Evaluate the following substitutions based on the two rules of substitution outlined in class:

   - $\{u/x\}(\lambda y.x)$
   - $\{u/x\}(\lambda u.x)$
   - $\{u/x\}(\lambda u.u)$
   - $\{u/x\}(\lambda y.u)$

2. **Reduction Sequences**

   This question is for practise on reduction sequences. It should help you to check your understanding of reduction sequences, as well as the syntax and strategies involved.

   - Using the α and β reductions discussed in the lecture, reduce the following expression:

     $T = (\lambda xyz.xz(yz))(\lambda x.x)(\lambda x.x)$

     The first step should be to identify *redexes* i.e. terms of the form $(\lambda x.M)N$. Remember that in the presence of several redexes in an expression, a reduction strategy guides you on which one to pick first. Apply both *call-by-name* and *call-by-value* strategies here and verify that the result is the same, hence upholding the Church-Rosser theorem.

   - An interesting case is the reduction of (Yf) where

     $Y = (\lambda f.(\lambda x.f(xx))(\lambda x.f(xx)))$

     A term Y of this form is known as a *fixed-point combinator* and is used to set up recursions in λ calculus. A detailed discussion on fixed-point combinators is beyond the scope of this exercise, nevertheless the reduction of (Yf) should help illustrate the concept to some extent. Prove the result
     $Yf = f(Yf)$