CSCI 2200 - Spring 2015

Exam 1

Name: _____

Instructions:

- Write your name on the **front and back** of this cover sheet and **sign the bottom of this page**.
- You have **100 minutes** to complete this exam. The exam is worth a total of 100 points.
- Put away laptop computers and other electronic devices. Calculators are NOT allowed. Cheating on an exam will result in an **immediate F** for the entire course.
- A one page, two-sided crib sheet is allowed. Rulers are also allowed.
- **Write your answers clearly and completely.**
- Logical equivalences and rules of inference are given on the last page.
- Please read each question carefully several times before beginning to work and especially before asking questions. We generally will not answer questions except when there is a glaring mistake or ambiguity in the statement of a question.

I will not discuss the contents of this exam in the presence of anyone who has not yet taken the exam.

Signature: _____ **Date:** _____

Name: _____

Problem 1 (12)	
Problem 2 (16)	
Problem 3 (7)	
Problem 4 (10)	
Problem 5 (20)	
Problem 6 (10)	
Problem 7 (15)	
Problem 8 (10)	
Total (100)	

1. (6+6=12 points)

   (a) Using propositional logic, write a statement that contains the propositions p, q, and r that is true only when exactly one of p, q, and r is false. Your statement should only use the logical connectives $\neg$, $\vee$ and $\wedge$.

   (b) Give a proposition that is equivalent to $(\neg p \wedge \neg q) \vee r$ that only uses the propositions p, q, and r, and the logical connectives $\neg$ and $\rightarrow$.

2. (4×4=16 points) Consider the following propositional functions:

$S(x, y)$: "x saw y."
$L(x, y)$: "x likes y."
$A(y)$: "y is an award winning movie."
$C(y)$: "y is a comedy."

Let the domain for the x be all people and the domain for y be all movies. Translate the following statements into predicate logic.

(a) There is an award-winning movie that is not a comedy.

(b) Someone has seen every award-winning comedy.

(c) No one liked every movie he has seen.

(d) Taz only likes movies that win awards.

3. (7 points) Determine whether the following argument is valid and prove your answer.

$$\begin{array}{ll} & p \to (q \vee r) \\ & q \wedge \neg r \\ \hline \therefore & p \end{array}$$

4. (10 points) Prove that the square root of any irrational number is irrational.

5. (20 points) Let n be an integer between 0 and 99. Note that $n = 10 \cdot a + b$, for some integers a and b, with $0 \leq a \leq 9$ and $0 \leq b \leq 9$. The integers a and b are the digits of n.
Prove that n is divisible by 3 if and only if the sum of its digits is divisible by 3.

6. (15 points) Prove that for all real numbers x and y, $|x - y| \geq |x| - |y|$.

7. (10 points) Give a proof by contradiction that for all positive real numbers x, $\frac{x}{(x+1)} < \frac{(x+1)}{(x+2)}$.

8. (3+3+4=10 points)

(a) Let A, B, and C be sets. According to the inclusion-exclusion principle, what is $|A \cup B \cup C|$?

(b) Let $A = \{0, 1, 2, 3, 4, 5\}$ and let $B = \{a, b, c\}$. What is $|\mathcal{P}(B) \times A|$?

(c) Disprove the following claim: for all sets X and Y, $\mathcal{P}(X \cup Y) \subseteq \mathcal{P}(X) \cup \mathcal{P}(Y)$.

[SCRATCH PAPER]