The Two Generals Problem

J. Gray,
Notes on Database Operating Systems,
In: *Operating Systems: an Advanced Course*,
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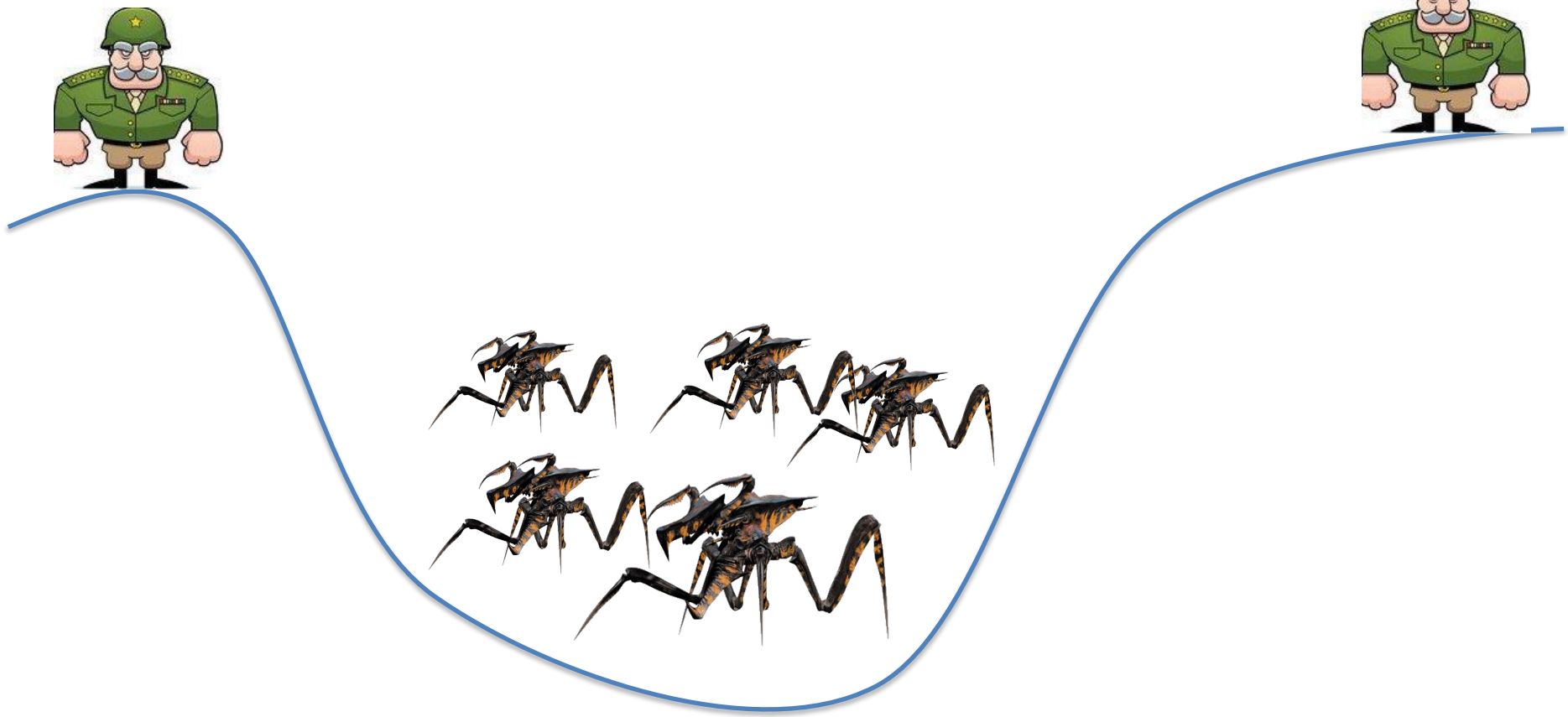
Based on Proof by E. A. Akkoyunlu, K. Ekanadham, and R. V. Huber, 1975

The Two Generals Problem

General A

General B



The N-Generals Problem
(aka: the Coordinated Attack Problem)

- Every process has an input in {0,1}
  - 1 is attack (commit)
  - 0 is don't attack (abort)

- Goal is for all processes to eventually decide 0 or 1
  - Once a process makes a decision, its decision cannot be changed

System Model

- N process, connected network
  - Each process knows the entire network graph
- There are **no process failures**
- Messaging is synchronous
- Messaging is not reliable (**messages may be lost**)

Algorithm Execution

(Abstraction of Algorithm for Synchronous System)

- Initially:
  - Each process in arbitrary state
  - All channels are empty

- Processes repeatedly perform following, in lock-step:
  1. Receive and process messages from incoming channels (if any), and update state
  2. Send messages on outgoing channels (optionally)

  * Messages may be lost
- These two steps are called a **round**

Correctness Conditions

A solution (algorithm) must satisfy the following:

- **Agreement**: No two processes decide on different values
- **Validity**:
    - 1. If all processes have input 0, then 0 is the only possible decision value
    - 2. if all processes have input 1, and all messages are delivered, then 1 is the only possible decision value
- **Termination**: All processes eventually decide

Impossibility Result

Theorem: There is no algorithm that solves the two generals problem.

There is no algorithm that guarantees agreement, validity, and termination.

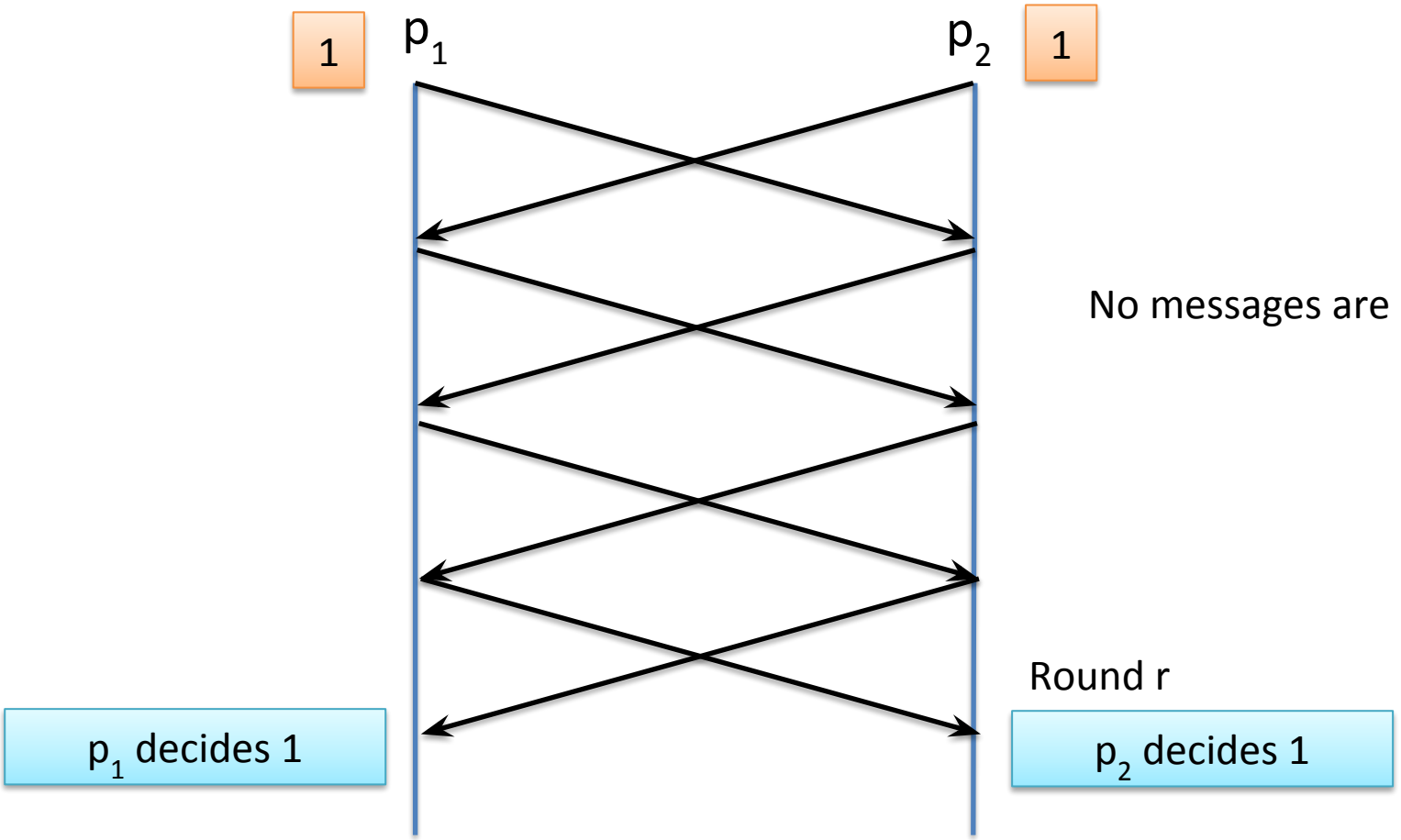
Proof (for N=2)

- Proof by contradiction

- Assume such an algorithm exists.
  - Agreement: p_1 and p_2 decide on same value.
  - Validity: If p_1 and p_2 both have input 0, they decide 0.
    If they both have input 1, and no messages are lost, they decide 1.
  - Termination: Both p_1 and p_2 eventually decide.

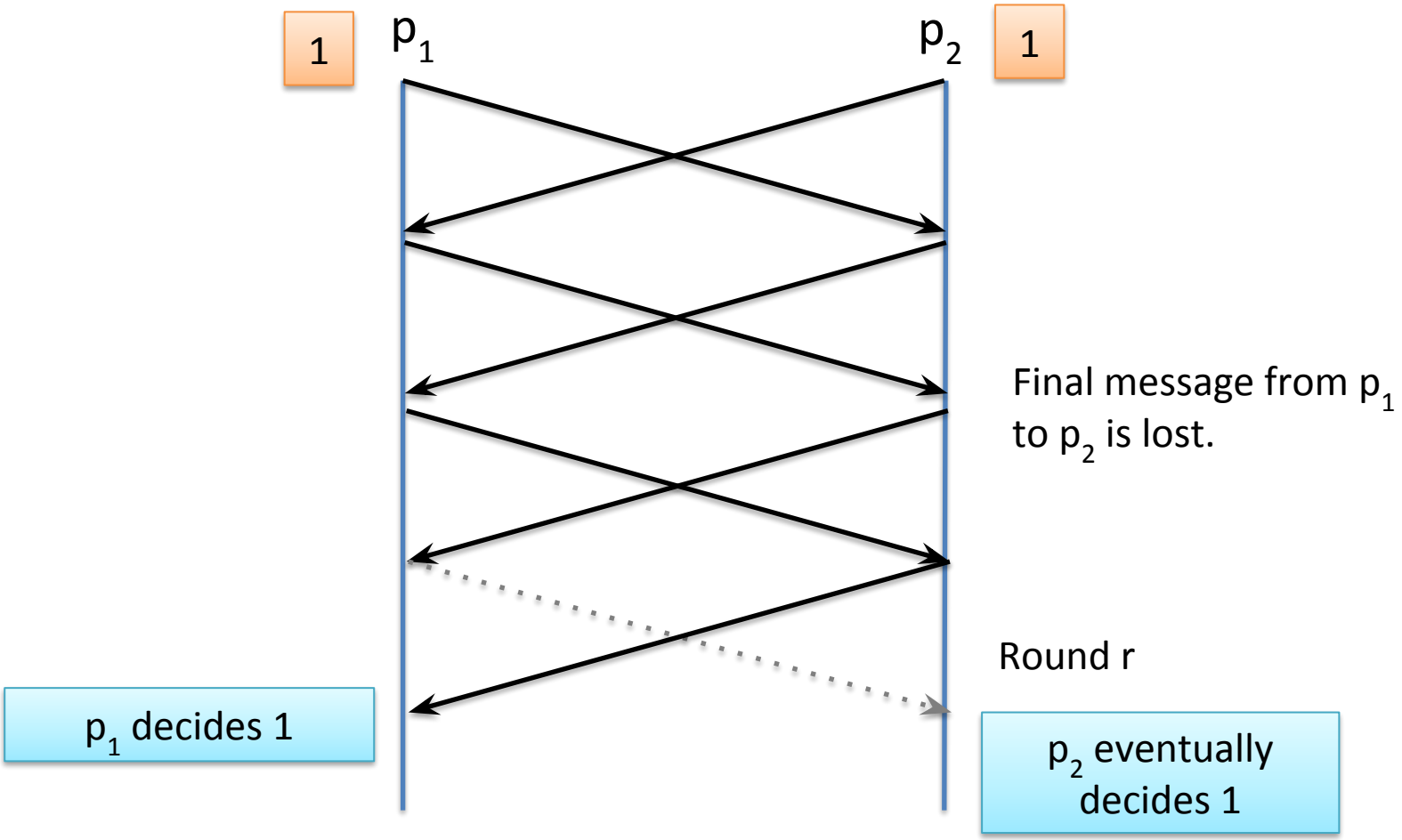
- Suppose the algorithm takes **r rounds to decide**.
  - w.l.o.g. p_1 and p_2 both send a message in each round.

Execution A



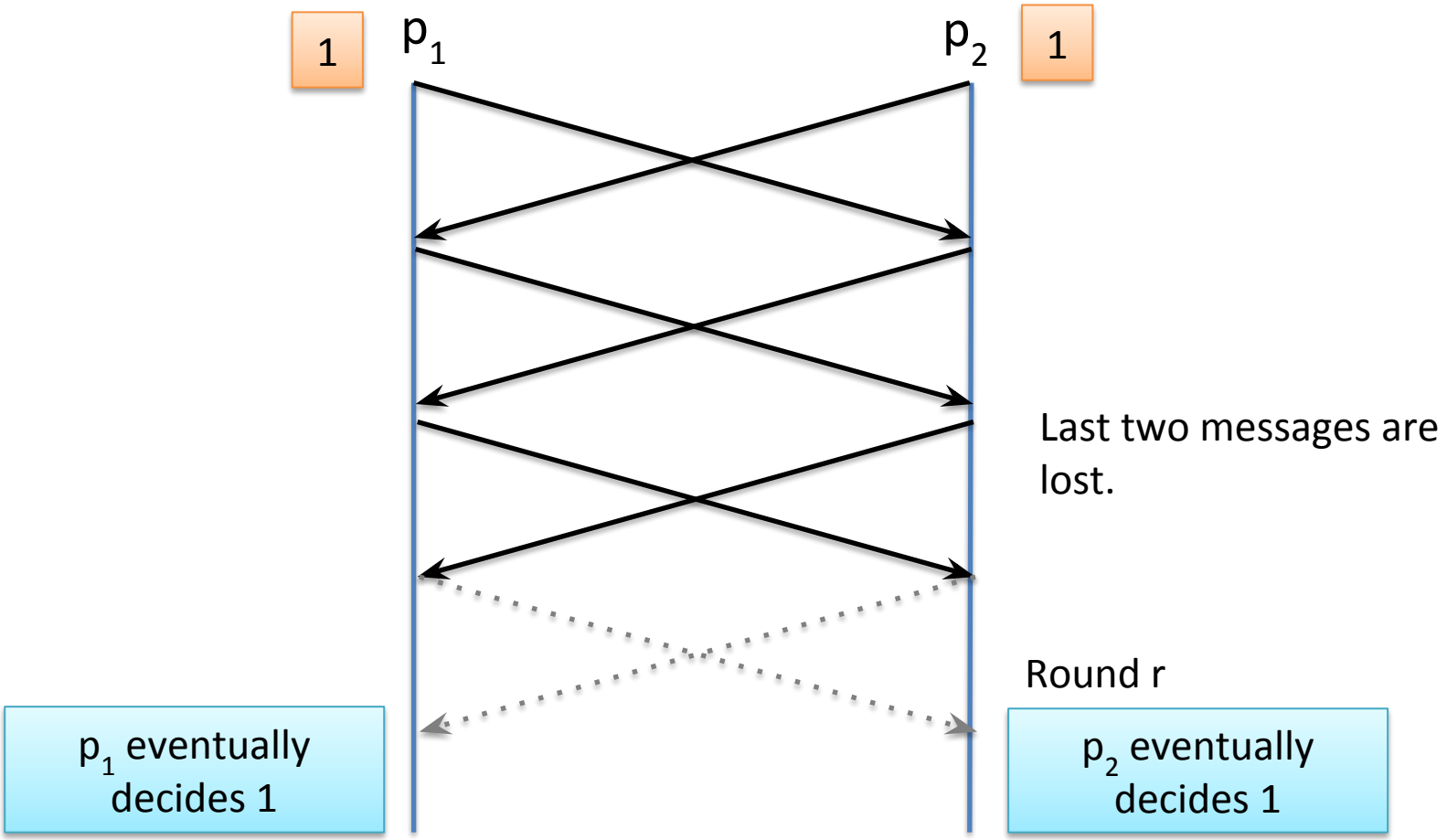
- By the termination requirement, both processes decide.
- By the validity requirement, both decide 1.

Execution B



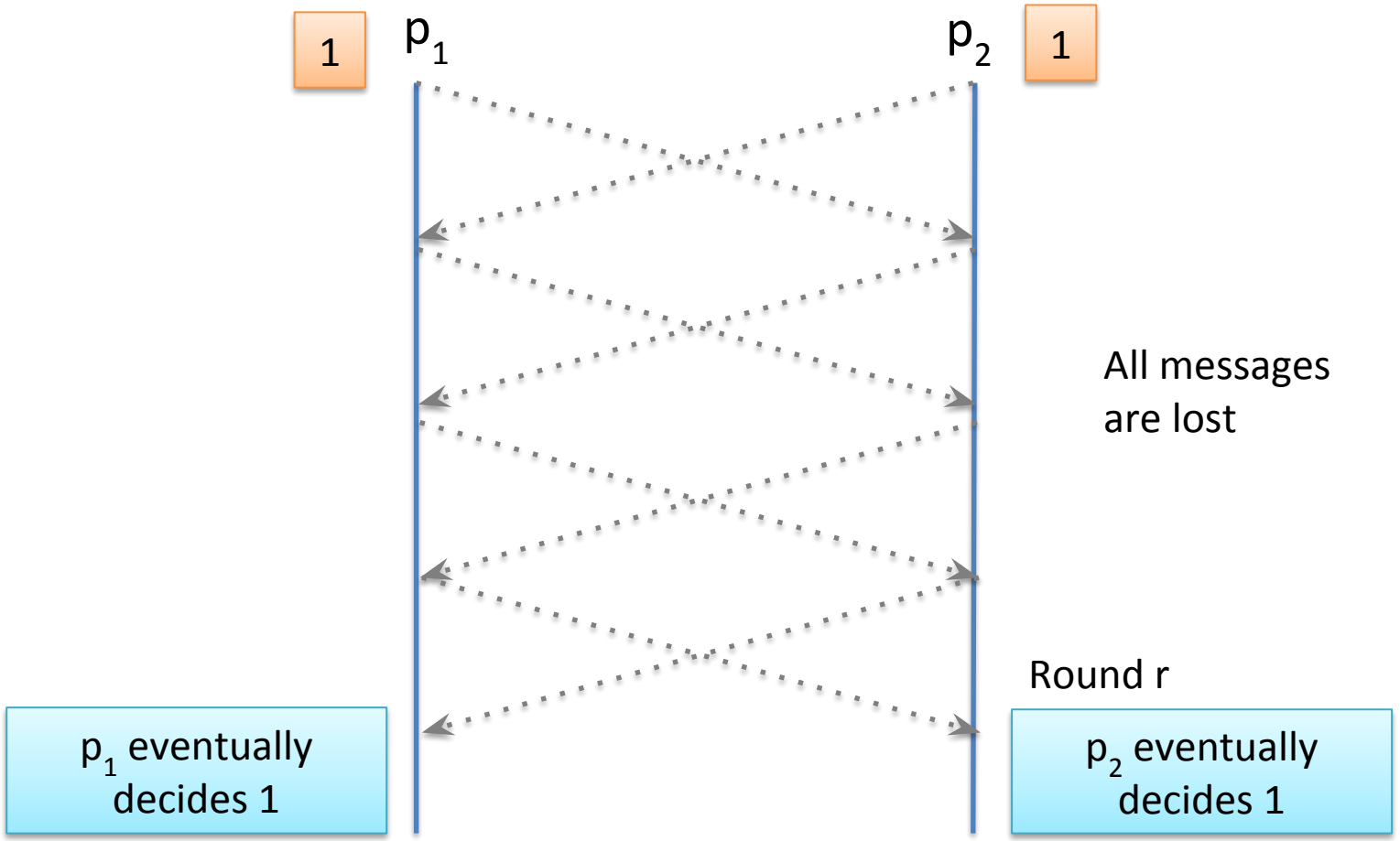
- Execution B looks exactly like Execution A to p_1
- Therefore, p_1 decides 1
- By termination and agreement, p_2 must also decide 1

Execution C



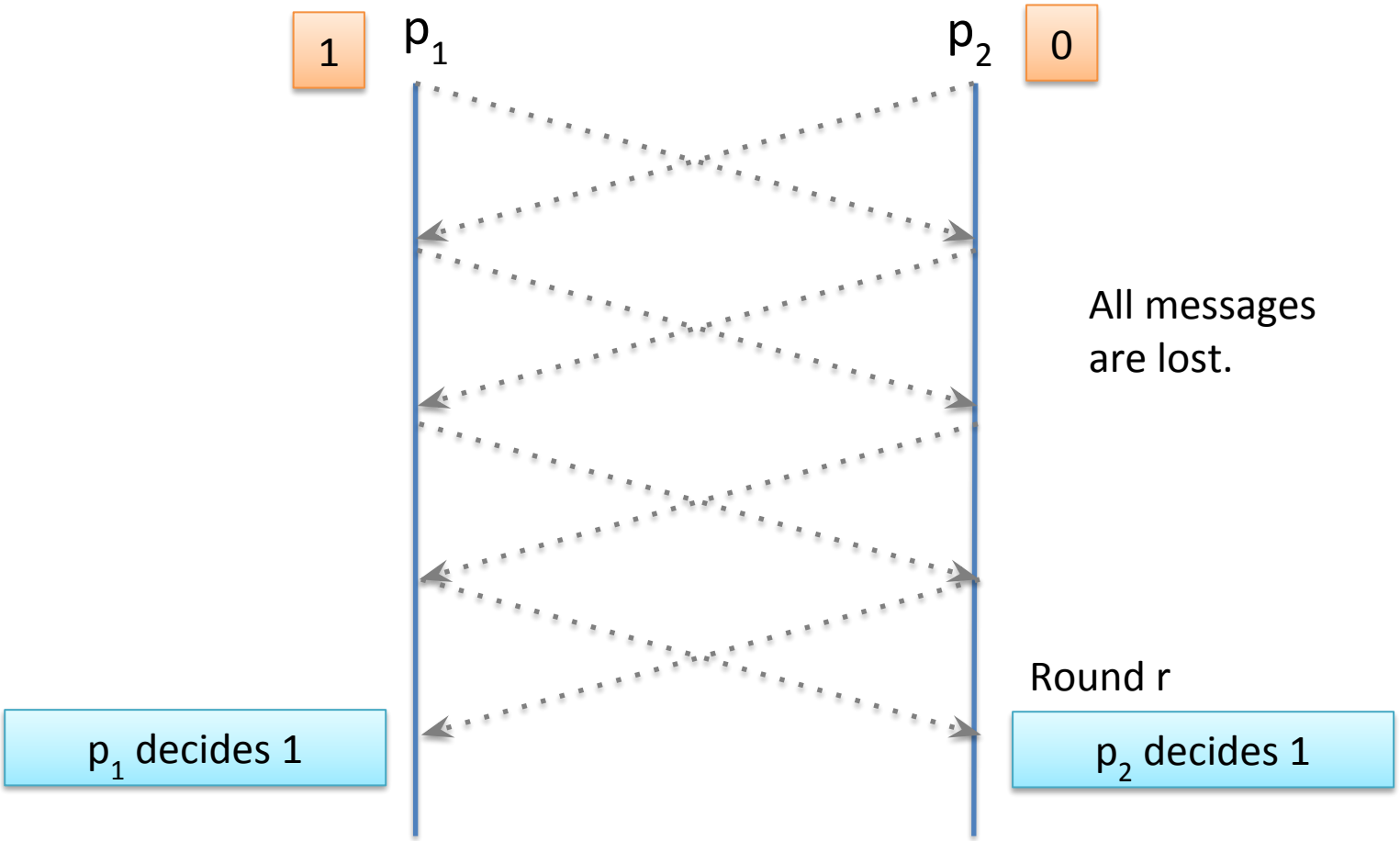
- Execution C looks exactly like Execution B to p_2
- Therefore, p_2 decides 1
- By termination and agreement, p_1 must also decide 1

Execution X



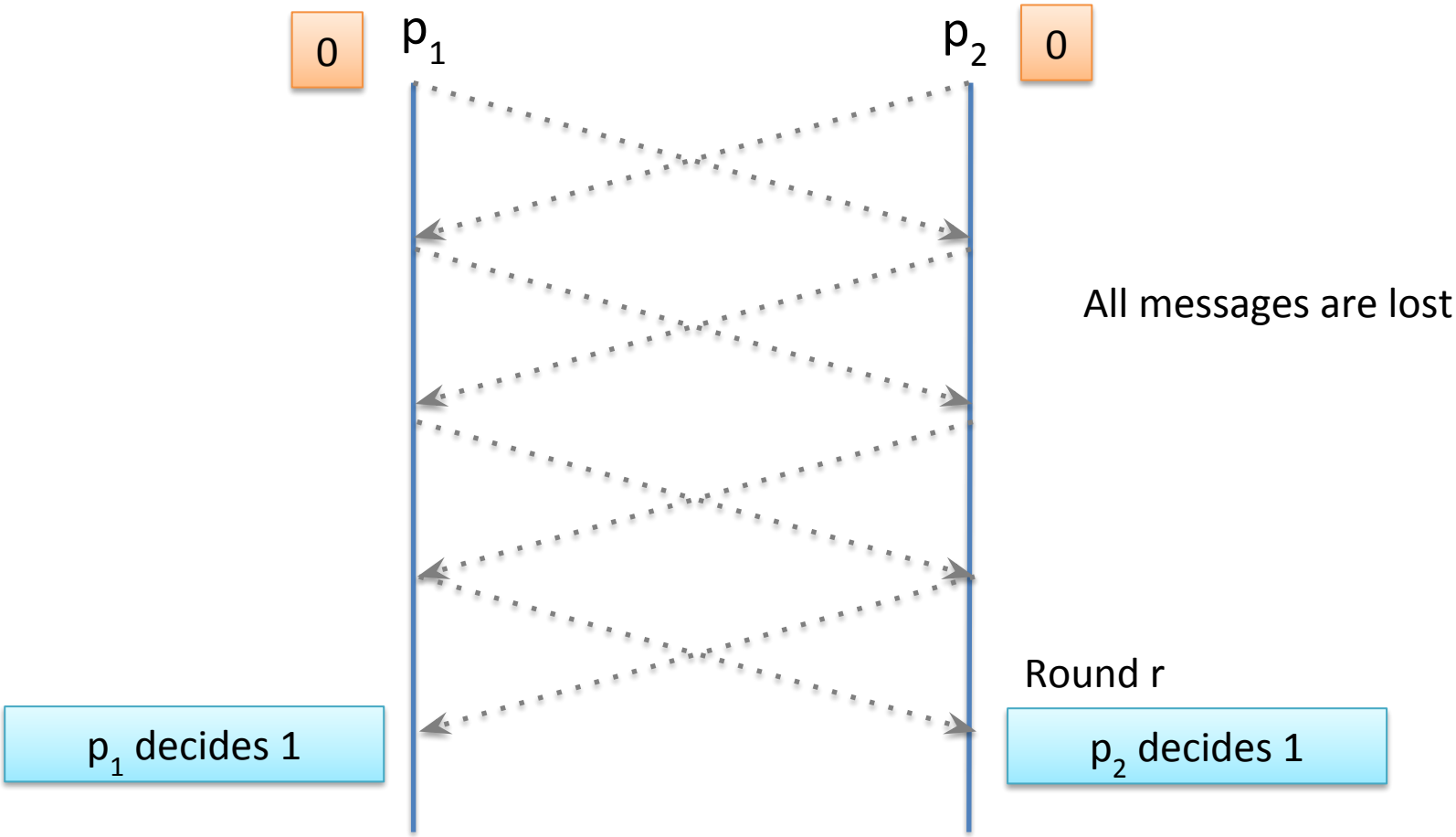
- Following this reasoning, we reach an execution where **no messages are delivered** and **both processes must decide 1**

Execution Y



- Execution Y looks exactly like execution X to p_1
- Therefore, p_1 decides 1
- By termination and agreement, p_2 must also decide 1

Execution Z



- Execution Z looks exactly like execution Y to p_2
- Therefore, p_2 decides 1
- By termination and agreement, p_1 must also decide 1
- This violates validity – both start with 0 and must decide 0

No Coordinated Attack?

- There is no algorithm that solves the coordinated attack problem in the present of message loss
  - Holds for N > 2 as well

- But we do need to solve this problem in real-world systems:
  - Must strengthen the model and/or relax the requirements

- Can make assumptions on probability of message loss:
  - There will be some chance of violating validity or agreement
- Can let processes use randomization:
  - There will be some chance of violating validity or agreement