Reducing $A$ to $MPCP$

Let $A = \{ \langle M, w \rangle \mid M$ is a Turing machine and $M$ accepts $w \}$.

Let $MPCP = \{ \langle P \rangle \mid P$ is an instance of the Post Correspondence Problem with a match that starts with the first domino $\}$.

We describe a function $f$ that given $\langle M, w \rangle$, generates an instance $P$ such that $\langle M, w \rangle \in A$ if and only if $P \in MPCP$.

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$. The function $f$ constructs $P$ as follows:

1. Put $\[#q_0w_1w_2\cdots w_n#\]$ into $P$ as first domino.
2. For every $a, b \in \Gamma$ and every $q, r \in Q$ where $q \neq q_{reject}$:
   - if $\delta(q, a) = (r, b, R)$ put $[\frac{qa}{by}]$ in $P$.
3. For every $a, b, c \in \Gamma$ and every $q, r \in Q$ where $q \neq q_{reject}$:
   - if $\delta(q, a) = (r, b, L)$ put $[\frac{cpa}{rcb}]$ into $P$.
4. For every $a \in \Gamma$:
   - put $[\frac{a}{a}]$ into $P$.
5. Put $[\#]$ and $[\#]$ into $P$.
6. For every $a \in \Gamma$:
   - put $[\frac{aq_{accept}}{q_{accept}a}]$ and $[\frac{q_{accept}a}{q_{accept}c}]$ into $P$.
7. Add $[\frac{q_{accept}##}{#}]$ to $P$. 