

Reducing A to $MPCP$

Let $A = \{\langle M, w \rangle \mid M \text{ is a Turing machine and } M \text{ accepts } w\}$.

Let $MPCP = \{\langle P \rangle \mid P \text{ is an instance of the Post Correspondence Problem with a match that starts with the first domino}\}$.

We describe a function f that given $\langle M, w \rangle$, generates an instance P such that $\langle M, w \rangle \in A$ if and only if $P \in MPCP$.

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$. The function f constructs P as follows:

1. Put $\left[\frac{\#}{\#q_0w_1w_2 \cdots w_n\#} \right]$ into P as first domino.
2. For every $a, b \in \Gamma$ and every $q, r \in Q$ where $q \neq q_{reject}$:
if $\delta(q, a) = (r, b, R)$ put $\left[\frac{qa}{br} \right]$ in P .
3. For every $a, b, c \in \Gamma$ and every $q, r \in Q$ where $q \neq q_{reject}$:
if $\delta(q, a) = (r, b, L)$ put $\left[\frac{cqa}{rcb} \right]$ into P .
4. For every $a \in \Gamma$:
put $\left[\frac{a}{a} \right]$ into P .
5. Put $\left[\frac{\#}{\#} \right]$ and $\left[\frac{\#}{\neg\#} \right]$ into P .
6. For every $a \in \Gamma$:
put $\left[\frac{aq_{accept}}{q_{accept}} \right]$ and $\left[\frac{q_{accept}a}{q_{accept}} \right]$ into P .
7. Add $\left[\frac{q_{accept}\#\#}{\#} \right]$ to P .