Homework 2
CSCI-4961/6961: 3D Computer Graphics
Fall 2006

Due: Monday, September 25, 2006

Homeworks are due at the **beginning** of lecture on Monday, September 25. **Late homeworks will receive no credit.** Homeworks are to be done individually and will be graded on the basis of correctness, clarity, and legibility. Show the steps in your work where appropriate. Each question is worth **10 points**, for a total of **50 points**.

Be sure to write your **name** and **RPI email address** on your homework submission.

1. (a) Compare the Digital Differential Analyzer (DDA) and Bresenham line drawing algorithms. What are the advantages of the Bresenham algorithm?
   (b) Consider a transformation that maps a square into a trapezoid. Is this an affine transformation? Justify your answer.
   (c) Suppose that a triangle is clipped to a rectangular window. After being clipped against the window, what is the maximum number of sides that the resulting clipped polygon might have? Draw an example to illustrate.

2. Suppose you are given an orthonormal frame F located at (f_x, f_y, f_z) and whose unit vectors are (u_x, u_y, u_z), (v_x, v_y, v_z), (n_x, n_y, n_z), expressed in the world coordinate frame W.
   (a) Write the 4×4 matrix that transforms points from the world coordinate frame W to the coordinate frame F.
   (b) Write the 4×4 matrix that transforms points from the frame F to the world coordinate frame W.
   (c) What are the coordinates of the origin of the world coordinate frame W in the frame F?

3. (a) Compare the relative advantages of orthographic and perspective projections.
   (b) What are the advantages of using perspective projection matrices of the form used by `glFrustum()` and `gluPerspective()` rather than the simple perspective projection matrix?
   (c) Consider a point located at (0, b, -far) in the view volume specified by the perspective transformation `glFrustum(-a, a, -b, b, near, far)`. Compute the corresponding coordinates of this point in the canonical view volume, which is the unit cube centered at the origin with sides of length 2 units. (Note: The perspective transformation matrix corresponding to `glFrustum()` is discussed in the class lecture notes and the OpenGL Blue book.)

4. Commencing from the position shown in Figure 1, a rectangular box is rotated about edge AB by 40° clockwise as viewed from A to B. Then the box is rotated about edge AC by 80° counterclockwise as viewed from A to C. Note that corner A is initially at the origin of the fixed XYZ frame. **Important:** Show the steps in your work to receive credit.

   (a) Determine the individual transformations and the composite transformation to accomplish this sequence of rotations.

   (b) Determine the location of corner D (in the fixed world frame) due to these rotations.

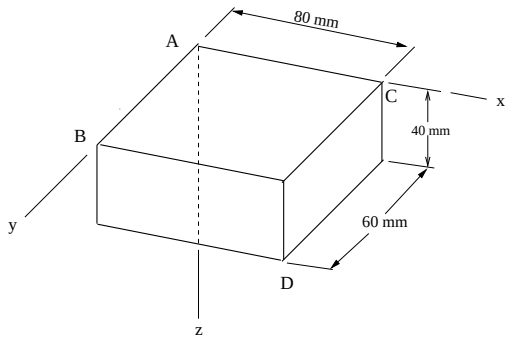

Figure 1: Initial configuration of box.

5. (a) What is the relationship between the quaternion representing a rotation of θ about the unit vector $\hat{u}$ and the quaternion representing a rotation of $-\theta$ about the unit vector $-\hat{u}$? Do they represent the same rotation or not? Justify your answer.

   (b) Starting from the position shown in Figure 2, the box is rotated by 40° about face diagonal AB, clockwise as viewed from corner B to corner A.

      i. Write the unit quaternion that represents this rotation.

      ii. Use the quaternion representation to determine the coordinates of corner C relative to the fixed reference frame XYZ after this rotation.

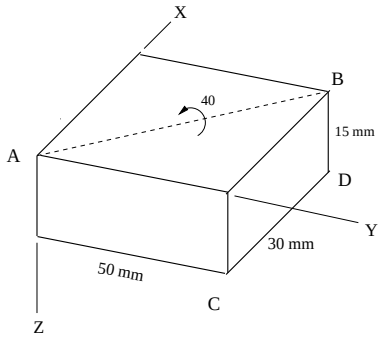

Figure 2: Initial configuration of box.