Homework 4
CSCI-4962: Three-Dimensional Computer Graphics
Fall 2002

Due: Thursday, November 14, 2002

Homeworks are due at the **beginning** of lecture on Thursday, November 14. **Late homeworks will receive no credit.** Homeworks are to be done individually and will be graded on the basis of correctness, clarity, and legibility. Show the steps in your work where appropriate. Each question is worth **10 points**, for a total of **50 points**.

Be sure to write your **name**, **section number**, and **RPI email address** on your homework submission.

1. Given an array $\mathbf{p}_{j,k}$ of control points, $0 \leq j \leq m$, $0 \leq k \leq n$, the Bezier surface patch is given by

$$\mathbf{P}(u,v) = \sum_{j=0}^{m} \sum_{k=0}^{n} \mathbf{p}_{j,k} Bez_{j,m}(u) Bez_{k,n}(v)$$

   for $0 \leq u, v \leq 1$, where $Bez_{j,m}(u)$ and $Bez_{k,n}(v)$ denote Bezier blending functions of degree m and n respectively. Consider a bicubic Bezier surface patch, that is, a patch with $m = n = 3$.

   (a) Given any fixed u_0, $0 \leq u_0 \leq 1$, define the u_0-slice to be the curve $\mathbf{c}(v) = \mathbf{P}(u_0, v)$. Show that $\mathbf{c}(v)$ is a Bezier curve of degree 3.

   (b) What are the four control points for the curve $\mathbf{c}(v)$?

   (c) What is the normal vector to the Bezier surface at $\mathbf{P}(u = 0, v = 0)$?

2. (a) What are the advantages of using a spline based representation of objects such as the Utah teapot instead of a polygonal mesh representation?

   (b) You want to know whether a given point $P = (x, y, z)$ lies on a surface. With which surface representation is it easier to answer this query: parametric or implicit? Justify your answer.

   (c) Give an implicit surface representation of a (right circular) cone with its vertex at the origin and with its central axis along the positive Z axis. That is, give an implicit function $f(x, y, z) = 0$ that describes the surface of the cone. Assume the cone makes an angle of α with the Z axis.

3. (a) Consider drawing an ellipsoid using OpenGL. Write a snippet of code to draw an axis-aligned ellipsoid centered at (1, 3, 5) such that its axis parallel to the X axis has length 2 units, its axis parallel to the Y axis has length 8 units, and its axis parallel to the Z axis has length 6 units. After completion, the modelview matrix should be unchanged from its original value.

   (b) Write the parametric representation of this ellipsoid, indicating the ranges of the parameters.

4. Consider mapping a texture onto a cylinder. The cylinder is aligned with the Z axis and has a radius of r units and a height of h units with its base at $z = 0$. You are given a texture map whose s coordinate ranges from 0 to 1, and whose t coordinate ranges from 0 to 1. You are to wrap three copies of this texture around the cylinder (so they are not overlapping and their vertical join lines are parallel to the Z axis). Assume the first texture copy starts at the positive X axis. Give the inverse mapping function, which maps a point (x, y, z) on the cylinder to the corresponding point (s, t) on the texture.

5. (a) Texture mapping significantly increases the memory bandwidth required per rendered pixel. Consider a simplified model of the memory bandwidth required due to texture reads when we perform 2D texture mapping with multitexturing. Consider a display with 1280×1024 pixels, and assume each pixel has two textures applied to it, where each texel is represented as a 32 bit RGBA value. Additionally assume that bilinear interpolation is performed at each pixel (that is, a linear average of the 2×2 array of texels nearest the center of the pixel must be computed). Finally, assume an average depth complexity of 3. (The depth complexity refers to the number of times each pixel is rendered, and depends on the number of surfaces that cover each pixel).

   What is the average memory transfer required per frame for the texture reads (in Mbytes)? At 60 frames per second, what is the memory bandwidth required (in Gbytes per second)?

   (Note that practical techniques to reduce the memory bandwidth requirements include the use of texture caches and texture compression.)

   (b) Why is mipmapping used?

   (c) If a texture map is stored as a 512×512 image with 24-bit color per texel, what is the additional storage required to store all mipmap levels down to the smallest 1×1 texel image?