Assignment 1
CSCI-4290/6290: Robot Motion Planning

Due: Friday, September 9, 2005

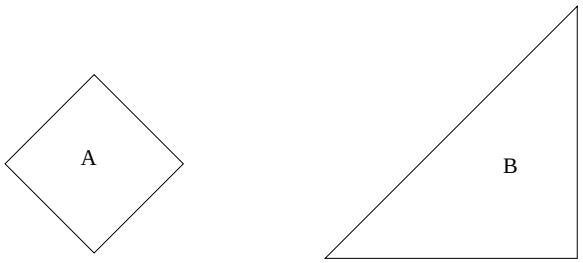
Questions 1–4 are due on September 9, and Questions 5–6 are due on September 16. Assignments are due at the beginning of class on the due date, and are to be done individually. Assignments will be graded on the basis of correctness, clarity, and legibility. See course syllabus for late submission policy.

1. Consider a polygon P in the plane, whose vertices in counterclockwise order have coordinates (0,4), (8, 12), (0, 10), and (-8, 12). Write a representation of the polygon P as a union of halfplane intersections.

2. Consider a convex polyhedron with vertices located at (0, 0, 0), (1, 0, 0), (0, 1, 0), and (0, 0, 1). Compute the equations of the planes that lie along the faces of this polyhedron, and represent the polyhedron as an intersection of halfspaces.

3. Consider a workspace with a rectangular obstacle, centered at the origin, with sides of length $2a$ and $2b$. Assume $a > b$ and that the longer edge is parallel to the X axis. Given a circular mobile robot of radius r, write a semi-algebraic representation of the resulting configuration space obstacle. Assume the reference point of the robot is at its center and that the robot can only translate.

4. You are given two convex polygons A and B in the plane, as shown in the figure. A is a square of side 8 units that has been rotated by 45 degrees. B is a right isosceles triangle where the two equal sides have length 16 units.



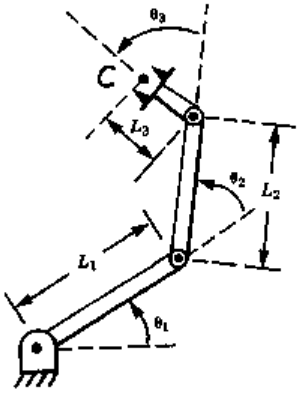
   a. Draw the configuration space obstacle when A is a robot that can only translate in the plane and B is a fixed obstacle in the workspace.

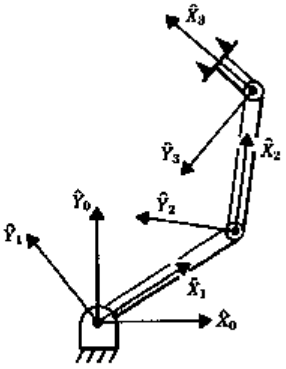
   b. If the polygon B is the robot and polygon A is the fixed obstacle, will the shape of the resulting configuration space obstacle be different (from that in part (a))?

5. Consider the manipulator with 3 revolute joints in the figure. Assume $L_1 = L_2 = 10$ units and $L_3 = 5$ units. Assume the positive Z axis points out of page, and that link A_2 is offset 10 units along the positive Z axis from link A_1, and that link A_3 is offset 10 units along the positive Z axis from link A_2.

   a. Use the Denavit-Hartenberg parameters to compute the homogeneous transformation matrix for points on link A_3.

   b. What are the coordinates of the point at the center of the gripper in the $X_0Y_0Z_0$ frame when $\theta_1 = 60$ degrees, $\theta_2 = 0$ degrees, and $\theta_3 = 30$ degrees?





6. **a.** Define the unit quaternion q_1 that represents a rotation of -90 degrees about the axis specified by the vector $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

   b. Define the unit quaternion q_2 that represents a rotation of 180 degrees about the axis specified by the vector $(0, 1, 0)$.

   c. Suppose the rotation represented by q_1 is performed, followed by the rotation represented by q_2. This combination of rotations can be represented as a single rotation around an axis given by a vector. Find this axis and the angle of rotation about this axis.