## Formulas

These formulas are provided for your reference. You may or may not find them necessary.

- $T_i$ is the $4 \times 4$ homogeneous transformation matrix specified using DH parameters that transforms points and vectors defined in frame $i$ to their description in frame $i-1$.

$$T_i = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & a_{i-1} \\ \sin(\theta_i)\cos(\alpha_{i-1}) & \cos(\theta_i)\cos(\alpha_{i-1}) & -\sin(\alpha_{i-1}) & -\sin(\alpha_{i-1})d_i \\ \sin(\theta_i)\sin(\alpha_{i-1}) & \cos(\theta_i)\sin(\alpha_{i-1}) & \cos(\alpha_{i-1}) & \cos(\alpha_{i-1})d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- A quaternion $q = s + ai + bj + ck$ can be represented as $q = (s, \vec{v})$ where $s$ is the scalar part and $\vec{v}$ is the vector $(a,\ b,\ c)$.

- The product $q_1q_2$ of two quaternions $q_1$ and $q_2$ where $q_i = (s_i, \vec{v_i})$ is given by:

$$q_1q_2 = (s_1s_2 - \vec{v_1} \cdot \vec{v_2},\ s_1\vec{v_2} + s_2\vec{v_1} + \vec{v_1} \times \vec{v_2})$$

- If $q = (s, v)$ is the unit quaternion corresponding to a rotation applied to a point $p$, the rotated point $p^{'}$ is given by

$$p^{'} = s\ ^2p + v(p.v) + 2s(v \times p) + v \times (v \times p)$$

- $(\sin(\theta))^2 + (\cos(\theta))^2 = 1$

- For $\theta$ expressed in degrees,
  $\sin(0) = 0$, $\cos(0) = 1$
  $\sin(90) = 1$, $\cos(90) = 0$
  $\sin(180) = 0$, $\cos(180) = -1$