Foundations of Trust and Distrust in Networks: Extended Structural Balance Theory [1]

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Modeling trust in very large social networks is a hard problem due to the highly noisy nature of these networks that span trust relationships from many different contexts, based on judgments of reliability, dependability and competence. Furthermore, relationships in these networks vary in their level of strength. In this paper, we introduce a novel extension of structural balance theory as a foundational theory of trust and distrust in networks. Our theory preserves the distinctions between trust and distrust as suggested in the literature, but also incorporates the notion of relationship strength which can be expressed as either discrete categorical values, as pairwise comparisons or as metric distances. Our model is novel, has sound social and psychological basis, and captures the classical balance theory as a special case. We then propose a convergence model, describing how an imbalanced network evolves towards new balance and formulate the convergence problem of a social network as a Metric Multidimensional Scaling (MDS) optimization problem. Finally, we show how the convergence model can be used to predict edge signs in social networks, and justify our theory through extensive experiments on real datasets.

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1. INTRODUCTION

Modeling trust in very large social networks is a hard problem due to the highly noisy nature of these networks. These networks span trust relationships from many different contexts, based on judgments of reliability, dependability and competence [Adalı 2013]. Furthermore, trust relationships vary in their level of strength as participants may or may not know each

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other well. These relationships change over time and only a subset of the existing relationships are declared explicitly in a given dataset. One specific problem of interest is inferring new relationships and making recommendations based on existing relationships. To this date, many algorithms for such prediction problems are based on machine learning methods. These methods generate multiple structural features from the known relationships, and then use these features to classify the unknown relationships [DuBois et al. 2011][Guha et al. 2004][Leskovec et al. 2010]. In particular, the concept of structural balance is widely applied when developing these algorithms. For example, Leskovec et al. generate a class of "triad features" in their prediction algorithm, i.e. a pair of relationships constraining a third relationship [Leskovec et al. 2010]. They also use the structural balance as a touchstone to see the congruence between their practical results and the long studied theory. In essence, the transitivity of trust and distrust within a triad can be easily inferred from the balance theory. Other algorithms [DuBois et al. 2011] make use of similar assumptions without explicit mention of balance theory in which trust and distrust relationships are mapped to metric distances on a continuous range. While these algorithms show strong prediction performance, there is no underlying theory that explains how structural balance can be mapped to metric distances. That is, a theory that stands as the cornerstone of trust prediction is missing. Without such a theory, it is not possible to develop principled algorithms and study to which degree a specific network conforms to the theory, and to illustrate the importance of various balance related properties of networks. Furthermore, in today's networks in which individuals interact with a large number of others, one expects that the relationships and their trust level will vary considerably. As a result, it is not clear how Cartwright-Harary's balance theory [Cartwright and Harary 1956] can be applied to large scale social networks in which nodes may have different types of trust and distrust relationships, ranging from strong to weak.

In this paper, we address all these concerns and make the following unique contributions.

— We introduce a new extended balance theory that allows arbitrary relationship strengths. We express balance with two simple principles that preserve the meanings of trust and distrust: positive and negative trust relationships. We show how balance can be modeled when the strength of relationships are represented with either discrete categorical values, pairwise comparisons or metric distances using the same principles. Our novel method has sound social and psychological basis, and captures the classical balance theory as a special case.
— We define the convergence model that describes precisely how an imbalanced network evolves towards new balance. The assumption behind our model is that in resolving tensions within imbalanced relationships, people tend to avoid the effort of changing relationships if possible. The introduction of extended balance theory allows us to formulate the convergence problem of a social network as a Metric Multidimensional Scaling (MDS) optimization problem. We show how the convergence model can be used to predict edge signs in social networks by applying stress majorization [Gansner et al. 2005]. We introduce an efficient algorithm for sparse networks that can be applied to large networks.
— We illustrate the convergence theory and our algorithmic solution on stylized networks. We show that neutral edges may become positive or negative edges based on the triads the edge is part of. Then, we show that the theory captures trust transitivity along positive paths naturally. The influence of a positive edge attenuates along long paths, even though all the edges overall remain positive. Finally, we show that as a node is part of more triads, the more it is effected by the relationships. All these results illustrate that our solutions follow the principles introduced by our theory.
— We justify our theory through an extensive experimental study on three real datasets. Our method consistently outperforms the state of the art. In particular, improvement of our method over the state of the art is even more significant in networks with a larger

proportion of negative edges. In such networks, it becomes increasinly crucial to capture the true meaning of negative edges. We show that we can achieve far superior prediction for strong trust edges, represented as bi-directional edges with the same sign. We illustrate the validity of our method with external data. We show that strong edges correspond to different rating behavior in Epinions for both trust and distrust. Additionally, whenever our algorithm predicts a distrust edge will change its sign after convergence, we show that the ratings for those edges actually increase more than average. Our algorithm is able to capture the expected change in ratings due to structural constraints. Finally, we show that our method can provide a promising first step towards solving the harder *link prediction problem* [Liben-Nowell and Kleinberg 2003].

The remainder of this paper is organized as follows. In Section 2, we describe the classical structural balance theory. We summarize the related work in trust prediction and its relation to the balance theory. Then, in Section 3, we describe our theory for structural balance. We show how to incorporate relation strength into the structural balance definition while preserving the basic principles of balance. Section 4 defines the notion of convergence as an optimization problem, how networks evolve towards a balanced state. We show how the converged state of a network can be computed. We then give an efficient algorithm for computing convergence approximately for sparse graphs that disregards neutral edges [2]. Section 5 illustrates convergence with the help of small stylized networks. Section 6 provides the setup of experiments involving three large networks that are used in prior literature in trust prediction. We illustrate different graphs that can be constructed from these networks to test different hypotheses. In Section 7, we provide an extensive experimental study of our algorithm, both comparing it to the state of the art and also illustrating its various properties. Finally, we conclude in Section 8 and discuss future work.

2. STRUCTURAL BALANCE THEORY & RELATED WORK

When modeling relationships between pairs of individuals, positive relationships are representative of liking, loving, valuing or approving someone, and negative relationships are representative of disvaluing, disapproving or negatively valuing someone [Cartwright and Harary 1956]. Suppose we consider trust from the perspective of trustworthiness where one trusts another if they are considered to be truthful, to have integrity and to have positive intentions towards the other. This definition of trustworthiness has a definite affective component that can easily be considered as an extension of the above definition. A positive relationship results in trust because the other person is considered to have positive values from the perspective of the trustor. Similarly, a negative relationship results in distrust because the other person is considered to have specific faults that would prevent them from being trusted [Adalı 2013].

Trust and distrust are not symmetric constructs. A person who is not trusted may eventually become trusted as a result of positive evidence. However, a distrusted person may not be trusted even after many positive experiences. As a result, one has to treat both types of relationships differently. Note that often trust and distrust relationships need not be mutual: A may trust B, but B may distrust A. We will illustrate later with experiments on real networks that it is possible to treat mutual trust relations as strong trust relations, and the rest as weak trust. However, for now we will treat all trust relations as simple undirected relationships.

Structural balance theory (SBT) is based on the assumption that certain types of relationships when viewed from a local perspective are more natural for psychological reasons [Easley and Kleinberg 2010]. The local level is defined as a triangle or triad consisting

[2]The implementation of the sparse graph algorithm used in this paper is available as open source code at *github.com/rpitrust/structuralbalance.*

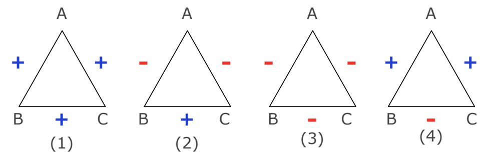


Fig. 1. Classic structural balance with four different types of triads. In strong balance theory, triads 1,2 are balanced and 3,4 are unbalanced. In weak balance theory, triads 1,2 and 3 are balanced and 4 is unbalanced.

of relationships between three people. It is natural for three people to be friends: Alice (A) is friends with Bob (B), Bob is friends with Chris (C) and Chris is friends with Alice. This triangle (marked (1) in Figure 1) is considered completely positive. Similarly, a relation where two friends, Bob and Chris have a common enemy Alice, is also natural (triangle (2) in Figure 1). It is considered that in such natural relationships, there is no tension in the interactions. However, it is less natural for Alice and Bob, and Alice and Chris to be friends, but Bob and Chris to be enemies (triangle (4) in Figure 1). This situation is likely to generate tension as now Alice must avoid spending time with Bob when Chris is around. As a result, all definitions of structural balance would consider triangles 1 and 2 as balanced, and 4 as imbalanced.

The last type of relationship is the one containing all negative edges. This type of relationship can be considered balanced as there is no specific conflict when three people all dislike each other but spend no time together. As a result, in Davis's weak structural balance theory (WSBT), triangle (3) is considered balanced [Easley and Kleinberg 2010][Davis 1967]. However, there is an opportunity for one of the pairs in this triangle to become friends, and team up against the common enemy. For this reason, (strong) SBT, considers triangle (3) unbalanced as well [Cartwright and Harary 1956][Easley and Kleinberg 2010].

A complete network in which all pairs of people are connected to each other satisfies the WSBT if all triangles in it are balanced with respect to WSBT. In this case, the network can be divided into a set of communities $C_1, \ldots, C_k$ such that within each community, nodes are positively connected, and across different communities, all connections are negative. This main result underlies many trust inference algorithms. Algorithms that attempt to solve the *edge sign prediction problem* which involves assessing which links in a network are positive and which links are negative, are often based on SBT/WSBT either implicitly or explicitly.

Guha et. al. [Guha et al. 2004] propose a method that models the propagation of both trust and distrust through various methods: direct propagation, co-citation and backwards propagation. They compute trust propagation by repeating matrix operations that combine the three types of propagations. They report an overall 85% prediction accuracy over data samples from Epinion that has equal number of positive and negative edges. Leskovec, Huttenlocher and Kleinberg [Leskovec et al. 2010] conduct a series of experiments on three large datasets: Epinions, Slashdot and Wikipedia. In particular, they collect two classes of features, one of which is based on degree and the other is based on triads. These relatively local features form a high dimensional space on which they perform standard machine learning methods and perform edge sign predictions. Similar to our work, they interpret some of their results in terms of the classical balance theory [Cartwright and Harary 1956], but unlike our work, they do not use balance theory as a starting point.

The recent work by DuBois et. al. [DuBois et al. 2011] is also related to our paper from an algorithmic point of view. This work stands out as it provides very good prediction performance for the edge sign prediction problem; $80-90\%$ on all of the three datasets used in [Leskovec et al. 2010] for both positive and negative edges. For this reason, this work represents the state of the art in trust prediction. In this work, two features are computed for each signed edge: the first one is based on path probability (PP, $O(n^3)$ in the worst case) [DuBois et al. 2009] and the second one uses a force directed algorithm (FD, $O(kn)$ at each iteration where k is the average degree of the network) [DuBois et al. 2011].

In the method of FD, the authors map trust and distrust relationships to metric distances: the larger the distance is, the more negative (less positive) the relation is. Then, the edge sign prediction problem is mapped to a graph drawing problem.

In this paper, show that some of the assumptions underlying these methods can be formally defined as part of an extended structural balance theory that not only works for simple positive and negative edges, but also takes into account trust strength when applicable. To the best of our knowledge, none of the existing methods provide a principled way to study the principles underlying trust and distrust relationships in very large networks with varying degrees of relationship strengths. It is unclear to which degree SBT or WSBT balance theory is valid for many large networks in which some or most relationships are simple acquaintances [Granovetter 1973] instead of close friendship relationships. An acquaintance may not result in the same type of structural constraints. For example, if Alice knows Bob, and Bob knows Chris, but Chris dislikes Alice, this may not cause much stress in the existing relationships if Alice, Bob and Chris rarely spend time with each other, i.e. their relationship is not strong. However, there are still some implications for the network overall when we consider acquaintances as well as friendships. We examine those in the next sections and provide a flexible theory of balance that generalizes WSBT. Being able to deal with strength also enables us to state the explicit optimization criteria in the metric space for the graph drawing problem. As a result, we are able to compare the prediction performance with respect to an optimal placement of nodes according to our theory. We show our method outperforms the state of the art for different data sets in prediction accuracy, and at the same time can be validated with an external ground truth evaluation. As a result, we not only show that we get better prediction by using structural information, but we also illustrate that our model of convergence based on network structure is meaningful in real networks with respect to the actions of the individuals in those networks.

Notice that both the force directed algorithm (FD) from [DuBois et al. 2011] and the stress majorization algorithm (SM) that we use in this paper have been used in the field of graph drawing [Gansner et al. 2005]. In FD, an attractive force is assigned between endpoints of each positive edge and a repelling force is assigned between endpoints of each negative edge. Nodes are initially randomly laid out, and the system is simulated until a stable equilibrium is reached when the total kinetic energy is below a certain threshold. The relation between every pair of nodes is represented by the distance between the two end nodes in the stable layout of the network.

While the original algorithm in [DuBois et al. 2011] uses a combination of FD and PP (path probability algorithm) that is computationally costly, we give results in this paper that show that FD alone is sufficient for achieving the performance reported in this work. The FD method is simple to implement. It operates on a local pairwise level, instead of a global level. However, this can lead to problems if the local forces end up not being sufficient to hold small groups together. Alternatively, if negative forces are too high, then the network may continuously expand in space and the algorithm may never converge. As a result, FD requires carefully tuned parameters for a specific network. In contrast, SM is a mature approach that guarantees monotonic convergence for drawing graphs. Moreover, in [DuBois et al. 2011], there is no force between pairs of unconnected nodes which can result in unintuitive distances for such pairs. In fact as we show in our results, the FD method maps unconnected nodes to a predominantly positive range. This presents a problem for using this algorithm for solving the *link prediction problem* [Liben-Nowell and Kleinberg 2003]. Link prediction is a harder problem since networks are often sparse and one needs to find the few edges that are true positive with high probability. We show that our results are very promising on this front.

Other methods to predict trust relations exist. Some of this work does not take structural information into account and most do not formulate the notion of convergence. For example, Tang et.al. [Tang et al. 2013] consider the similarity (homophily) in rating information as a way to predict (positive) trust relations. In this work, authors start with the assumption that users with trust relations have more similar ratings than those who do not, and users with similar ratings are more likely to establish trust relations. They do not consider structural information, which is the only information that we consider here. They also only predict positive relations, while we predict both positive and negative relations. Homophily and influence are two frequently studied factors that can provide a secondary explanation for the changes in trust relations [Aral et al. 2009]. However, an interesting application of such work to ours would be to use the external rating information as a way to gauge link strength or persistence that our algorithm can use to improve precision further.

There are many trust inference algorithms that rely upon the concept of the transitivity of (positive) trust, which our theory also supports. In TrustDavis [do B. DeFigueiredo and Barr 2005], trust transitivity is described in terms of contract. The truster's trust in the trustee takes the form of a debt guaranteed by a trusted third party. When inferring trust between two parties, they study the lowest network-flow cost for the specific debt as the network capacity. Similar algorithms include Advogato [Levien and Aiken 1998], Appleseed [Ziegler and Lausen 2004], Sunny [Kuter and Golbeck 2010], and Moletrust [Avesani et al. 2005]. There are other algorithms that treat direct trust in terms of probability. Inferring trust is then computation by different metrics, including probabilistic [DuBois et al. 2009] [Hang et al. 2008] [Patel et al. 2005] [Josang et al. 2006] and other numerical methods [Yao et al. 2013]. These methods do not handle negative distrust as a separate cognitive concept, but as lack of sufficient trust. As a result, they are more effective in inferring positive trust relations.

3. EXTENDED STRUCTURAL BALANCE THEORY (ESBT)

In this section, we introduce a new and more fine-tuned way to define structural balance that we will call extended structural balance theory, or ESBT for short. The psychological explanation for weak structural balance relies on the concept of stress. Certain situations cause stress in interactions and as a result, are not considered natural. So, in balanced situations such stress must not exist. We define this stress more precisely as a function of the strength of relationships.

In classical balance theory, relations are restricted to binary values (+/−), which we interpret as trust and distrust. When Cartwright and Harrary first formalized the theory of structural balance, they also suggested that relationships of interest exist in varying degrees, and that their theory is built on the incomplete representation of strengths of relations [Cartwright and Harary 1956]. Tie strength is a well-studied concept in social psychology. A person may have close friends and acquaintances (strong/weak ties), with different trust expectations. A strong tie may represent a trust relation corresponding constructs relevant to high risk situations, while a weak tie may be trusted for low risk situations like providing private information [Granovetter 1973].

To model this distinction, we consider a scenario where relationships have varying strengths. A strong positive link represents a close friendship or family tie, i.e. strong trust constructs, and a strong negative link represents hatred, i.e. strong distrust constructs. However, many other types of trust relationships may exist in between the spectrum of (strong) trust and (strong) distrust. For example, a negative bias may be considered a weak distrust relationship and a weak tie may represent a weak (positive) trust relationship.

A complete balance theory should be able to deal with relationhips with strengths. As a first step, we need to have a measurement of relations with various strengths. While it is arguable whether such strengths can be expressed by numerical values, it is fairly clear that the strength of any two relations can be compared. For positive relations such as liking, valuing or approving, two relationships are comparable in terms of which one is stronger than the other. Similar argument applies to two negative relations. Finally, a positive relation and a negative relation are comparable by their signs. Hence, relations with strengths by nature inherit a total ordering.

Let the collection of relations with strengths be E. An edge (A, B) and a relation with associated strength e will be used interchangeably in later discussion. We pick the ordering $\preceq$ such that, $e_1 \preceq e_2$ denotes that e_1 is positively equivalent to or stronger than (or negatively equivalent to or weaker than) e_2. That is, if e_1, e_2 are both positive relations, then $e_1 \preceq e_2$ is true if e_1 is at least as positive as e_2 in terms of strengths; if e_1, e_2 are both negative relations, then $e_1 \preceq e_2$ is true if e_1 is at most as negative as e_2 in terms of strengths; and finally if e_1, e_2 are of different signs, then $e_1 \preceq e_2$ is true if e_1 is positive and e_2 is negative. In the simplest case where we have only positive and negative relations, we have that $+ \preceq -$.

We also consider a **neutral relationship** as one that is unbiased, which will be denoted as O. Basically a neutral relationship is a non-negative and non-positive relationship, corresponding to no opinion and no bias. In the case of incomplete networks, classic balance theory implicitly considered two types of triads with neutral relations balanced: "$+, +, O$", "$+, -, O$" [Easley and Kleinberg 2010]. With the introduction of neutral relations, E can be partitioned into three subsets: positive relationships P, negative relations N and neutral relations O. Following the definition of ordering $\preceq$, it is clear that for any $e_+ \in P$, $e_O \in O$, $e_- \in N$, $e_+ \preceq e_O \preceq e_-$ holds. We use $\langle e_1, e_2 \rangle$ to denote the set of relations

$$\langle e_1, e_2 \rangle = \{e \mid e_1 \preceq e \preceq e_2\}$$

Hence, given $\langle e_1, e_2 \rangle$, the lower bound e_1 represents the strongest possible relationship and the upper bound represents e_2 represents the weakest possible relationship in this range.

3.1. Principles of Structural Balance

A triad is the smallest unit in balance theory, within which two of its relations cause influence over the third one. Such an influence will limit the range of comfortable relation types the third relation can have in a balanced state; and if it goes out of range, tension occurs and participants will suffer from **stress**. Participants will seek relationship changes to resolve this type of stress. We call such range of relations **tolerance**, with which we interpret structural balance at a finer level.

Given a network of nodes $G = (V, E)$, there exists a tolerance for each pair (A, B) of nodes of the form $\langle e_1, e_2 \rangle$ which is constrained by the triads (A, B) is part of. When there is no constraint, i.e. stress, on a specific relationship, the tolerance includes any relationship in E. We propose two principles regarding tolerance: transitivity and heterophily.

PRINCIPLE 1 (TRANSITIVITY OF POSITIVE RELATIONSHIPS). *Let $T = \langle e_1, e_2 \rangle$ denote the tolerance of (A, C) based on relations $(A, B), (B, C)$, and $T' = \langle e'_1, e'_2 \rangle$ denote the tolerance of (A, C) based on $(A, B)', (B, C)$. If $(A, B)' \preceq (A, B)$ then we have that $e'_2 \preceq e_2$.*

Furthermore, there exists a strength value $e_{sp} \in P$ such that if $(A, B) \preceq e_{sp}$ and $(B, C) \preceq e_{sp}$, then $e_2 \preceq e_O$ holds for all $e_O \in O$. That is, T will only have positive relations beyond a certain threshold e_{sp}.

The stress that is based on positive relations has been frequently defined by SBT and WSBT. Positive relations in a triad cause stress for the remaining relations to be positive. As a result, both in SBT and WSBT, a balanced network consists of communities that are connected to each other with positive ties. When we consider the strength of relations, we generalize this by saying that the more positive two of the relations are in a tie, there is lesser tolerance for negative trust values.

There is a point when the strength of the two positive relations are strong enough such that it is imbalanced for (A, C) to remain unfriended, i.e. neutral. This observation is inspired by the "strong triadic closure" in [Easley and Kleinberg 2010]. In trust literature, it is often referred as the "transitivity of trust" though transitivity is also used in other contexts.

PRINCIPLE 2 (HETEROPHILY IN RELATIONSHIPS). *Let $T = \langle e_1, e_2 \rangle$ denote the tolerance of (A, C) based on relations $(A, B), (B, C)$, and $T' = \langle e'_1, e'_2 \rangle$ denote the tolerance based on $(A, B)', (B, C)$. We have that if $(A, B)' \preceq (A, B) \preceq (B, C)$, or $(B, C) \preceq (A, B) \preceq (A, B)'$, then $e_1 \preceq e'_1$.*

Furthermore, suppose there is a well-defined concept of difference of relation strength values. Then, the larger difference between (A, B) and (B, C) is, the less positive e_1 (the lower bound) is.

In other words, given individuals A, B, C in a network, if the relationship between (A, B) and the relation between (B, C) differs to some extent, then the tolerance is geared towards the negative. Furthermore, the more different the strength of the relationships are, the tolerance is geared towards more negative values.

The second type of stress is an interpretation of homophily. We note that homophily, i.e. having common friends or enemies, may sometimes cause stress (as in $-, -, -$ for SBT) but sometimes it does not (in $+, +, +$ for SBT and WSBT). However, lack of homophily, which we call heterophily does cause stress. For example, consider the case $+, +, -$ for (A, B), (B, C) and (A, C). There is stress on (A, C) to be positive due to transitivity. But, there is also stress on (A, B) and (B, C) to be negative. Either way, the result will be more desirable: either all being friends, or having two friends with a common enemy. We call the second type of stress the principle of heterophily. The more different the ties are (the strongest difference is between distrust and trust, and the weakest the difference is between two identical trust ties), the more pressure there is for the tie to be negative. In other words, neutral relations do not cause stress. The difference in the strong beliefs about another is a cause for stress. At a point when the difference between (A, B) and (B, C) is significant enough, we argue that a positive value for (A, C) will cause imbalance. This is inspired by the observation that two people who have severely conflicted relationships with a common neighbor, e.g., one is the other's close friend's enemy, are not likely to be friends.

Table I. Tolerance rules in structural balance theory

(A, B)	(A, C)	Tolerance for (B, C)	
$+$	$+$	$\langle +, \mathrm{O} \rangle$	Transitivity
$+$	$-$	$\langle \mathrm{O}, - \rangle$	Heterophily
$-$	$-$	$\langle +, - \rangle$	No stress

The two principles help interpret balance in terms of relations with strengths precisely. A triad is balanced if all relationship strengths are within the tolerance implied by the other relationships in the triad.

Definition 3.1 (*Balance*).

The tolerance rules for the Davis's balance theory are given in Table I. Notice that neutral relations "O" are also added. This is because triangles "+, +, O" and "+, −, O" are allowed implicitly in their theory in the general case of incomplete graphs. According to the table, triads (1), (2) and (3) from Figure 1 are balanced as each relation strength is within the tolerance, but triad (4) is not balanced. Hence, our theory generalizes WSBT in the classical balance theory.

3.2. Balance theorems with weak and strong ties.

In this section, we show how our reasoning can be applied to a network with multiple types of relationships. We consider a set of discrete labels (shown in Table II) that have been discussed in previous literature and show that we can reason about balance in such a network using our two principles.

Strong positive ties, s+ (trust) are similar to a close friendship. There is a strong expectation of reciprocity, similarity of tastes (homophily), common intentions and benevolence towards each other [Tomasello et al. 2005]. The traditional definition of SBT is based on these types of positive relationships.

Strong negative ties, s- (distrust) are generally explained as having negative experiences with someone which is indicative of their negative intentions, unreliability and overall belonging to groups that are not considered trustworthy [Fiske et al. 2007]. Even when a distrusted person behaves in a trustworthy way, this could be considered a trick to get one to trust them.

Table II. Strength of relations: $s+ \preceq w+ \preceq O \preceq w- \preceq s-$.

Relation Type	Interpretation
Strongly positive (s+)	close friendship, trust
Weakly positive (w+)	aquiantance
Neutral (O)	unbiased relation, no relation
Weakly negative (w-)	minor disagreement, negative bias
Strongly negative (s-)	hatred, distrust

However, these do not constitute the only types of relationships that one might consider in a network. While one might consider a continuum of tie strength, we summarize some additional discrete classes. **Weak positive ties, w+ (weak trust)** can be considered a utilitarian type of trust. Interacting with someone who is only trusted partially is more risky, but can be acceptable in certain situations. For example, Uzzi [Uzzi 1996] uses the term embedded vs. arm's length ties to distinguish between the two types of trust. While a close friend is highly trusted, they may not have access to the resources a more risky contact may provide. Granovetter [Granovetter 1973] uses the term weak tie to talk about a relationship that is an acquaintance, not a close friend. Weak ties give access to less privileged information than strong ties, but come from outside of one's close network. In both cases, there is a trust relationship between two people, but this does not imply a continuous interaction or a strong affective component as in trust.

We also introduce **weak negative ties, w- (weak distrust)** to model cases in which there is a certain amount of distrust as a result of biases stemming from social groups people belong to or heresay that may not be as strong as distrust [Ames et al. 2011]. In essence, the burden of proof of one's trustworthiness is lower in weak distrust, but in both types of distrust, positive evidence is not evaluated in the same way as in trusting relations. These five types of relationships are summarized in Table II.

Table III. Tolerance of strong, weak & neutral relations

(A,B)	(A,C)	(B,C)'s tolerance	(A,B)	(A,C)	(B,C)'s tolerance
$s+$	$s+$	$\langle s+, w+\rangle$	$w+$	$w-$	$\langle w+, s-\rangle$
$s+$	$w+$	$\langle s+, \mathrm{O}\rangle$	$w+$	$s-$	$\langle \mathrm{O}, s-\rangle$
$s+$	O	$\langle s+, w-\rangle$	O	O	$\langle s+, s-\rangle$
$s+$	$w-$	$\langle \mathrm{O}, s-\rangle$	O	$w-$	$\langle s+, s-\rangle$
$s+$	$s-$	$\langle w-, s-\rangle$	O	$s-$	$\langle w+, s-\rangle$
$w+$	$w+$	$\langle s+, w-\rangle$	$w-$	$w-$	$\langle s+, s-\rangle$
$w+$	O	$\langle s+, s-\rangle$	$w-$	$s-$	$\langle s+, s-\rangle$
			$s-$	$s-$	$\langle s+, s-\rangle$

Given these relationship types, we now describe the tolerance for different triads in Table III and the resulting imbalanced triads or structures in Table IV. Notice that the triads with two positive relations and one negative relation are imbalanced as they are in classic balance theory, except for the cases in which all three relations as weak. In fact, the types of triads that consist of weak relations and neutral relations only are not considered to be imbalanced structures. The argument here is that when all relations are weak or neutral, the influence inside the triad is not significant enough to draw tension. Also, triads of type “$s+$ $s+$ O” and “$s+$ $s-$ O” are considered to be imbalanced structures. The arguments against each type of imbalanced structure is listed in Table IV.

Table IV. Imbalanced triadic structures in the presence of strong and weak ties.

Triads and the argument for stress	
$(s+s+s-)$ $(s+s+w-)$ $(s+w+w-)$	my two friends cannot get along with each other
$(s+w+s-)$ $(w+w+s-)$	my two friends cannot get along with each other
$(s+s+\mathrm{O})$	my two close friends do not friend each other
$(s+s-\mathrm{O})$	my enemy’s close friend does not pick a side

3.3. Relation Distance and General Expression of Balance

The concept of extended balance is meaningful only if the tolerance rules can be explicitly defined, so that whether a triad and a network is balanced or not can be determined. Whenever relations are drawn from a finite and small set, this is easy to do. However, it is considerably more complex in the general case when the strengths of relations are drawn from arbitrary numerical values.

To handle such cases, we refine the measurement of relations with strengths from a total ordering to positive real values. In particular, we define function $\psi : E \rightarrow R^+$ such that, for two relations $e_1, e_2 \in E$, $\psi(e_1) \leq \psi(e_2)$ if and only if $e_1 \preceq e_2$. Since positive values can be seen as metric distances, we call $\psi(e)$ the relation distance of e. In other words, relations with varying strengths are represented by distances with different lengths. More negative strengths are represented by larger distances and more positive strengths are represented by smaller distances. We propose the following general rule of tolerance with the concept of relation distance.

Definition 3.2. Given adjacent relations (A, B) and (B, C), the tolerance of (A, C) is given by $[|\psi(A, B) - \psi(B, C)|, \psi(A, B) + \psi(B, C)]$.

It can be easily checked that the general tolerance rule agrees with Principle 1 and 2 by substituting $\leq$ for $\preceq$. Immediately, we have the following theorem.

THEOREM 3.3. *Given a triad (A, B, C), if $\psi(A, B)$, $\psi(B, C)$, $\psi(A, C)$ satisfies the metric triangle inequality, then (A, B, C) is balanced.*

$b_+ < b_-$ such that if $\psi(e) \geq b_-$ then the relationship e is negative. Similarly, if $\psi(e) \leq b_+$, then e is a positive relationship. For any value $b_+ < \psi(e) < b_-$, the relationship is considered neutral. We can see that as long as $b_- > 2b_+$, Table I is equivalent to Definition 3.2.

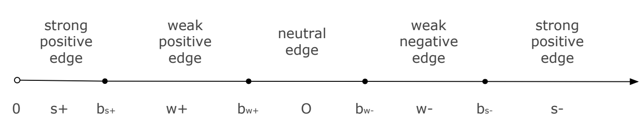


Fig. 2. Partitioning of distances: $[0, b_{s+}) : s+$, $[b_{s+}, b_{w+}) : w-$, $[b_{w+}, b_{w-}) : \mathrm{O}$, $[b_{w-}, b_{s-}) : w-$, and $[b_{s-}, \infty) : w+$.

We can see that if the following conditions are satisfied for the boundary parameters:

$$
\begin{aligned}
b_{w+} &> 2b_{s+} \\
b_{s-} &> b_{w-} + b_{s+} \\
b_{s-} &> 2b_{w+} \\
b_{w-} &> b_{w+} + b_{s+}
\end{aligned}
$$

the tolerance given in Table III are equivalent to Definition 3.2, and all triads shown in Table IV are imbalanced according to metric triangle inequality.

4. CONVERGENCE MODEL

Researchers have long argued that every social network has a tendency towards a balanced state [Doreian 2002]. The next question of interest is if an imbalance rises, in what way will a social network change towards a new balance. It is noted in social psychology literature that people are reluctant to make changes in relations as they tend to avoid the effort needed to make such changes. In a balanced triadic relation, participants are likely to do nothing and keep their pairwise relations as what they were. In an imbalanced triadic relation, participants are likely to make the smallest effort possible to regain triadic balance. We define the concept of relation cost as the effort one needs to take to accomplish a certain relation change. Our convergence model is established based on a unified assumption: every social network converges in a way that requires as little total change in relations as possible to reach a balanced state.

With the concept of relation distance, we are able to express the structure of a social network by drawing it in the Euclidean space. The strength of each relation is expressed by the distance between their locations. Notice that every layout in the Euclidean space automatically satisfies the metric triangle inequality, and hence corresponds to a balanced state of the network. For an imbalanced social network, it is not possible to draw it using its initial relation distances. Hence, our convergence model produces a layout of the social network with minimum total relation cost from the original one.

Let $G = (V, E)$ denote an arbitrary social network , and $G^* = (V, E^*)$ denote a balanced state of G. Let $n * n$ matrix X denote the layout of G^*, with each row vector x_i denoting node i's location in m-dimensional space. For each pair (i, j), $\psi(i, j)$ denotes its relation distance in G, and $d_{i,j}(X)$ denotes distance between i and j in X, i.e., its relation distance in G^*. Given an edge $(i, j) \in E$, the relation cost on (i, j) is given by:

$$c_{i,j}(X) = w_{\psi(i,j)} * (d_{i,j}(X) - \psi(i, j))^2$$

where the weight value is a function of the original distance. The weight function can take into account the difficulty of changing a relation. For example, it is generally easier to change a neutral relation than a positive or a negative relation that incorporates an initial bias. The study of optimal weights is beyond the scope of this paper. However, we consider three main classes of weights:

$$w_{\psi(i,j)} = \begin{cases} w_+ & \text{if } \psi(i,j) \text{ is a positive edge} \\ w_O & \text{if } \psi(i,j) \text{ is a neutral edge} \\ w_- & \text{if } \psi(i,j) \text{ is a negative edge} \end{cases}$$

If $w_O << w_+$ and $w_O << w_-$, then neutral edges would have very little influence on the already established positive/negative relations.

Definition 4.1.

$$\sigma(X') = \min_X \sum_{i<j\leq n} w_{\psi(i,j)} * (d_{i,j}(X') - \psi(i, j))^2$$

4.1. Computing Convergence

The optimization of relation cost is in fact a Metric Multidimensional Scaling problem (MDS) by assigning nodes a location in metric space. The total cost function is called stress in MDS, and is often minimized through an optimization strategy called *stress majorization (SM)* [Gansner et al. 2005]. Stress majorization is an iterative method that guarantees monotonically decreasing stress in each iteration, and returns a locally minimum solution. It is recognized as a principled technique in the field of graph drawing. The algorithm, however, requires $O(n^3)$ time and $O(n^2)$ space. Due to its complexity, stress majorization is applicable on graphs with limited size when missing edges are explicitly represented as neutral ties.

To address the issue of complexity, we also introduce an implementation of stress majorization for sparse graphs, *SM/SG* by modifying the approach in [Gansner et al. 2005]. Following Gansner's description, we denote the d-dimensional location matrix as X [Gansner et al. 2005]. Let node i's location be X_i. The MDS problem is to minimize the following stress function

$$stress(X) = \sum_{i<j} w_{ij}(||X_i - X_j|| - d_{ij})^2 \,, \tag{1}$$

where w_{ij} are weights and d_{ij} denote the ideal distance between i and j.

There are many ways to minimize $stress(X)$. The iterative majorization method at each step minimizes a simple convex function, which guarantees a monotonically convergent solution [Leeuw 1977]. In application, the majorization process involves solving the following equation iteratively:

$$L^w X(t+1) = L^{X(t)} X(t), \tag{2}$$

where L^w and $L^{X(t)}$ are given by

$$L^w_{i,j} = \begin{cases} -w_{ij} & i \neq j \\ \sum_{k\neq i} w_{ik} & i = j \end{cases}, \quad L^{X(t)}_{i,j} = \begin{cases} -w_{ij}d_{ij}/||X(t)_i - X(t)_j|| & i \neq j \\ -\sum_{j\neq i} L^{X(t)}_{i,j} & i = j \end{cases}.$$

At each iteration, L^w is constant throughout the process, whereas matrix $L^{X(t)}$ needs to be computed recursively. Solving Equation 2 in each iteration requires the computation of a $n \times n$ matrix, which in total requires $O(n^3)$ time and $O(n^2)$ space. For network of size $n > 10^5$, direct implementation of stress majorization becomes impossible.

Fortunately, most social networks appear to be sparse. If we treat all unknown edges as zero weighted, we are able to proceed. In particular, the right side of recursive Equation 2 can be rewritten as

$$(L^{X(t)}X(t))_i = \sum_{j\neq i} w_{ij}d_{ij}\frac{X_i - X_j}{||X_i - X_j||}. \tag{3}$$

Hence, we will only need to sum up the terms corresponding to non-zero weighted edges. Let E denote the number of edges of a social network, computing $L^{X(t)}X(t)$ only requires $O(E)$.

For a sparse undirected network, the corresponding L^w is sparse and symmetric. Weighted Laplacian L^w is known to be semi-positive definite with a one-dimensional null space spanned by $1_n = (1, ..., 1) \in R^n$. We use $\bar{L^w} = L^w + \epsilon I$ to approximate L^w, where $\epsilon << w_O$ is a small positive number. Since $\bar{L^w}$ becomes strictly diagonal dominant and hence positive definite, Cholesky factorization is guaranteed to work very efficiently.

On a single 2.8GHz processor running 64-bit Ubuntu 11.04 with 12GB of RAM, it takes about one minute to reach convergence on our largest data set containing 840,000 edges, requiring $O(|E|)$ space.

Table V. Algorithms used in this paper for solving the convergence problem.

Algorithm	Description
SM	Stress majorization, solution that takes into account neutral edges
SM/SG	Stress majorization for sparse graphs, neutral edges are given zero weight.
FD	Force directed graph drawing algorithm introduced in [DuBois et al. 2011]
PP	Path probability algorithm used in conjunction with FD in [DuBois et al. 2011]

5. STYLIZED NETWORKS

In this section, In particular, we test the impact of positive and negative trust relations and the transitivity of positive relations using small networks for illustration. We will study real and large scale networks in Section 7. For this section, we have chosen the boundary parameters shown in Table VI. The initial distance for each range is set to the middle point of each range (and 1.2 for strong negative). The weights are chosen as follows: $w_+ = w_- = 1$, $w_O = 0.01$. We have found the converged layout of nodes in each graph using the SM algorithm.

Table VI. Boundary parameters chosen for the stylized networks given in Section 5

Type:Bounds				
s+: [0,0.1)	**w+**: [0.1, 0.5)	**O**: [0.5, 0.7)	**w-**: [0.7,1.1)	**s-**: [1.1, ∞)

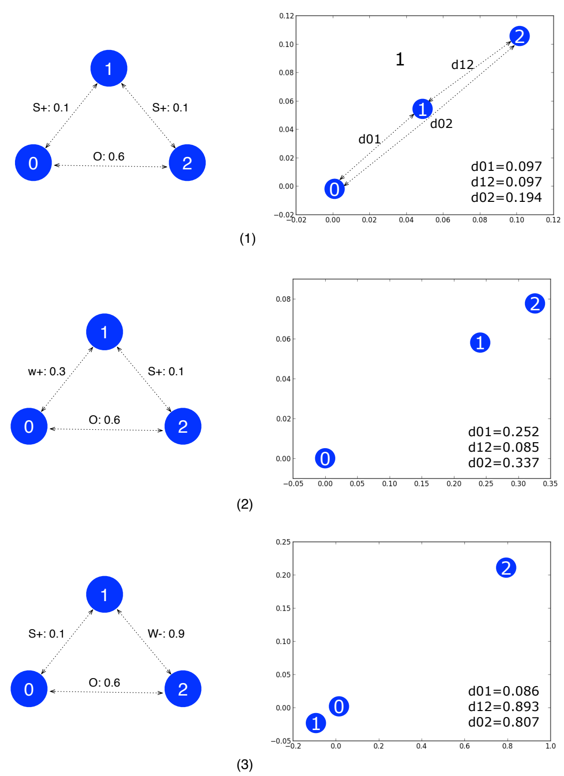


Fig. 3. Convergence of networks containing neutral edges.

5.1. Convergence of networks with neutral edges.

First, we look at convergence for different networks containing neutral edges. We consider three networks in Figure 3 containing a single neutral edge between nodes 0 and 2. The initial values for relation strengths is given in the graph on the left. The converged location for the nodes is shown on the right. The first graph contains two strong positive edges and a neutral edge. After convergence, the neutral edge has changed to a weak positive node. As a result, the two strong positive edges were above the threshold that requires the remaining edge to be a positive relation. These two edges remain strong positive after convergence. This graph illustrates principle 1.

Next, we look at graph 2 in Figure 3 which contains a strong positive and a weak positive edge in addition to the neutral one. As expected, the neutral edge becomes a weak positive edge after convergence. However, as expected from principle 1, since the pull is lower in graph 2 than in graph 1, the strength of edge (0,2) is also weaker in graph 2 than in graph 1 after convergence.

Finally, we look at graph 3 in Figure 3 which contains a strong positive and a weak negative edge. Based on principle 2, due to the large difference in strengths, we expect that the relation between 0 and 2 cannot remain neutral and it will become negative. In fact, after convergence, edge (0,1) remains strong positive, edge (1,2) remains weak negative, and the edge (0,2) becomes weak negative.

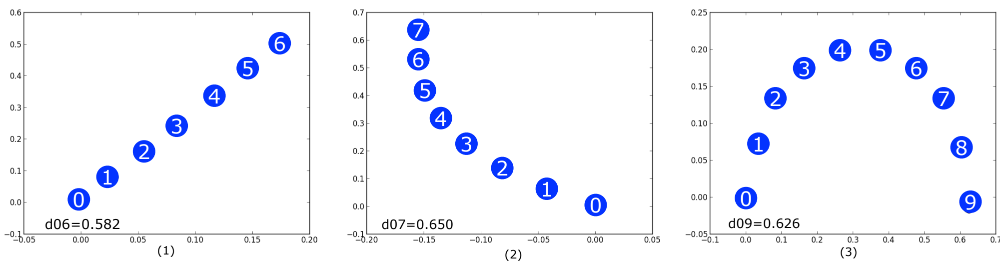


Fig. 4. Transitivity of positive edges.

5.2. Transitivity

Next, we consider transitivity of positive edges. This is an often assumed property of trust relationships in networks. we consider graphs of n nodes arrange on a single line, e.g. (0,1), (1,2), (2,3), etc. Each edge is a strong positive. Graphs 1,2 and 3 in Figure 4 show transitivity of 7,8 and 10 nodes. Note that these graphs are very sparse, containing only $n-1$ edges. The converged relations display a number of desirable properties. First, pairwise distances of existing edges remain strong positive. Secondly, the longer the path between two nodes in the graph is, the larger is the Euclidian distance between them. This follows the often hypothesized property of attenuation of trust along longer paths. We also note that the largest distance, between nodes (0,6) in graph 1 (as well as (0,7) and (0,9) in graphs 2 and 3) remain a neutral edge. In fact, this is achieved by curving the underlying placement of nodes in graph 3.

5.3. Aggregation

Finally, we study the aggregate effect of having multiple paths between two nodes. We are first given graph 1 in Figure 5. Given two triangles containing strong positive edges, the originally neutral edge (0,3) becomes weak positive after convergence. In fact, the strength of the edge is 0.104, an almost strong positive edge.

Next, we consider adding other paths between nodes (0,3) and investigate the effect. In graph 2, we add another triangle with two strong positive edges. The effect of this is to cause (0,3) to come even closer and become a strong positive edge. In other words, as more positive paths between the two nodes are added, the more positive they become.

In graph 3 of Figure 5, we add a path containing a strong positive and a weak negative edge. Due to the difference in the strengths of these edges, we expect that they would exert a negative influence on (0,3). In the converged network, the edge (0,3) is weak positive with

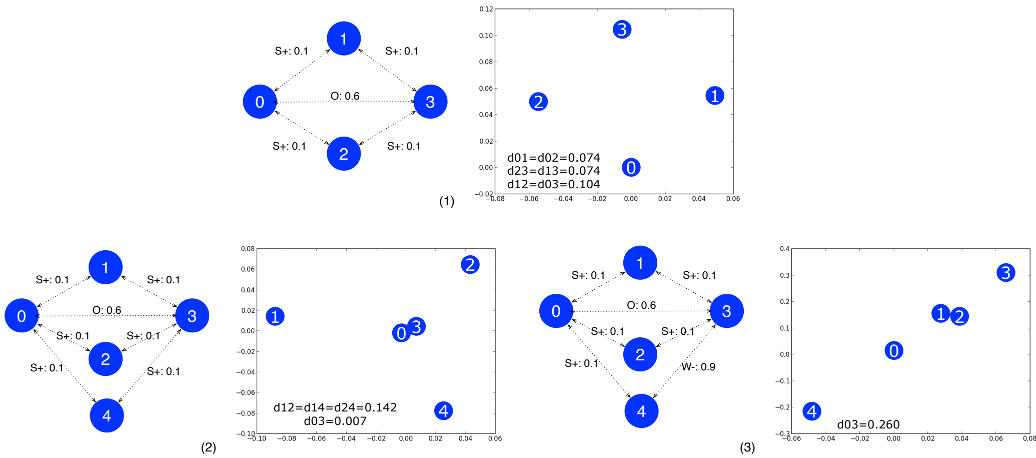


Fig. 5. Impact of increasing connectivity in convergence.

distance 0.26. In fact, the strength of edge (0,3) is weaker than in both graphs 1 and 2. This is an expected result as there is no negative influence in any of the other networks.

This indicates that adding a common friend between two people will grant a better chance to them to become friends. However, adding a conflicted common neighbor between two people will deteriorate their relationship.

Overall, our results in stylized networks illustrate that the convergence formulation results in intuitive results.

Data: $\mathcal{A}$, M, k, deg, G
Result: Pt, Nt
Get G' = generate-subgraph(M, k, deg, G);
Partition G' into 10 groups of test and training samples;
Create two empty sets Pt, Nt;
for *each of the groups* **do**
  run algorithm $\mathcal{A}$ on the training sample and get the layout;
  for *each edge e in the testing sample* **do**
    compute its distance in the layout;
    if *e is a positive edge* **then**
      add its distance to Pt;
    else
      add its distance to Nt;
    end
  end
end

Algorithm 1: Edge Sign Prediction Methodology

6. EXPERIMENTAL SETUP

In the next section, we study the various properties of our framework, in particular its application to the *edge sign prediction problem* for various real networks. The *edge sign prediction problem* is defined as follows. Suppose we are given a social network with signs, but a small fraction of the edge signs are “hidden”. How can we predict these signs with the information provided by the rest of network? The convergence model is able to predict these "hidden" signs. Let's denote the original social network with all signed edges as G, the network consisting of hidden edges as G_h, and the network consisting of the remaining edges as G_r. The edges (relations) between each pair of nodes is measured by $\{+, -, O\}$. We run the convergence model on G_r, and denote the network after convergence as $G_r^{'}$. We expect that the signs of the hidden edges in $G_r^{'}$ largely agree with the true signs.

By the assumption that every social network has a tendency towards balance, it can be inferred that G is largely balanced at any moment. Hence, the majority of G_r is balanced. The only exceptions are the components with hidden edges, which are of sign O in G_r. By the principle of total relation cost minimization, the changes mostly occur on the O-sign hidden edges during the convergence. We expect the hidden edges in G_h to have their true signs in $G_r^{'}$ if G is largely balanced. Hence, we test the performance of an algorithm by first randomly sampling 90% of the edges of a given graph as the training graph (e.g. G_r) and set aside the 10% as the test data (e.g. G_h). We then find the distance between nodes in G_h with respect to the converged version $G_r^{'}$ of G_r. We use the algorithms given in Table V for finding a converged graph. We repeat each experiment 10 times.

The distances of testing edges are computed by the layout of the training data. Given a distance threshold, the sign of each edge is predicted as positive if and only if its distance is smaller than the threshold. In the previous work, such threshold is computed from the (distance,sign) pairs of the training samples using standard machine learning techniques [DuBois et al. 2011][Leskovec et al. 2010]. In this paper, however, we do not concentrate on the learning process. The issue of interest is how well the convergence model performs in separating hidden positive edges from negative ones in terms of distance. Instead of making predictions based on a particular threshold, we draw ROC curves for evaluation which capture the performance of sign prediction for both positive and negative edges across all thresholds and compute the false and true positive rates based on the computed Pt (Nt) values returned by the Algorithm 1. The ROC curves are drawn upon the Pt (Nt) values from the accumulation of all testing samples.

In the implementation of SM and SM/SG, the weight of each type of edge satisfies:

$$w_O << w_+ < \frac{w_-}{2}.$$

The first inequality has been argued in the previous section. The second one is chosen empirically, indicating that a negative edge has larger influence than a positive one. We repeat the experiments on various weights and original distance configurations under the above two constraints, the prediction performance turns to be very consistent. We present sensitivity results in the next section.

The partitioning of the distance domain satisfies

$$b_+ < \frac{b_O}{2} < \frac{b_-}{2}$$

for a positive, neutral and negative edge, conforming to our theory. Hence, the initial values for distances of a positive edge d_+, a neutral edge d_O and a negative edge d_- should at least satisfy $b_+ < \frac{d_O}{2} < \frac{d_-}{2}$. One of the recommended parameter configurations is given as the following.

Table VII. The recommended parameter configuration.

Parameter	w_+	w_O	w_-	d_+	d_O	d_-
Configuration	1.0	0.01	3.0	0.1	0.6	1.1

Note that SM/SG does not consider neutral edges and as a result it implicitly assumes that $w_O = 0$. We use the same setting for all the networks and do not employ any other adjustable parameters unless explicitly specified. In Section 7.7, we employ a version of our algorithm that considers tie strength as well. We give the parameters used for this version in the relevant section.

6.1. Datasets studied

We use the same three datasets used in [DuBois et al. 2011] and [Leskovec et al. 2010] to conduct our experiments, all provided by the Stanford Large Network Dataset Collection.

(1) *Epinions* is a product review website where users give reviews and ratings on product articles. Users can choose to trust or distrust others. The network contains more than 100,000 users and over 700,000 trust/distrust edges. There are also 13,668,320 ratings for 1,559,803 articles. We use the ratings only for external validation.
(2) *Wikipedia* elections collects the votes by Wikipedia users in elections for promoting candidates as administrators. Each user can give a supporting (positive) or opposing (negative) vote on the promotion of another. The dataset has about 7,000 users and around 100,000 votes (edges) for 2,794 elections.
(3) *Slashdot* is a technology news website where users rate each other as friends or foes. The dataset released in February 2009 contains over 77,000 users and over 900,000 friend/foe edges.

Overall, we note that the trust ratings in Epinions is more of a vote of competence, whereas in Slashdot, it is more of a rating of trustworthiness (constructs like reliability, friendliness, etc.). The ratings in Wikipedia fall somewhere in between the two, incorporating both aspects of competence and trustworthiness. Wikipedia administrators are expected to be both competent in their job, but also not subvert the power of their position. We expect structural balance to be more applicable to trust ratings of trustworthiness. While the affective aspect of trust relations based on trustworthiness impact the competence judgments, there is no parallel notion of stress in trust relations based on competence. As a result, we expect our theory to be most applicable to Slashdot.

There is also difference in the fraction of positive votes to negative votes in the data sets. In Epinions, there is a negative vote for every 5.8 positive vote. This number is 3.63 for Wikipedia and 3.43 for Slashdot. As a result, capturing the meaning of distrust relations becomes even more important for Wikipedia and Slashdot.

6.2. Graphs used in the experiments

Using these networks, we construct different graphs to test the different hypotheses made in this paper. Even though the edges in the underlying networks are directed, we construct several undirected graphs out of these networks. In this process, we consider four types of edges:

(1) *Bi-directional edges* are reciprocal signed edges. For each edge (A, B), the network contains both directed edges (A, B) and (B, A) with the same sign.
(2) *Single directional edges* are non-reciprocal edges. For each edge (A, B), there is a directed edge (A, B), but no edge (B, A).
(3) *Known neutral edges* are edges (A, B) such that either there are the directed edges (A, B) and (B, A) with conflicting signs, or one of (A, B) or (B, A) is explicitly a neutral edge. Neutral edges only exist in Wikipedia where the votes can be positive, negative or neutral.
(4) *Unknown edges* are any pair of nodes with no edge in between.

Given these edges, we construct the graphs given in Table VIII. They are roughly ordered from general to specific. Notice that for each pair of reciprocal signed edges, we add a single undirected edge in the constructed graph to avoid duplications.

The sizes of the various graphs for each network are given in Table IX. We note that for Wikipedia, there are only around 1/33 edges bi-directed. If a node A has voted on B,

Table VIII. The different graphs constructed from the given networks in our experiments.

Graph	Positive/Negative Edges	Neutral Edges
G^*	Single and bi-directional edges are signed as positive or negative, no tie strength.	Unknown and known neutral edges.
G	Same as G^*	Only the known neutral edges.
$G^{\#}$	Same as G^*	The known neutral edges and unknown edges between any pair of nodes that are connected by a path of length 2.
G-bi	Only bi-directional edges are signed as positive or negative.	Only the known neutral edges.
G-di	Bi-directional edges are considered strong ties, single directional edges are considered weak ties.	Only the known neutral edges.

then A is an admin. Hence, even if B becomes an administrator, it is extremely rare for B to vote on A later on. As a result, the graphs in G-bi in Wikipedia are too small for representative results. In comparison, around 1/6 of the edges in Epinions are bi-directed and around 1/10 of the edges in Slashdot are bi-directed. For these graphs, it is possible for edges to independently review each other in either direction.

Table IX. The size of the different graphs (in terms of number of edges) constructed from the given networks in our experiments. Note: for G^* we only provide the number of known edges in sampled graph sizes.

Graph	Epinions	Wikipedia	Slashdot
G^*	180,000	160,000	65,000
G	841,000	100,000	549,000
$G^{\#}$	45,400,000	1,600,000	14,200,000
G-bi	128,000	3,000	48,000
G-di	704,000	105,000	495,000

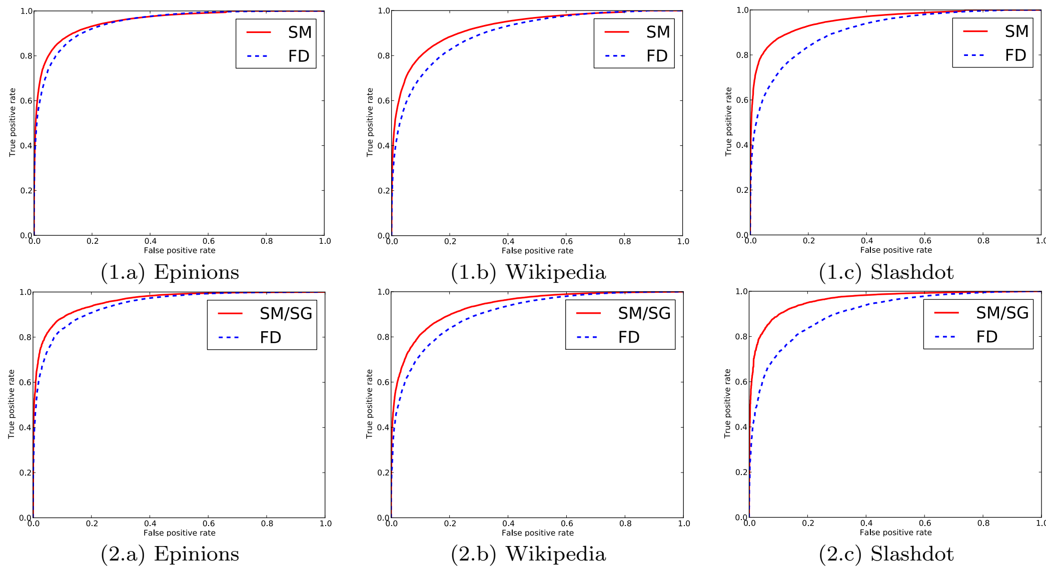
The graphs considered in this paper, in particular G^*, are dense (similar in size to $G^{\#}$). After we add the unknown neutral edges, G^* becomes a complete graph. It is practically infeasible for SM to run on the entire dataset for any version of the graphs. This is due to large number of neutral edges that must be explicitly represented to run SM. In practice, we will evaluate the performance of SM on small sub-networks of G^* computed by sampling only. To accomplish this, we generate random samples of our datasets using the snowball sampling method in which a small number k of seeds with degree greater than a given threshold deg are selected at random, then all nodes that are adjacent to the seed node are selected iteratively until the desired network size is reached. In our practice, the size of the resulting graph is in the range 3,000-5,000 nodes, k is chosen from 2-10 randomly and deg is chosen from 7-20 randomly. For each dataset, we generate 10 sub-networks and perform 10-fold cross validation. The number of known edges in a sub-network of Epinions is around 180,000, for Slashdot 65,000 and for Wikipedia 160,000.

In our first set of results, we compare the performance of SM to our baseline algorithm, FD using sampled graphs as well as performance of SM/SG on the full graph.

7. EXPERIMENTAL RESULTS

In this section, we focus on the *edge sign prediction problem* unless otherwise stated. We compare our algorithm with the force directed algorithm (FD) in [DuBois et al. 2011] as this algorithm's performance has been shown by the authors to be the state of the art for measuring trust and distrust. Note that we have tuned our implementation of FD to provide similar performance reported in this work. Even though this work combines two algorithms, FD and PP, in our comparison experiments, we find that FD alone gives equally

Fig. 6. The ROC curves are drawn upon distances of hidden edges for samples of G* for Epinions, Slashdot and Wikipedia datasets. Comparison between SM in Figures 1.a, 1.b, 1.c, and SM/SG in Figures 2.a, 2.b, 2.c.



(1.a) Epinions (1.b) Wikipedia (1.c) Slashdot

(2.a) Epinions (2.b) Wikipedia (2.c) Slashdot

good prediction performance on all three datasets as the combination. The comparison is shown in Table X. As a result, we will only compare our method with FD in the following discussion. Note that all the results reported in this section are highly significant. The variation in the values reported (with the exception of tests involving the last 10% of the edges) have very small variation, below 0.001 in most cases. For this reason, we will not put error bars on our graphs.

Table X. The prediction performance by FD and FD&PP on G*, in terms of prediction accuracy on positive and negative edges, on all three datasets. The probability parameter p is set to 0.05 as it is in [DuBois et al. 2011].

	Epinions		Slashdot		Wikipedia	
Method	Positive	Negative	Positive	Negative	Positive	Negative
FD	90%	87%	83%	80%	82%	79%
FD&PP	88%	88%	82%	80%	82%	80%

7.1. Performance of SM on G*.

We first compare the performance of our algorithm to the state of the art on the largest network, G*. As mentioned earlier, due to the complexity of computing with all neutral edges, we can only run SM on sampled graphs. We compare the performance of both algorithms on the same graphs. The ROC curves on the top line in Figure 6 are drawn upon the Pt (Nt) values from the accumulation of all testing samples based on Algorithm 1.

For all three datasets, we find the ROC curve of SM is on the "northwest" side of the one of FD, which indicates SM is consistently better than FD in separating hidden positive edges from negative ones. Notice that the improvement for Slashdot is the most significant one among the three, possibly due to the fact that Slashdot edges represent "friends" or "foes", which is by nature a more clear identification of trust/distrust compared to votes in Wikipedia or distrust for reviews in Epinions that also include ratings of competence. As a result, our convergence model produces a very good prediction performance. On the

Epinions and Slashdot datasets, the best thresholds on ROC curve give $88-90\%$ accuracy on both positive and negative hidden edges. For Wikipedia, SM achieves $83-85\%$ at the best threshold. The accuracy rates of Epinions and Wikipedia match the best results from previous work, and Slashdot appears to be the best so far.

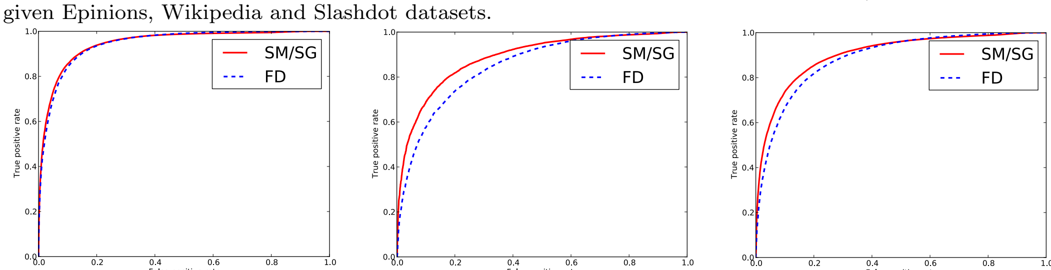
7.2. The effect on neutral edges, comparison between SM and SM/SG

Next, we compare the performance of SM to SM/SG on the same graphs, sampled from G* to see if SM/SG provides a good alternative to SM and to quantify the effect of neutral edges on performance. We compare the ROC curves on the top and bottom line of Figure 6.

We essentially achieve the same prediction performance. In our theory, indirect relationships could also have influence on the convergence. By considering all neutral edges, we have added some missing factors in the original network, but also a vast amount of nonexistent influence on the convergence. Consequently, we do not necessarily achieve better results by considering unknown edges. In fact, the performance improves slightly in Slashdot. This could be due to the fact that Slashdot is the most representative dataset for our model, and hence is most effected by the addition of incorrect neutral edges.

Given the similar performance, we concentrate on evaluating SM/SG on the full version of the graphs from now on. We also compare SM/SG to FD on G, in Figure 7. Note that previous results corresponded to running SM/SG on a sample. In this part, we are considering the performance on the full graph, but ignoring any missing links. The only neutral edges in G are those that are known neutral edges. The performance is still superiour to FD, but the difference is diminished slightly. Clearly, the full graph can be considered more noisy, especially since most stress resulting from different relationships are likely to be of a local nature. Furthermore, one expects most relationships in the network to be weak and applying transitivity to them as if they were strong possibly causes problems. Our tests currently do not take relation strength into account. We will explore this issue in the later sections. However, we simply note here that SM/SG matches and exceeds performance of FD in all three datasets.

Fig. 7. The ROC curves are drawn upon distances of hidden edges, generated by SM/SG and FD for G given Epinions, Wikipedia and Slashdot datasets.



Epinions Wikipedia Slashdot

Table XI. Total number of edges in each class (S for single (S) edges and Bi for bi-directional edges) for Epinions using graph G.

Edges	S(+)	Bi(+)	S(-)	Bi(-)	Total
All trust edges	465,350	249,614	116,271	4,731	835,966
Flipped by SM/SG	28,428	5,623	6,554	193	40,798
Flipped by FD	31,706	6,027	8,755	280	46,768

7.3. External Validation

In this section, we consider an external validation of our methodology. Our algorithm solely considers the structural information. However, additional information in the form of user ratings of each other's reviews is available in the Epinions dataset that can be used to validate the results of our algorithm. We showed that our algorithm results in better prediction accuracy than FD. However, there are various papers that report quite strong prediction performance in Epinions such as in [DuBois et al. 2011] by using structural information and in [Tang et al. 2013] by using homophily in the ratings. We believe this is due to the fact that most distrust relationships in this dataset are weak. As a result, fairly good prediction is possible even without considering negative relations.

It is likely that most trust relationships in Epinions correspond to a rating of competence (good/bad reviews) than untrustworthiness (friend/enemy). When people judge competence, they consider accumulation of positive evidence diagnostic. When somebody is considered competent, their work is not scrutinized closely. However, incompetent people are watched. If they perform well, trust for them grows. As a result, even if a distrust rating exists, the relationship may grow towards trust over time which should show up as improved ratings for their articles. Structural balance in the network may effect ratings in a few ways. For example, suppose Alice distrusts Bob, but other friends of Alice trust Bob. This may cause Alice to have more opportunities to judge Bob as her friends promote Bob's reviews. It is also possible that Alice starts to view Bob's reviews more positively as she values her friends' judgment. Our objective is to check which algorithm captures this effect more accurately.

To test this, we first run both algorithms to find a converged version of the training data as before. Our running hypothesis is that the relationships were close to convergence before running the algorithm and hence only a minimal set of relations will change sign after convergence. Using the test cases, we determine the threshold that gives the best separation between positive and negative predictions. Based on these predictions, we now consider which edges in the *training set* have flipped their sign after convergence. In other words, flipped edges after convergence were those edges that were most likely to change sign according to a specific algorithm. We compare the flipped edges from the training set for both algorithms SM/SG and FD in Tables XI and XII. Note that in this case, we are only reporting on edges that are either single directional or bi-directional with the same sign. The number of edges in each category follow a similar pattern in SM/SG and FD, but fewer edges are flipped in SM/SG. This is expected as FD does not operate on an explicit constraint to preserve the network relations as much as possible. The more important question is if there is a difference in the characteristics of the flipped edges. For these, we look at the ratings Alice gave to Bob *after she reported on her trust rating.* These are given on the right side of the table.

We first look at the overall trend, if Alice trusts Bob, she also gives her overall higher rating values (4.60/4.76 for S(+)/Bi(+) edges) than if she distrusts him (3.76/3.32 for S(-)/Bi(-) edges). Furthermore, after the trust rating is given, the review ratings for trusted people go up (4.98/4.99) and distrusted people goes down (3.20/3.40). We can also see that for bi-directional edges, trusted people have higher ratings (4.76 versus 4.60) and distrusted people have lower ratings (3.32 versus 3.76). This shows the correlation between trust ratings and reviews, as well as justification to treat bi-directional edges as stronger relationships.

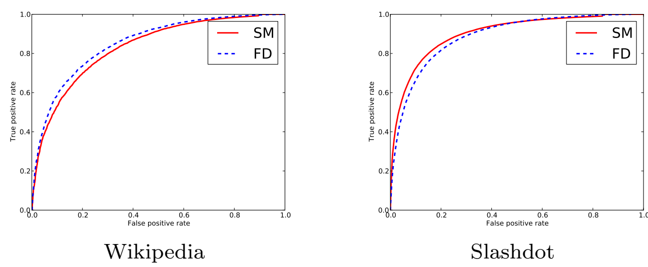
How about the people who were flipped by the different algorithms (see Table XII)? For both algorithms, the flipped people had overall slightly lower positive ratings, which might signal that the overall strength of positive relationships were not as high as the average link. There is no difference in the ratings after the trust edge is created. Hence, the reviews become more positive after the rating for flipped edges as well. This could just be the norm for this data set.

The distinction becomes clear in the negative edges. For both algorithms, the flipped negative edges (i.e. those that have become positive) have higher ratings after the trust rating is given when compared to the full data. SM/SG is able to better capture the expected change in relationships by finding negative edges that are weak overall and that have evolved towards a more positive relationship over time. In fact, the structural information and stress in relations provide significant information that can be used to predict such change. Note that for this experiment, we are treating all edges the same to obtain a fair comparison with FD that does not consider edge strength. In Section 7.7, we will look at prediction when considering edge strength as well.

Table XII. Average value of ratings along a specific type of trust edge in the whole Epinions dataset and after the trust edge was created (S for single (S) edges and Bi for bi-directional edges). For each edge, the average ratings for that edge is considered, and then the value is averaged for the specific group of edges. Results based on graph G.

	Average rating value				Average rating value after trust edge is created			
Edges	S(+)	Bi(+)	S(-)	Bi(-)	S(+)	Bi(+)	S(-)	Bi(-)
All trust edges	4.60	4.76	3.76	3.32	4.98	4.99	3.20	3.40
Flipped by SM/SG	4.43	4.80	4.13	3.75	4.98	5.00	4.41	3.93
Flipped by FD	4.42	4.81	4.03	3.63	4.99	5.01	4.39	3.32

Fig. 8. The ROC curves are drawn upon distances of hidden edges, generated by SM/SG and FD for $G^{\#}$ given Slashdot and Wikipedia datasets.



7.4. Impact of Indirect Neutral Edges

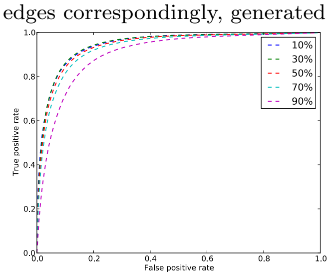
As we have seen earlier with the sampled graphs, disregarding unknown neutral edges by using SM/SG does not result in a performance penalty. The question is whether there is a better definition of neutral edges. Instead of considering all unknown edges as neutral, we can restrict neutral edges to those pairs who have a chance to know each other. We define this as pairs of nodes for which the shortest path between them is of length 2. This is given in graph $G^{\#}$. In the resulting graphs, the expanded Epinions has around 70 times its original size, the expanded Slashdot has around 23 times its original size and the Wikipedia has around 15 times its original size. Due to its large size, running SM/SG on Epinions is not feasible. We only report on Wikipedia and Slashdot.

We compare the performance of SM/SG to FD in Figure 8. The performance of FD is the same as it is for G since it does not consider neutral edges. The performance is slightly worse in Wikipedia. In fact, this is the only case SM/SG does worse than FD. If (A,B) and (A,C) are both weak ties, then it is very likely that the existence of these ties would have an effect on the relationship between (A,C). Hence, adding such a relationship as a neutral edge is likely to hurt performance. We hypothesize that this is the reason for the reduced performance. In fact, despite the relatively small percentage of edges added, Wikipedia is the worst effected from this change. A reasonable conclusion in this case is that Wikipedia data set has the highest concentration of weak edges. Given Wikipedia is based on voting, the relationships have a more hierarchical and organizational flavor than the other datasets. This might explain why the relationships between voters (incumbents) and those who are voted on (newcomers) are mostly weak relationships.

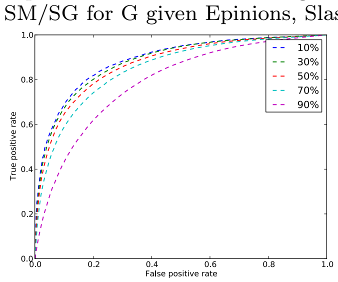
7.5. Coverage

We now consider the accuracy of prediction by removing 10%, 30%, 50%, 70% and 90% of the edges. The remaining edges are used as the training set, and the removed edges are used as the testing set. Notice that there are edges in the testing set with endpoints that are not shown in the training set. We cannot predict such edges because the locations of the untrained endpoints are unknown. We simply ignore such edges in testing. Again, we employ 10-fold validation for each experiment. The results are shown in Figure 9. In all data sets, the prediction remains quite high even when we remove 10%-50% of the edges. There is slight degradation at 70%, and a much higher drop at 90%. At 90%, the prediction is around 83.7%, 71.8% and 73.4% on Epinions, Wikipedia and Slashdot respectively at the best cut-off. The degradation is better than reported in [DuBois et al. 2011]. We also note that Wikipedia and Slashdot are impacted more by this change, as prediction is harder for these data sets.

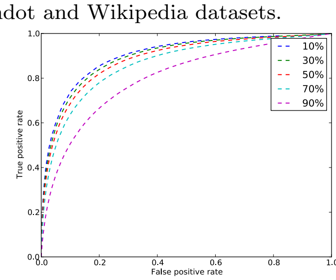
Fig. 9. The ROC curves are drawn upon distances of hidden edges after removing 10%, ..., 80% of the edges correspondingly, generated by SM/SG for G given Epinions, Slashdot and Wikipedia datasets.



Epinions



Wikipedia



Slashdot

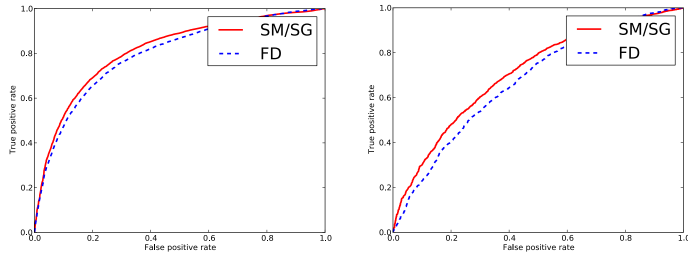
7.6. Effect of Time

In this section, we consider the accuracy of our algorithm in predicting the future. In other words, given the relations as they exist in the database right now, what will be the relations for the newcomers? This is a slightly harder problem. Newcomers are unlikely to have many connections to the existing network. They are also very unlikely to link to each other. To test this, we take the first 90% of the edges in Epinions as the training set, and use the remaining as the test case. In Wikipedia, we take the first 90% of the elections as training set, and the remaining as the test set. As there is no time information in Slashdot, we are unable to apply this test to this dataset. The results are given in Figure 10. The prediction is worse than random removal case. In fact, it is almost random choice for Wikipedia. But SM/SG does better than FD in both data sets. This is a case in which additional information like homophily based on reviews can be expected to make a big difference in improving the accuracy of predictions.

7.7. Impact of considering weak and strong edges

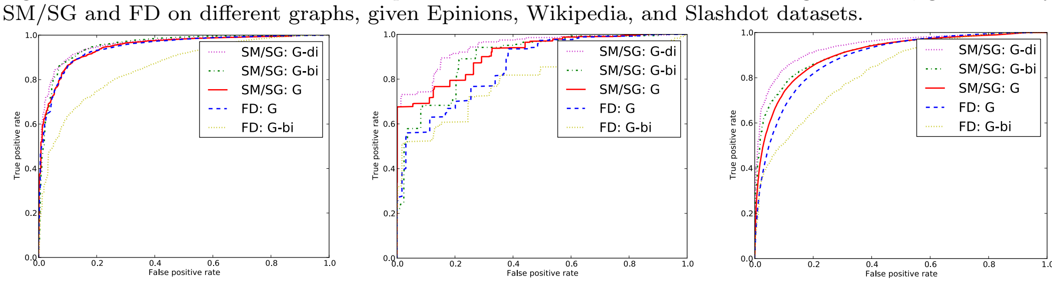
We repeat the experiments by considering strong and weak ties. We check whether it leads to an increased performance using different graphs. In particular, bi-directional edges are

Fig. 10. The ROC curves are drawn upon distances of hidden edges after removing the **last** 10% with respect to time of the edges in Epinions and Wikipedia, generated by SM/SG for G.



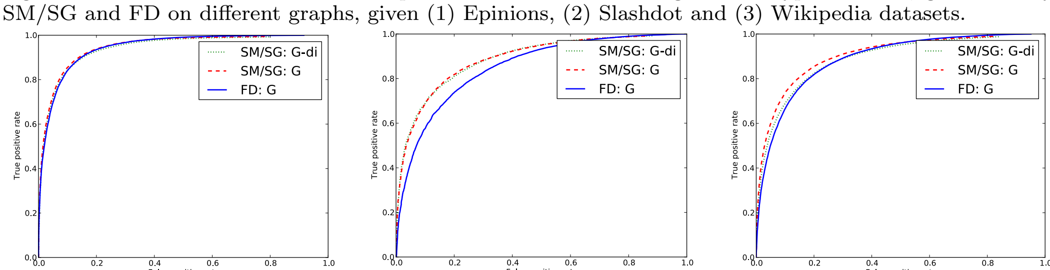
Epinions Wikipedia

Fig. 11. The ROC curves are drawn upon distances of hidden bi-directional edges in G-di, generated by SM/SG and FD on different graphs, given Epinions, Wikipedia, and Slashdot datasets.



Epinions Wikipedia Slashdot

Fig. 12. The ROC curves are drawn upon distances of hidden edges of all types in G-di, generated by SM/SG and FD on different graphs, given (1) Epinions, (2) Slashdot and (3) Wikipedia datasets.



Epinions Wikipedia Slashdot

considered to be of strong strength and are assigned with large weights while single directional edges are considered to be of weak strength and are assigned with small weights. In our practice, we set the weight of a strong edge to be 10 times the weight of a weak edge.

As it is shown in Figure 11, we first compare the prediction performance on testing samples consisting of **bi-directional edges only** in different graphs. For all three datasets, we find SM/SG on G$-di$ has consistently the best performance, followed by SM/SG on G$-bi$, SM/SG on G, FD on G and FD on G-bi. In Wikipedia, the variance of the prediction is relatively big because the testing sample is small ($n \approx 3000$). As argued earlier, relationships based on voting do not naturally lend themselves to bi-directional edges. But, even in this case, the existence of votes of the same type in both directions lend themselves to a dramatic improvement in the prediction of bi-directional edges.

At the best thresholds, SM/SG on G$-di$ achieves 90% $-$ 91% accuracy for Epinions, 87% $-$ 88% accuracy for Slashdot and 84% $-$ 85% accuracy for Wikipedia on both positive and negative hidden edges. On the one hand, the fact that SM/SG on both G$-di$ and G$-bi$ have better performance than it is on G implies that bi-directional edges are indeed of strong strengths, which provide significant influence on network convergence. On the other hand, the superior performance of SM/SG on G$-di$ indicates that both strong and weak ties influence the convergence, and that our convergence model provides an accurate way to model such combined influence.

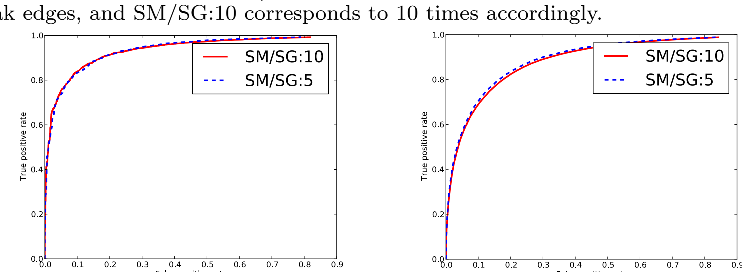
We repeat the comparing experiments on testing samples consisting of both bi-directional edges and single directional edges. As it is shown in Figure 12, SM on G$-di$ does not give better predictions than it is on G. As the single directional edges are assigned with small weights, their influences on convergence is weakened which impacts the prediction accuracy negatively. Considering the fact that single directional edges are the dominant majority in all three datasets, it is reasonable to see compromised overall prediction performance.

We note however that while the number of positive edges is about 3.5-5 times the number of negative edges overall, the number of strong positive edges to the number of negative edges is much more unbalanced. Mutual hatred is generally uncommon. For example, in Epinions, the ratio of strong positive edges to strong negative edges is 50. Hence, both the prediction and the convergence is strongly effected by the strong positive edges as a result. This could be the reason behind the poor performance of FD on G$-bi$ that relies only on the strong edges. We repeat the same ground truth comparison that was shown in Table XII of Section 7.3 for Epinions, this time for G-bi, using the strong edges to determine the optimal threshold. The comparison is shown in Table XIII. The results remain almost the same except there is a slight improvement in all cases except for the strong negative edges. A possible way to counteract the effects of such an imbalance is to increase the weights of strong negative edges.

Table XIII. Average value of ratings along a specific type of trust edge in the whole dataset and after the trust edge was created (S for single (S) edges and Bi for bi-directional edges). Comparing SM/SG on G and G-di.

	Average rating value				Average rating value after trust edge is created			
Edges	S(+)	Bi(+)	S(-)	Bi(-)	S(+)	Bi(+)	S(-)	Bi(-)
Flipped by SM/SG on G-di	4.46	4.82	4.17	3.71	4.99	5.01	4.48	3.49
Flipped by SM/SG on G	4.43	4.80	4.13	3.75	4.98	5.00	4.41	3.93

Fig. 13. The ROC curves for Slashdot. SM/SG:5 corresponds to a run in which strong edges have 5 times the weight of weak edges, and SM/SG:10 corresponds to 10 times accordingly.



(a) Bi-directional edge prediction on G-di (b) All edge prediction on G-di

7.8. Sensitivity to parameter choice

In the previous sections, we have presented a number of results with a specific collection of parameters. In this section, we discuss briefly to which degree the algorithm depends on the selection of these parameters.

We have run our algorithm with various combinations of weights. Even though the weights provide an adjustable parameter for our algorithm, the results do not change considerably in response to small weight changes (e.g. setting $w_- = w_+$). However, w_O must be set to a much smaller value ($w_O << w_-, w_+$), otherwise neutral edges have too much influence in the results. These neutral edges are very noisy as most correspond to pairs who do not know each other. In fact, SM/SG assigns zero weights to these edges without significant penalty.

Note that by significantly changing the weights, it is possible to favor the influence of positive relations over negative, and vice versa. It is also possible to tune the weights in SM/SG at the edge level by incorporating prior information about the underlying network. It is not obvious how such changes can be applied to FD in a principled way and without jeopardizing converge. To illustrate the effect of changes in weight, we run a comparison between two different versions of assigning weights to edges with strong ties. In SM/SG:5, the strong ties have five times the weight as weak ties, and in SMS/SG:10, strong ties have 10 times the weight as weak ties (our original setting). The comparison is shown in Figure 13. We only show slashdot for brevity, but the other results are similar. We note that by lowering the weight of strong edges to 5 times, the strong edge prediction gets slightly worse, but the general edge detection improves slightly. Hence, the weights provide a way to influence which factors the algorithm should prioritize.

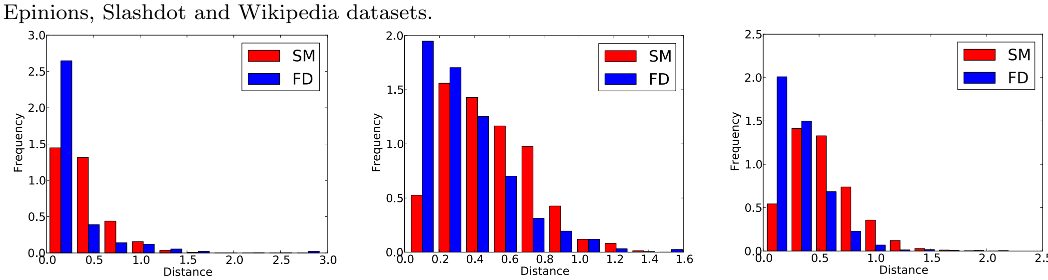
We have also investigated the importance of changing the initial values for distances. We have argued that the initial distances should at least satisfy: $d_+ < \frac{d_O}{2} < \frac{d_-}{2}$ where d_+, d_O, d_- are original distances of a positive edge, a neutral edge and a negative edge. We have run tests on $0.1, 0.6, 1.1$, $0.1, 0.3, 0.9$ and $0.05, 0.5, 2.0$ for d_+, d_O, d_-. The prediction curves remained identical. Overall, the algorithm is not sensitive to the constraints for positive and negative distances as long as the overall above relationship between the distances are preserved.

7.9. General Link Prediction.

We finish this section by discussing a different application of our framework and algorithm. So, far we have been discussing the *edge sign prediction* which deals with the cases in which we already know that an edge exists in the original network. A more general and harder problem is to predict whether there is a positive or a negative edge between a pair of nodes (link prediction [Liben-Nowell and Kleinberg 2003]). The difficulty of these problems stem from the fact that large social networks are not only sparse and are incomplete, but most of the edges in such networks are of weak strength that do not convey much useful information. Our convergence model should be able to make general edge predictions based on distances. If larger distances represent more negativeness (less positiveness), then the distance of a neutral relation should be smaller than a negative one and larger than a positive one. As a consequence, the distribution of neutral edges in terms of distance should concentrate in the middle range. We study this distribution, as a preliminary step towards solving the general edge prediction.

For each dataset, we generate samples based on random source nodes as in samples of G^* we have studied earlier, except that we exclude the edges between the k source nodes. Instead of cross validation, we use the entire sub-network for training, and use the $k(k-1)/2$ edges between the k source nodes as testing data, whose signs are available in the original dataset (positive, negative or neutral, i.e. no link). After convergence the distances of these edges should be representative of their true signs. We repeat the experiments 50 times over all three datasets, and collect the distances for only the neutral testing edges. Figure 14 shows that the distances of neutral testing edges generated by SM do relatively concentrate in the middle-range of values following an almost Gaussian distribution. In contrast, the majority of neutral testing edges's distances by FD have small values, implying a positive prediction is much more likely for FD than for SM. However, SM provides more flexibility as the distances are distributed over a larger range with an almost Gaussian distribution, allowing us to test different tunable algorithms. As a result, our model is a good starting point for developing algorithms for solving the general *link prediction problem*.

Fig. 14. The histograms are drawn upon distances of neutral testing edges, generated by SM and FD for Epinions, Slashdot and Wikipedia datasets.



Epinions Wikipedia Slashdot

8. CONCLUSIONS

In this paper, we introduced a general model for structural balance theory that can handle relation strengths and generalizes the classical balance theory. Our theory builds on the hypothesis that both the strong similarity and the strong dissimilarity in relationships can cause stress in triads, but in different directions. We showed that our theory naturally extends to cases involving relationships with differing strength, allowing us to represent strong and weak ties, as well as trust and distrust relationships. We showed that our theory can handle arbitrary relation strengths drawn from a set of values with a total ordering. Our notion of balance can also be mapped to triangular inequality when relation strengths are modelled with metric distances. Our extended balance theory allows us to formally state the issue of convergence as an optimization problem, in which individuals try to make the smallest change in their relationships to resolve the inherent stress in the triads that they are part of. This problem can be modeled as the metric multidimensional scaling problem for which stress majorization provides minimal solutions.

With the help of both stylized networks and an extensive experimental study, we have shown that our theory can be used to effectively solve the edge sign prediction problem. Its performance exceeds state of the art for this problem. This is due to the fact that positive and negative edges are mapped to a continuous range of strengths based on the constraints provided by the other nodes. However, in contrast with previous work, our method is aware of global constraints based on balance which results in better results overall. We have shown that by considering bi-directional edges as strong ties, the prediction accuracy can be improved considerably for these edges. We provided external validation that relation strength is correlated with rating behavior, strong positive ties resulting in overall higher ratings and string negative ties resulting in higher negative ties. Furthermore, external validation shows that our convergence hypothesis correctly predicts the negative edges that will becomes more positive over time based on rating behavior. These edges are those that change sign by our algorithm during the convergence process. The algorithm is robust to parameter choice and computationally feasible for sparse graphs. Furthermore, we showed that the solutions provided by our method provide a first step towards solving the harder *link prediction problem*.

We are investigating various avenues of future work. Our ability to tailor the weights of different edges allows us to better incorporate external data such as homophily of interests and level of supporting evidence in trust relations into the framework. In such a framework, edges with high homophily are less likely to change their sign from positive to negative for example. Similarly, trust relations with a great deal of past history are less likely to change, if this information can be obtained from the underlying network. Given such measures have been shown to be effective in past work, incorporating them can significantly improve prediction accuracy.

Our method also has applications to many related problems like clustering and link prediction, which we are currently investigating. Our method also allows us to study and compare the characteristics of existing networks towards balance such as the ratio between positive and negative distances, and the distribution of neutral edges. These measures can help us develop new insights into the nature of adversarial relationships in different networks.

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