Today’s Learning

1. Computations of Graphs
2. OpenMP refresher
3. Hands-on: Breadth-First Search
Computations of Graphs

Overview

▶ Vertex-centric Model
▶ Bulk-Synchronous Parallization
▶ Push vs. Pull updating
▶ Storing graphs in memory
Bulk Synchronous Parallel Model

Slides from Rob Bisseling
Parallel computer: abstract model

Bulk synchronous parallel (BSP) computer.
BSP computer

- A BSP computer consists of a collection of processors, each with its own memory. It is a distributed-memory computer.
- Access to own memory is fast, to remote memory slower.
- Uniform-time access to all remote memories.
- No need to open the black box of the communication network. Algorithm designers should not worry about network details, only about global performance.
- Algorithms designed for a BSP computer are portable: they can be run efficiently on many different parallel computers.
Parallel algorithm: supersteps
BSP algorithm

- A BSP algorithm consists of a sequence of supersteps.
- A computation superstep consists of many small steps, such as the floating-point operations (flops) addition, subtraction, multiplication, division. In scientific computing, flops are the common unit for expressing computation cost.
- A communication superstep consists of many basic communication operations, each transferring a data word such as a real or integer from one processor to another.
- In our theoretical algorithms, we distinguish between the two types of supersteps. This helps in the design and analysis of parallel algorithms.
- In our practical programs, we drop the distinction and mix computation and communication freely in each superstep.
Vertex-centric Model

*Slides from Wenfei Fan, QSX: Advanced Topics in Databases*
Vertex-centric models
Bulk Synchronous Parallel Model (BSP)


- Processing: a series of supersteps
- Vertex: computation is defined to run on each vertex
- Superstep \( S \): all vertices compute in parallel; each vertex \( v \) may
  - receive messages sent to \( v \) from superstep \( S - 1 \);
  - perform some computation: modify its states and the states of its outgoing edges
  - Send messages to other vertices (to be received in the next superstep)

Vertex-centric, message passing
Pregel: think like a vertex

- Input: a directed graph G
  - Each vertex v: a node id, and a value
  - Edges: contain values (associated with vertices)

- Vertex: modify its state/edge state/edge sets (topology)

- Supersteps: within each, all vertices compute in parallel

- Termination:
  - Each vertex votes to halt
  - When all vertices are inactive and no messages in transit

- Synchronization: supersteps

Asynchronous: all vertices within each superstep
Example: maximum value

3 6 2 1

Superstep 0

6 6 2 6

6 6 6 6

6 6 6 6

Superstep 1

Superstep 2

Superstep 3

Shaded vertices: voted to halt

message passing
Pushing vs. Pulling
Push vs. Pull

General idea

- We have a graph structure we want to compute on
- We have an algorithm we want to run
- That algorithm utilizes stored per-vertex data
- We iteratively update that data with a vertex-centric computation
- We can update that data by having vertices *push* data updates to their neighbors or *pull* in data updates
  - Either the vertices’ own data gets updated or the neighbors’ data gets updated
Push vs. Pull

Pushing:

▶ Information is pushed – a vertex updates its neighbor’s data

▶ **The Good:**
  ▶ Can be work-optimal – only push needed updates

▶ **The Bad:**
  ▶ Synchronization concerns – race-conditions updating neighbor’s data

▶ **The Algorithms:**
  ▶ Standard breadth-first search – push “discovery” to neighbors and update distance/level/parent data
  ▶ Color Propagation connectivity algorithm – push colors to neighbors
Pull vs. Pull

Pulling:

Pulling:
- Each vertex pulls in information from neighbors to update their own value

The Good:
- Minimal synchronization concerns, only updating own value
- Easier to parallelize – can often get better scaling

The Bad:
- Not necessarily work-optimal – but there exist ways to make it close

The Algorithms:
- Standard PageRank – pull in neighbors’ PageRanks, update own value
- Label Propagation – find max label count among neighbors, update own value to it
An Introduction to OpenMP

Ruud van der Pas
An Introduction Into OpenMP

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Outline

- The OpenMP Programming Model
- OpenMP Guided Tour
- OpenMP Overview
  - Clauses
  - Worksharing constructs
  - Synchronization constructs
  - Environment variables
  - Global Data
  - Runtime functions
- Wrap-up
The OpenMP Programming Model
Shared Memory Model

- **Programming Model**
  - All threads have access to the same, globally shared, memory
  - Data can be shared or private
  - Shared data is accessible by all threads
  - Private data can be accessed only by the threads that owns it
  - Data transfer is transparent to the programmer
  - Synchronization takes place, but it is mostly implicit
About Data

- In a shared memory parallel program variables have a "label" attached to them:

  - **Labelled "Private"** ➣ Visible to one thread only
    - Change made in local data, is not seen by others
    - Example - Local variables in a function that is executed in parallel
  
  - **Labelled "Shared"** ➣ Visible to all threads
    - Change made in global data, is seen by all others
    - Example - Global data
The OpenMP execution model

Fork and Join Model

Master Thread

Parallel region

Synchronization

Worker Threads

"threads"
Example - Matrix times vector

```c
#pragma omp parallel for default(none) \
  private(i,j,sum) shared(m,n,a,b,c)
for (i=0; i<m; i++)
{
    sum = 0.0;
    for (j=0; j<n; j++)
        sum += b[i][j]*c[j];
    a[i] = sum;
}
```

... etc ...

**TID = 0**

```
for (i=0,1,2,3,4)

  i = 0
  sum = \sum b[i=0][j]*c[j]
  a[0] = sum

  i = 1
  sum = \sum b[i=1][j]*c[j]
  a[1] = sum
```

**TID = 1**

```
for (i=5,6,7,8,9)

  i = 5
  sum = \sum b[i=5][j]*c[j]
  a[5] = sum

  i = 6
  sum = \sum b[i=6][j]*c[j]
  a[6] = sum
```

... etc ...
OpenMP Guided Tour
When to consider using OpenMP?

- The compiler may not be able to do the parallelization in the way you like to see it:
  - A loop is not parallelized
    - The data dependency analysis is not able to determine whether it is safe to parallelize or not
  - The granularity is not high enough
    - The compiler lacks information to parallelize at the highest possible level
- This is when explicit parallelization through OpenMP directives and functions comes into the picture
About OpenMP

- The OpenMP programming model is a powerful, yet compact, de-facto standard for Shared Memory Programming
- Languages supported: Fortran and C/C++
- Current release of the standard: 2.5
  - Specifications released May 2005
- We will now present an overview of OpenMP
- Many details will be left out
- For specific information, we refer to the OpenMP language reference manual (http://www.openmp.org)
Terminology

- **OpenMP Team** := Master + Workers

- A **Parallel Region** is a block of code executed by all threads simultaneously
  - The master thread always has thread ID 0
  - Thread adjustment (if enabled) is only done before entering a parallel region
  - Parallel regions can be nested, but support for this is implementation dependent
  - An "if" clause can be used to guard the parallel region; in case the condition evaluates to "false", the code is executed serially

- A **work-sharing construct** divides the execution of the enclosed code region among the members of the team; in other words: they split the work
A loop parallelized with OpenMP

```c
#pragma omp parallel default(none) \ 
    shared(n,x,y) private(i)
{
    #pragma omp for
    for (i=0; i<n; i++)
        x[i] += y[i];
} /*-- End of parallel region --*/

!$omp parallel default(none) \ &
!$omp shared(n,x,y) private(i)
!$omp do
    do i = 1, n
        x(i) = x(i) + y(i)
    end do
!$omp end do
!$omp end parallel
```
Components of OpenMP

Directives
- Parallel regions
- Work sharing
- Synchronization
- Data scope attributes
  - private
  - firstprivate
  - lastprivate
  - shared
  - reduction
- Orphaning

Environment variables
- Number of threads
- Scheduling type
- Dynamic thread adjustment
- Nested parallelism

Runtime environment
- Number of threads
- Thread ID
- Dynamic thread adjustment
- Nested parallelism
- Timers
- API for locking
Directive format

- **C**: directives are case sensitive
  - **Syntax**: `#pragma omp directive [clause [clause] ...]`
  - **Continuation**: use `\` in pragma
  - **Conditional compilation**: `_OPENMP` macro is set

- **Fortran**: directives are case insensitive
  - **Syntax**: sentinel directive [clause [clause] [clause] ...]
  - **The sentinel is one of the following**:
    - ✓ `!$OMP` or `C$OMP` or `*$OMP` (fixed format)
    - ✓ `!$OMP` (free format)
  - **Continuation**: follows the language syntax
  - **Conditional compilation**: `!$` or `C$` -> 2 spaces
A more elaborate example

```c
#pragma omp parallel if (n>limit) default(none) \ 
    shared(n,a,b,c,x,y,z) private(f,i,scale)
{
    f = 1.0;
    #pragma omp for nowait
    for (i=0; i<n; i++)
        z[i] = x[i] + y[i];
    #pragma omp for nowait
    for (i=0; i<n; i++)
        a[i] = b[i] + c[i];
    #pragma omp barrier
    ....
    scale = sum(a,0,n) + sum(z,0,n) + f;
    ....
} /*-- End of parallel region --*/
```

- **parallel loop** (work will be distributed)
- **synchronization**
- **parallel region**
- **Statement is executed by all threads**
Another OpenMP example

```c
1 void mxv_row(int m, int n, double *a, double *b, double *c)
2 {
3    int i, j;
4    double sum;
5
6    #pragma omp parallel for default(none) \ 
7        private(i, j, sum) shared(m, n, a, b, c)
8    for (i=0; i<m; i++)
9    {
10       sum = 0.0;
11      for (j=0; j<n; j++)
12         sum += b[i*n+j]*c[j];
13      a[i] = sum;
14    } /*-- End of parallel for --*/
15 }
```

% cc -c -fast -xrestrict -xopenmp -xloopinfo mxv_row.c
"mxv_row.c", line  8: PARALLELIZED, user pragma used
"mxv_row.c", line 11: not parallelized
OpenMP performance

Matrix too small *

*) With the IF-clause in OpenMP this performance degradation can be avoided
Some OpenMP Clauses
About OpenMP clauses

- Many OpenMP directives support clauses
- These clauses are used to specify additional information with the directive
- For example, `private(a)` is a clause to the for directive:
  - `#pragma omp for private(a)`
- Before we present an overview of all the directives, we discuss several of the OpenMP clauses first
- The specific clause(s) that can be used, depends on the directive
The if/private/shared clauses

- **if (scalar expression)**
  - Only execute in parallel if expression evaluates to true
  - Otherwise, execute serially

- **private (list)**
  - No storage association with original object
  - All references are to the local object
  - Values are undefined on entry and exit

- **shared (list)**
  - Data is accessible by all threads in the team
  - All threads access the same address space

```c
#pragma omp parallel if (n > threshold) 
    shared(n,x,y) private(i)
{
    #pragma omp for
     for (i=0; i<n; i++)
        x[i] += y[i];
} /*-- End of parallel region --*/
```
About storage association

- **Private variables are undefined on entry and exit of the parallel region**
- **The value of the original variable (before the parallel region) is undefined after the parallel region!**
- **A private variable within a parallel region has no storage association with the same variable outside of the region**
- **Use the first/last private clause to override this behaviour**
- **We will illustrate these concepts with an example**
main()
{
    A = 10;

#pragma omp parallel
{
    #pragma omp for private(i) firstprivate(A) lastprivate(B)...
    for (i=0; i<n; i++)
    {
        ....
        B = A + i;  /*-- A undefined, unless declared firstprivate --*/
        ....
    }
    C = B;  /*-- B undefined, unless declared lastprivate --*/
}
}  /*-- End of OpenMP parallel region --*/
The first/last private clauses

**firstprivate (list)**

- All variables in the list are initialized with the value the original object had before entering the parallel construct.

**lastprivate (list)**

- The thread that executes the sequentially last iteration or section updates the value of the objects in the list.
The default clause

**default ( none | shared | private )**

**default ( none | shared )**

- **none**
  - ✔ No implicit defaults
  - ✔ Have to scope all variables explicitly

- **shared**
  - ✔ All variables are shared
  - ✔ The default in absence of an explicit "default" clause

- **private**
  - ✔ All variables are private to the thread
  - ✔ Includes common block data, unless THREADPRIVATE

Fortran

C/C++

Note: default(private) is not supported in C/C++
The reduction clause - example

```
sum = 0.0
!$omp parallel default(none) &
!$omp shared(n,x) private(i)
!$omp do reduction (+:sum)
    do i = 1, n
        sum = sum + x(i)
    end do
!$omp end do
!$omp end parallel
print *,sum
```

☞ Variable SUM is a shared variable

☞ Care needs to be taken when updating shared variable SUM

☞ With the reduction clause, the OpenMP compiler generates code such that a race condition is avoided
The reduction clause

reduction ( [operator | intrinsic] ) : list )

Fortran

reduction ( operator : list )

C/C++

✔ Reduction variable(s) must be shared variables

✔ A reduction is defined as:

Fortran
\[
\begin{align*}
x &= x \text{ operator } expr \\
x &= expr \text{ operator } x \\
x &= \text{ intrinsic } (x, \text{ expr_list}) \\
x &= \text{ intrinsic } (\text{expr_list}, x)
\end{align*}
\]

C/C++
\[
\begin{align*}
x &= x \text{ operator } expr \\
x &= expr \text{ operator } x \\
x &= ++x, ++x, x--, --x \\
x &= \text{binop} = expr
\end{align*}
\]

✔ Note that the value of a reduction variable is undefined from the moment the first thread reaches the clause till the operation has completed

✔ The reduction can be hidden in a function call

Check the docs for details
The nowait clause

- To minimize synchronization, some OpenMP directives/pragmas support the optional nowait clause.
- If present, threads will not synchronize/wait at the end of that particular construct.
- In Fortran the nowait is appended at the closing part of the construct.
- In C, it is one of the clauses on the pragma.

```
#pragma omp for nowait
{
    :
}
```

```
!$omp do 
   :
   :
!$omp end do nowait
#pragma omp for nowait
```
The parallel region

A parallel region is a block of code executed by multiple threads simultaneously

```c
#pragma omp parallel [clause[[], clause] ...]
{
    "this will be executed in parallel"
}
(implied barrier)

!$omp parallel [clause[[], clause] ...]

    "this will be executed in parallel"

!$omp end parallel (implied barrier)
```
A parallel region supports the following clauses:

- if (scalar expression)
- private (list)
- shared (list)
- default (none|shared) (C/C++)
- default (none|shared|private) (Fortran)
- reduction (operator: list)
- copyin (list)
- firstprivate (list)
- num_threads (scalar_int_expr)
Worksharing Directives
The **OpenMP work-sharing constructs**

- **The work is distributed over the threads**
- **Must be enclosed in a parallel region**
- **Must be encountered by all threads in the team, or none at all**
- **No implied barrier on entry; implied barrier on exit (unless nowait is specified)**
- **A work-sharing construct does not launch any new threads**

```c
#pragma omp for
{
    ....
}
!$OMP DO
    ....
!$OMP END DO

#pragma omp sections
{
    ....
}
!$OMP SECTIONS
    ....
!$OMP END SECTIONS

#pragma omp single
{
    ....
}
!$OMP SINGLE
    ....
!$OMP END SINGLE
```
The WORKSHARE construct

Fortran has a fourth worksharing construct:

```
!$OMP WORKSHARE
    <array syntax>
!$OMP END WORKSHARE [NOWAIT]
```

Example:

```
!$OMP WORKSHARE
    A(1:M) = A(1:M) + B(1:M)
!$OMP END WORKSHARE NOWAIT
```
The omp for/do directive

The iterations of the loop are distributed over the threads

```c
#pragma omp for [clause[,] clause] ... ]
    <original for-loop>
```

```c
$omp do [clause[,] clause] ... ]
    <original do-loop>
$omp end do [nowait]
```

Clauses supported:

- private
- firstprivate
- lastprivate
- reduction
- ordered*
- schedule
- nowait

*) Required if ordered sections are in the dynamic extent of this construct
The omp for directive - example

```c
#pragma omp parallel default(none)\  
   shared(n,a,b,c,d) private(i)  
{
    #pragma omp for nowait
    for (i=0; i<n-1; i++)
    b[i] = (a[i] + a[i+1])/2;

    #pragma omp for nowait
    for (i=0; i<n; i++)
    d[i] = 1.0/c[i];

} /*-- End of parallel region --*/

(implied barrier)
```
Load balancing

- **Load balancing is an important aspect of performance**

- For regular operations (e.g., a vector addition), load balancing is not an issue.

- For less regular workloads, care needs to be taken in distributing the work over the threads.

- **Examples of irregular workloads:**
  - Transposing a matrix
  - Multiplication of triangular matrices
  - Parallel searches in a linked list

- For these irregular situations, the schedule clause supports various iteration scheduling algorithms.
The schedule clause/1

```
schedule ( static | dynamic | guided [, chunk] )
schedule (runtime)
```

static [, chunk]

- Distribute iterations in blocks of size "chunk" over the threads in a round-robin fashion
- In absence of "chunk", each thread executes approx. N/P chunks for a loop of length N and P threads

**Example: Loop of length 16, 4 threads:**

<table>
<thead>
<tr>
<th>TID</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>no chunk</td>
<td>1-4</td>
<td>5-8</td>
<td>9-12</td>
<td>13-16</td>
</tr>
<tr>
<td>chunk = 2</td>
<td>1-2</td>
<td>3-4</td>
<td>5-6</td>
<td>7-8</td>
</tr>
<tr>
<td></td>
<td>9-10</td>
<td>11-12</td>
<td>13-14</td>
<td>15-16</td>
</tr>
</tbody>
</table>
The schedule clause/2

- **dynamic [, chunk]**
  - Fixed portions of work; size is controlled by the value of chunk
  - When a thread finishes, it starts on the next portion of work

- **guided [, chunk]**
  - Same dynamic behaviour as "dynamic", but size of the portion of work decreases exponentially

- **runtime**
  - Iteration scheduling scheme is set at runtime through environment variable OMP_SCHEDULE
The experiment

500 iterations on 4 threads

Thread ID

0 1 2 3

Iteration Number

0 50 100 150 200 250 300 350 400 450 500

static
guided, 5
dynamic, 5

500 iterations on 4 threads
Synchronization Controls
Suppose we run each of these two loops in parallel over i:

```c
for (i=0; i < N; i++)
   a[i] = b[i] + c[i];
```

```c
for (i=0; i < N; i++)
   d[i] = a[i] + b[i];
```

This may give us a wrong answer (one day)

Why?
We need to have updated all of a[] first, before using a[]

```
for (i=0; i < N; i++)
    a[i] = b[i] + c[i];
```

```
for (i=0; i < N; i++)
    d[i] = a[i] + b[i];
```

All threads wait at the barrier point and only continue when all threads have reached the barrier point
Each thread waits until all others have reached this point:

```c
#pragma omp barrier

!$omp barrier
```
When to use barriers?

- When data is updated asynchronously and the data integrity is at risk

- Examples:
  - Between parts in the code that read and write the same section of memory
  - After one timestep/iteration in a solver

- Unfortunately, barriers tend to be expensive and also may not scale to a large number of processors

- Therefore, use them with care
If sum is a shared variable, this loop can not be run in parallel

```c
for (i=0; i < N; i++){
    ......
    sum += a[i];
    ......
}
```

We can use a critical region for this:

```c
for (i=0; i < N; i++){
    ......
    sum += a[i];
    ......
    one at a time can proceed
    next in line, please
}
```
- Useful to avoid a race condition, or to perform I/O (but which still will have random order)

- Be aware that your parallel computation may be serialized and so this could introduce a scalability bottleneck (Amdahl's law)
All threads execute the code, but only one at a time:

```c
#pragma omp critical [(name)]
 {<code-block>}

!$omp critical [(name)]
 <code-block>
 !$omp end critical [(name)]
```

There is no implied barrier on entry or exit!

```c
#pragma omp atomic
 <statement>

!$omp atomic
 <statement>
```

This is a lightweight, special form of a critical section

```c
#pragma omp atomic
 a[indx[i]] += b[i];
```
This construct is ideally suited for I/O or initialization

for (i=0; i < N; i++)
{
    
    "read a[0..N-1]";
    
}

"declare A to be be shared"

#pragma omp parallel for
for (i=0; i < N; i++)
{
    
    "read a[0..N-1]";
    
}

one volunteer requested

thanks, we're done

May have to insert a barrier here
- Usually, there is a barrier needed after this region
- Might therefore be a scalability bottleneck (Amdahl's law)

Threads wait in the barrier
**SINGLE and MASTER construct**

**Only one thread in the team executes the code enclosed**

```
#pragma omp single [clause[[], clause] ...]
{
    <code-block>
}
```

```
!$omp single [clause[[], clause] ...]
    <code-block>
!$omp end single [nowait]
```

**Only the master thread executes the code block:**

```
#pragma omp master
{<code-block>}
```

```
!$omp master
    <code-block>
!$omp end master
```

There is no implied barrier on entry or exit!
More synchronization directives

The enclosed block of code is executed in the order in which iterations would be executed sequentially:

```c
#pragma omp ordered
{<code-block>}

!$omp ordered
<code-block>
!$omp end ordered
```

Ensure that all threads in a team have a consistent view of certain objects in memory:

```c
#pragma omp flush [(list)]

!$omp flush [(list)]
```

Expensive!

In the absence of a list, all visible variables are flushed.
OpenMP provides for a compact, but yet powerful, programming model for shared memory programming

OpenMP supports Fortran, C and C++

OpenMP programs are portable to a wide range of systems

An OpenMP program can be written such that the sequential version is still “built-in”
Graph Representations, Computing for Data Analytics: Methods and Tools

Da KuangG, Polo Chau
Sparse matrix: Graph adjacency matrix

How to represent a graph?

A node in a graph is typically connected to only a small fraction of nodes.

Source: www.cs.umn.edu/~metis
Sparse matrix is often very sparse

Term-document matrix for 4.5M English Wikipedia articles:
0.05% nonzeros

DBLP co-authorship network for 300,000 academic authors:
0.0007% nonzeros

→ We need efficient storage for sparse matrices.
Storage of a sparse matrix

We store only the nonzeros and their positions
  ◦ (row, column, value)-triplet

Use the same example:

(1, 2, 1) (1, 3, 1) (1, 5, 1)
(2, 1, 1) (2, 3, 1) (2, 4, 1)
(3, 1, 1) (3, 2, 1) (3, 4, 1) (3, 5, 1)
(4, 2, 1) (4, 3, 1) (4, 6, 1) (4, 7, 1)
(5, 1, 1) (5, 3, 1) (5, 6, 1)
(6, 4, 1) (6, 5, 1) (6, 7, 1)
(7, 4, 1) (7, 6, 1)

This is the “edge list” format; in this case, an array of tuples of length 3.

Viewing indices of the matrix as graph nodes, these triplets are edges.

Symmetric sparse matrix \((A = A^T)\) \(\iff\) Undirected graph

What about the adjacency matrix of **directed graph**? And **Bipartite graph**?
Coordinate list (COO) format

The triplets can be stored as 3 arrays: rows, cols, values.

rows = [0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6]
cols = [1, 2, 4, 0, 2, 3, 0, 1, 3, 4, 1, 2, 5, 6, 0, 2, 5, 3, 4, 6, 3, 5]
values = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

Note: 0-based arrays
Compressed sparse row (CSR) format

Suppose a sparse matrix has \( \text{nnz} \) nonzero entries.

\[
\text{rows} = [0, 0, 0, 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6] \\
\text{cols} = [1, 2, 4, 0, 2, 3, 0, 1, 3, 4, 1, 2, 5, 6, 0, 2, 5, 3, 4, 6, 3, 5] \\
\text{values} = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] 
\]

The COO format needs \( 3\text{nnz} \) elements to store the matrix. Can we do better?

When the nonzeros are stored row by row (and row IDs start at 0), we can compress the above storage:

\[
\text{rowptr} = [0, 3, 6, 10, 14, 17, 20, 22] \\
\text{colind} = [1, 2, 4, 0, 2, 3, 0, 1, 3, 4, 1, 2, 5, 6, 0, 2, 5, 3, 4, 6, 3, 5] \\
\text{values} = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] 
\]

This CSR format needs \( 2\text{nnz}+n \) elements to store the matrix.

Similarly, we have compressed sparse column (CSC) format.
Breadth-First Search

Overview

- General Algorithm
- “Pushing”
- “Pulling”
- C++ demonstration
Why BFS? Prototypical graph algorithm, high memory access/communication to computation ratio. Has been used as an example for extreme optimization (Graph500.org)

- We select a root
- We want to figure out the number of hops/distance of every vertex reachable from the root
- Naturally iterative – one level/hop from the root at a time
- Algorithm concludes when no new vertices are found on a level
Breadth-first search - pushing

1: procedure BFS($G(V, E), \text{root}$)
2: for all $v \in V$ do
3: $\text{Levels}(v) \leftarrow -1$ \hfill $\triangleright$ Initialize levels
4: $level \leftarrow 0$
5: $Q \leftarrow \text{root}$
6: $\text{Levels}(\text{root}) \leftarrow level$
7: while $Q \neq \emptyset$ do \hfill $\triangleright$ Finishing when queue is empty
8: $level \leftarrow level + 1$
9: for all $v \in Q$ do
10: for all $\langle v, u \rangle \in E$ do
11: if $\text{Level}(u) < 0$ then \hfill $\triangleright$ Have we discovered $u$?
12: $\text{Level}(u) \leftarrow level$ \hfill $\triangleright$ $v$ pushes update to $u$
13: $Q_{next} \leftarrow u$
14: Swap($Q, Q_{next}$)
15: $Q_{next} \leftarrow \emptyset$
Breadth-first search - pulling

1: procedure BFS(G(V, E), root)
2:   for all v ∈ V do
3:     Levels(v) ← −1
4:   level ← 0
5:   Q ← root
6:   Levels(root) ← level
7:   size = 1
8:   while size > 0 do               ▷ Instead of a queue, just track level size
9:     level ← level + 1
10:    size ← 0
11:   for all v ∈ V do
12:     if level(v) < 0 then            ▷ We haven’t discovered v yet
13:       for all ⟨v, u⟩ ∈ E do
14:         if Level(u) = level − 1 then
15:           Level(v) ← level           ▷ v pulls update from u
16:             size ← size + 1
17:           break                    ▷ No need to go further
C++ Demonstration – Blank code and data available on website

www.cs.rpi.edu/~slotag/classes/FA16/index.html