1 Degrees

For simplicity, we're going to use variables n and m regularly as:

$$n = |V(G)|, m = |E(G)|$$

The **degree** of a vertex is the number of incident edges. We write degree of vertex v_i as $d(v_i)$ or sometimes d_i. For a graph G, the maximum degree is $\Delta(G)$ and the minimum degree is $\delta(G)$. A graph is **regular** if $\Delta(G) = \delta(G)$. A graph is k**-regular** if $k = \Delta(G) = \delta(G)$.

The degree sum formula shows that the sum of the degrees of all vertices in a graph is always even:

$$\sum_{v \in V(G)} d(v) = 2m$$

So it follows that there can only be an even number of vertices of odd degree in G.

The average degree of a graph G is $\frac{2m}{n}$. Therefore:

$$\delta(G) \leq \frac{2m}{n} \leq \Delta(G)$$

A **hypercube**, or k**-dimensional hypercube** Q_k is a simple graph whose vertices are k-tuples of $\{0, 1\}$ and whose edges are the pairs of k-tuples that differ by one.

Prove a hypercube is a (regular) bipartite graph.

Prove that any k-regular bipartite graph has the same number of vertices in each partite set.

2 Extremal Problems

An **extremal problem** asks for the maximum or minimum value of a function over a class of objects. We'll do a couple extremal proofs related to degrees and connectivity.

Prove the minimum number of edges in a connected graph is $(n - 1)$.

Prove a graph must be connected if $\delta(G) \geq \frac{(n-1)}{2}$.

3 Graphic Sequences

The **degree sequence** of a graph is the list of vertex degrees, usually in decreasing order: $d_1 \geq d_2 \geq \ldots \geq d_n$.

A **graphic sequence** is a list of nonnegative numbers that is the degree sequence of a simple graph. A simple graph G with degree sequence S *realizes* S.

A sequence $S = \{d_1, d_2, \ldots, d_n\}$ is a graphic sequence iff sequence $S' = \{d_2 - 1, \ldots, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n\}$ is a graphic sequence, where $d_1 \geq d_2 \geq \ldots \geq d_n$ and $n \geq 2$ and $d_1 \geq 1$.