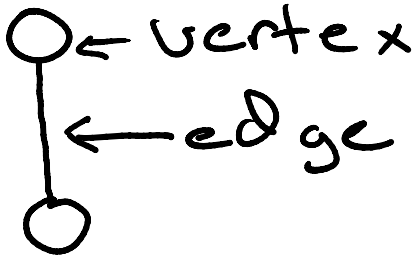
Graph Theory - Lecture 1

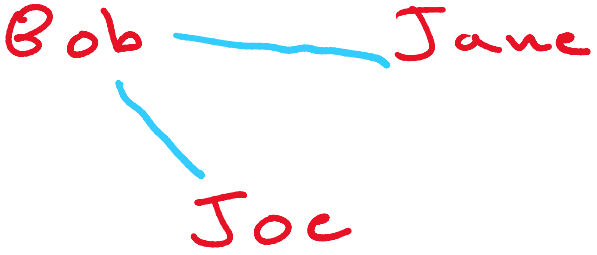
Tuesday, January 26, 2021 11:53 AM

What is a graph?

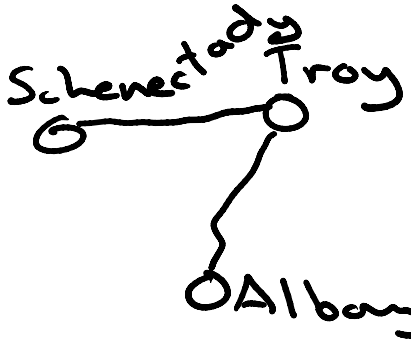


Real-world graphs

social networks



Road networks



Web

Properties: #edges #vertices

- Sparsity → $|E| << |V|^2$
- Degree Skew → $d_{min} << d_{max}$
- Hubs → have some importance
- Irregularity → no inherent structure
- Small-world → low diameter

Basic Definitions

Graph G → tuple of vertices $V(G)$

$G = \{V(G), E(G)\}$ edges $E(G)$

for some $e \in E(G) \rightarrow e = (u, v)$

where $u, v \in V(G)$

u, v are endpoints of e

e joins u, v

u, v are incident on e

u, v are adjacent to each other

u, v are in each others' neighborhood

$u \in N(v), v \in N(u)$

The degree of v is the number of edges incident on v

$d(v) = |N(v)|$

order of $G = |V(G)| = n$

size of $G = |E(G)| = m$

If $|V|$ and $|E| < \infty$

$\rightarrow G$ is finite

If $|V|$ and $|E| = 0$ $\quad V = \phi$ ← empty set

$\rightarrow G$ is null

If $|V| = 1$ and $|E| = 0$

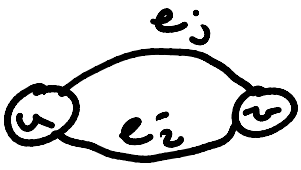
$\rightarrow G$ is a trivial graph

If $|V| \geq 1$ and $|E| = 0$

$\rightarrow G$ is an empty graph

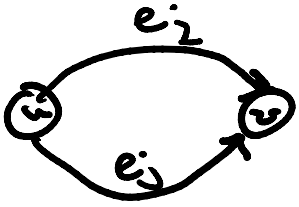
If $\exists\, e_i, e_j \in E(G)$ s.t. $e_i = (u, v)$

$e_j = (u, v)$



$\rightarrow G$ is a multi-graph

e_i, e_j are multi-edges



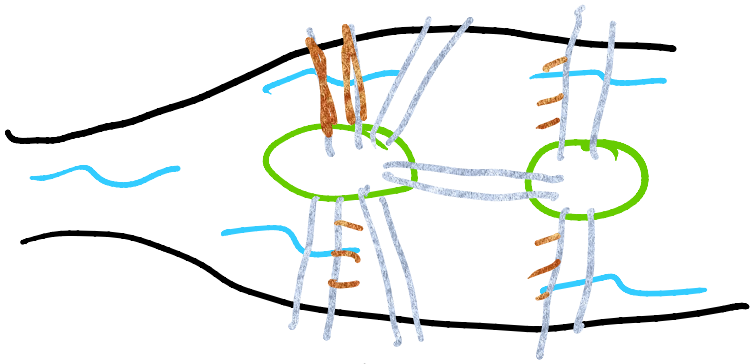
→ self-loop → $e = (u, u)$

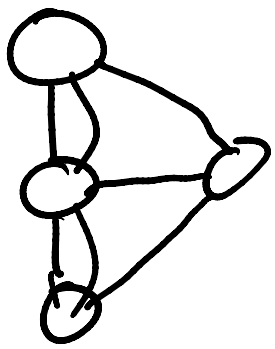
→ undirected self-loop

If G is simple ↔ G has no multi-edges

History of Graph theory

Königsberg





Euler: 

Can I start at one point, and traverse each bridge exactly once → return to start

Solution:

→ Invent graph theory

"Euler tour"