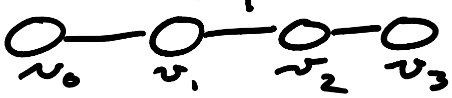
Lecture 2

Friday, January 29, 2021 12:12 PM

Types of Graphs

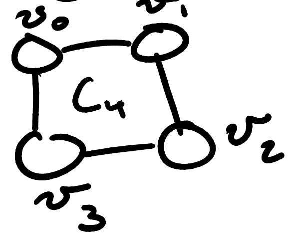
Path: P_4



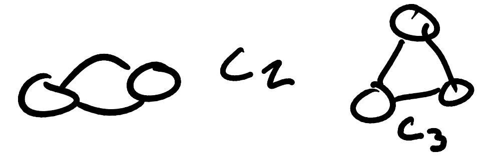
P_n $\{v_0 v_1 v_2 v_3\}$

→ Graphs that can have their vertices listed s.t. if two vertices are adjacent in the list → adjacent in the graph

Cycle:



$C_n \to |V(C_n)| = n$



→ Can also have their vertizes ordered s.t. " " " "

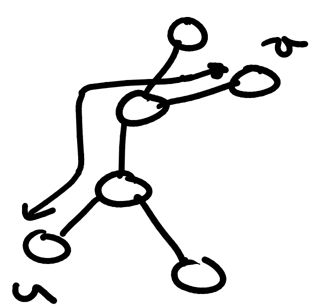
BUT: start and end vertices in the list are the same

Tree: simple acyclic graph and connected

all paths are trees

no cycles are trees

acyclic → $\nexists C_n \in T$



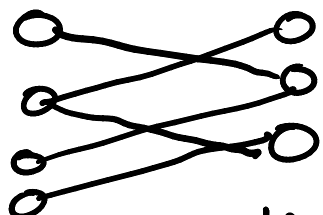
Note: no root

Bipartite graph: a graph containing

Bipartite graph: a graph containing two disjoint independent vertex sets

$B_{X,Y} \to V(B_{X,Y}) = X + Y$

$\forall x_1, x_2 \in X \to e = (x_1, x_2) \notin E(B_{X,Y})$



X Y

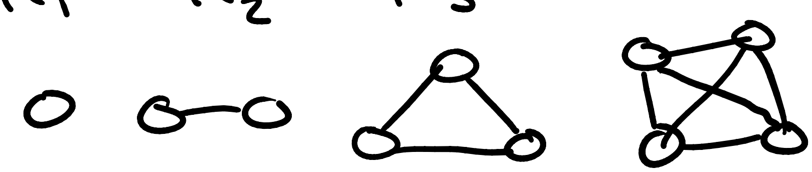
Complete graph: a clique

$K_n \to \forall u, v \in V(K_n) \to (u, v) \in E(K_n)$

(simple)

→ all possible edges exist

K_1 K_2 K_3 K_4



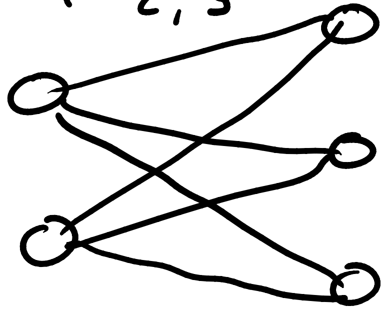
Complete bipartite graph

$K_{n,m} : |X| = n, |Y| = m$

$\forall x \in |X|, \forall y \in |Y| \to e = (x, y) \in E(K_{n,m})$

$K_{2,3}$

$K_{2,3}$

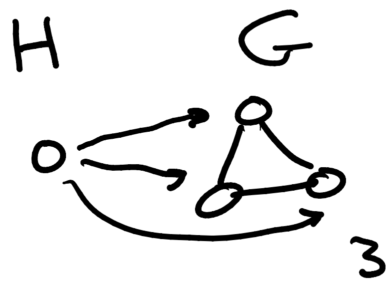


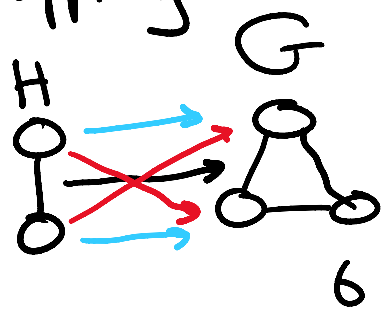
Subgraph: H is a subgraph of G if H is entirely contained within G

$\rightarrow \forall v \in V(H) \rightarrow u \in V(G)$

$(u,v) \in E(H) \rightarrow e \in E(G)$

$\exists$ this mapping





$\forall$: for every

$\exists$: there exists

$\in$: within

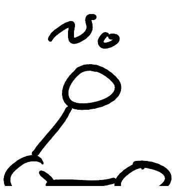
$\Rightarrow$: implies

$\Leftrightarrow$: if and only if

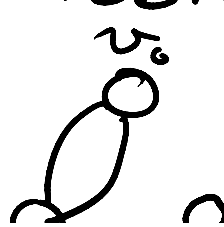
Adjacency matrix representation

A : $n \times n$ matrix representing some graph G, $|V(G)| = n$

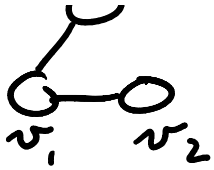
$\rightarrow$ if a_{ij} is nonzero $\Leftrightarrow \exists (i,j) \in E(G)$



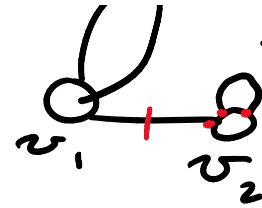
$$\begin{array}{c|ccc} & v_0 & v_1 & v_2 \\ \hline v_0 & 0 & 1 & 0 \\ v_1 & 1 & 0 & 1 \end{array}$$



$$\begin{array}{c|ccc} & v_0 & v_1 & v_2 \\ \hline v_0 & 0 & 2 & 0 \\ v_1 & 2 & 0 & 1 \end{array}$$



$$\begin{array}{c} v_1 \\ v_2 \end{array}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



$$\begin{array}{c} v_1 \\ v_2 \end{array}\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & \boxed{2} \end{bmatrix}$$

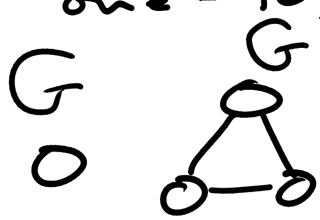
$\sum row_i \to d(v_i)$ $\quad d(v_2) = 3$

multigraph $\to$ For some $u, v \in V(G)$

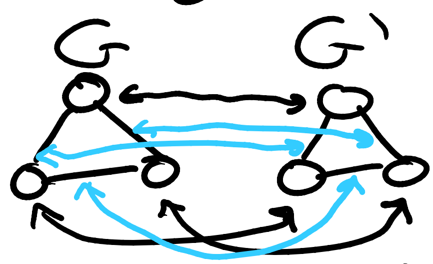
$\exists\, e_i e_j$ s.t. $e_i = (u, v)$
$i \neq j$ $\qquad e_j = (u, v)$

Graph Isomorphism

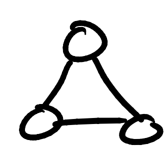
$G = (V, E)$ and $G' = (V', E')$ are isomorphic if there exists a one-to-one mapping $\underline{\forall} v \in V \to u \in V'$ (all vertices)
$\underline{\forall} e = (u, v) \in E$ $\to$ all edges in E'



not isomorphic



yes isomorphic



isomorphic

If $G \cong G'$ $\qquad$ 2, 2, 2

$\Rightarrow |V| = |V'|, \; |E| = |E'|$

$\Rightarrow$ sorted degree sequence is equivalent

degree sequence $\to$ listing of degrees degree sequence → listing of degrees

⇒ Equivalent diameter
diameter → longest shortest path in some graph

⇒ Equivalent girth
girth → shortest cycle

Note: these properties are <u>necessary</u> but not <u>sufficient</u>

Isomorphic relation between G, G'

$G \cong G$ → reflexive

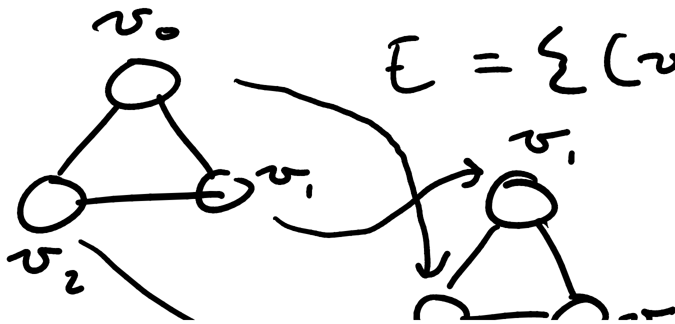
$G \cong G'$ and $G' \cong G$ → symmetric

if $G \cong G'$ and $G' \cong H \Rightarrow G \cong H$
→ transative property

Automorphism

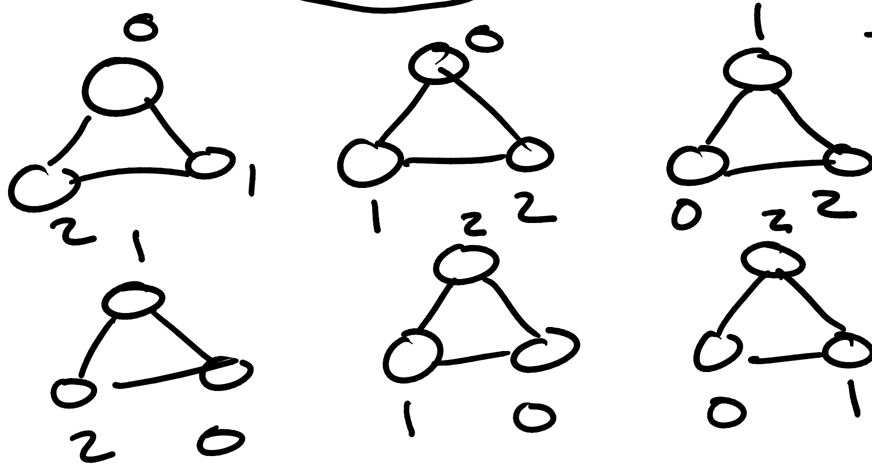
→ an isomorphism of G to itself s.t. the edge list is preserved

$E = \{(v_0, v_1), (v_0, v_2), (v_1, v_2)\}$



K_n: any vertex can be to any of the

be to any of the n vertices

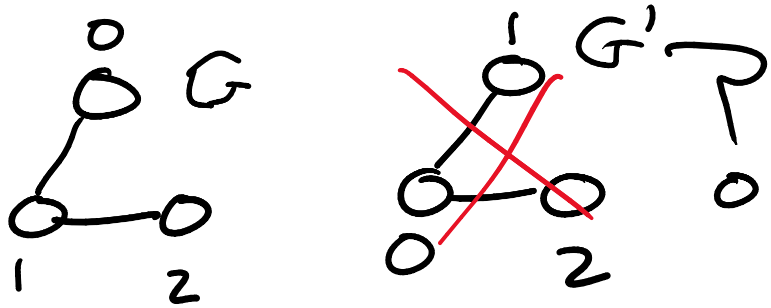


→ next vertex can be mapped to $n-1$ vertices

...

n factorial possible automorphisms

$3! \to 6$ automorphisms



not an automorphic mapping

$E \neq E'$

$E = \{(0,1),(1,2)\}$ $E' = \{(0,1),(0,2)\}$

G G' $E = \{(0,1),(1,2)\}$

$0 \to f(0) = 1$

$1 \to f(1) = 0$

$2 \to f(2) = 2$

$E' = \{(f(0), f(1)), (f(1), f(2))\}$

$E' = \{(1,0),(0,2)\}$