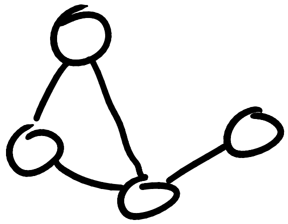
Lecture 6 - Degree sequences and trees

Friday, February 12, 2021 1:16 PM

Graphic sequence: a sequence of degrees that can realize a simple undirected graph

$\exists$ some G w/ the degrees



$S = \{1, 2, 2, 3\}$

degree sequence which is graph

How can we tell if a sequence is graphic?

* sum of degrees must be even
  → Necessary but not sufficient

* Sum $\leq 2|E(K_n)|$ with $|S| = n$

woof

Big Dawg of GT

Havel-Hakimi theorem

A sequence $S = \{d_1, d_2, \dots, d_n\}$ is graphic iff sequence

$$S' = \{d_2 - 1, d_3 - 1, \dots, d_{d_1+1} - 1, \dots d_n\}$$

$S - \{d_2 - 1, d_3 - 1 \ldots, \underbrace{d_{d_1+1}} - 1, \ldots d_n\}$

is graphic

where $d_1 \geq d_2 \geq \ldots \geq d_n$

$S = \{3, 2, 2, 1\}$ $\quad d_{d_1+1} \rightarrow d_{3+1}$
$\quad\quad\quad\ \ d_1 \ \ d_2 \ \ d_3 \ \ d_4 \quad\quad = d_4$

$\uparrow$ graph

$\quad\quad\quad\quad d_1 \ \ d_2 \ \ d_3$

$S \quad S' = \{1, 1, 0\}$

$\quad\quad\quad\quad d_2 - 1 \ \ d_3 - 1 \ \ d_4 - 1$

asking if this is graphic, is same as asking if this is graphic

$\uparrow$ $d_2 = d_{d_1+1}$

$S' \quad S'' = \{0, 0\}$ ✓

$\quad\quad\quad\quad 0 \quad 0 \leftarrow$ vertices

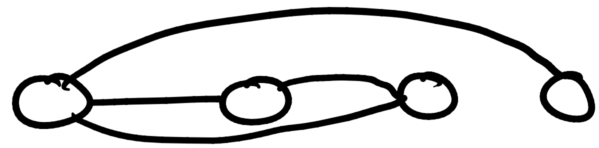
Graph realization $\rightarrow$ constructing a graph given a degree sequence

To realize using H-H: map each value in the sequence to a vertex and then draw an edge (d_i, d_j) same d_i is decremented by the removal of d_j from $S \rightarrow S'$

$S = \{\cancel{3}\ 2\ 2\ 1\}$

$S = \{\cancel{3}, 2, 2, 1\}$

$-1 \quad -1 \quad -1$



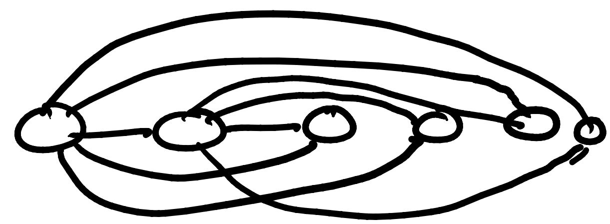
$S' = \{\cancel{1}, 1, 0\}$

-1

$S'' = \{0, 0\}$ D

$S = \{\cancel{5}, 5, 2, 2, 2, 2\}$

$-1 \quad -1 \quad -1 \quad -1 \quad -1$



$S' = \{\cancel{4}, 1, 1, 1, 1\}$

$-1 \quad -1 \quad -1 \quad -1$

$S'' = \{0, 0, 0, 0\}$

$S = \{\cancel{6}, 5, \cancel{4}, 3, 2\}$

$-1 \quad -1 \quad -1 \quad -1 \quad +1$

$S = \{3, 2, 1, 1\}$

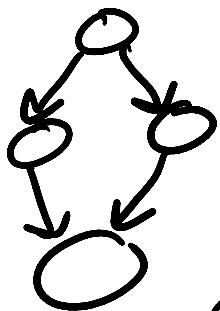
$S' = \{1, 0, \cancel{0}\}$ σ

Trees

Tree - a connected undirected simple acyclic graph

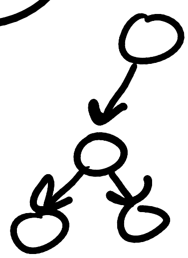
Forest - a disconnected undirected simple acyclic graph simple acyclic graph

Acyclic - no cycles



DAG - directed acyclic graph

Polytree - a DAG where the underlying graph is a tree



For Tree T - Necessary conditions

T is minimally connected

- removing any edge will disconnect T

T is maximally acyclic

- adding any edge will create a cycle

T has $|E(T)| = |V(T)| - 1$

T has a single u,v-path $\forall u,v \in V(T)$

Prove: T is a tree $\rightarrow$ T is bipartite

Prove: T is a tree → T is bipartite

using induction on edge or vertices

Basis $P(n=1) \rightarrow$ ○ ✓ ← $P(m=0)$

$P(m=1) \rightarrow$ ○—○

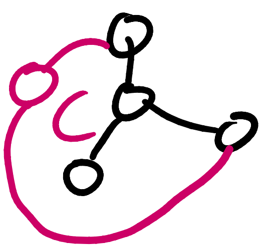
$P(k)$ case $k = |V(T)|$ or $|E(T)|$

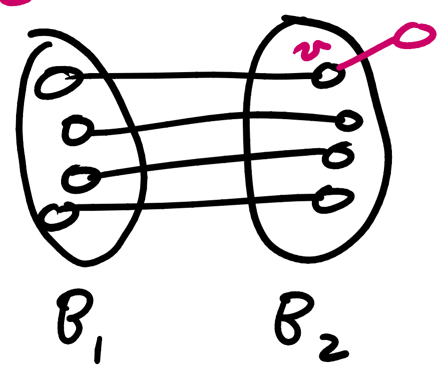
→ $P(k+1)$ case: add an edge and a vertex

↳ By definition of a tree, must be a leaf

Assume $P(k)$ is bipartite

Note: not true representation





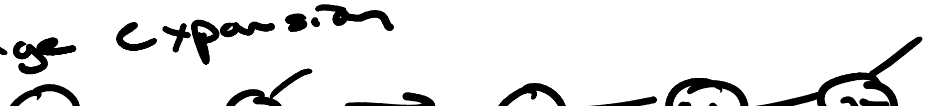
Add an edge and leaf vertex

leaf → degree-1 vertex

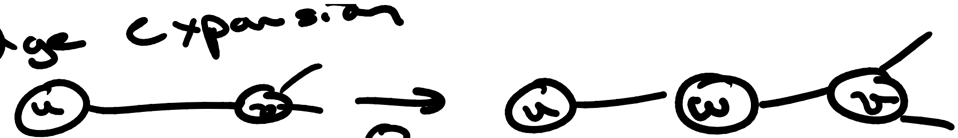
connected to some vertex v wlog

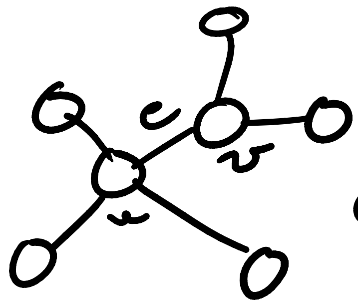
Note: a new single leaf vertex can be placed in the opposite bipartite set from v → $P(k+1)$ is bipartite □

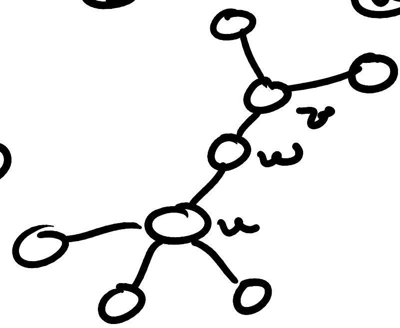
Edge expansion



Edge expansion

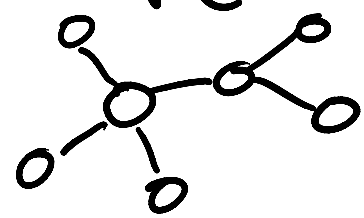






P(k)

P(k+1)



w is in v's original set and we swap sets for v and all vertices in v's component of T-w

---

Extra fun definition

distance - $d(u,v)$ = length of shortest u,v-path

diameter - $D(G)$ = length of longest shortes u,v-path

= max $d(u,v)$ over all $u,v \in V(G)$

epsilon

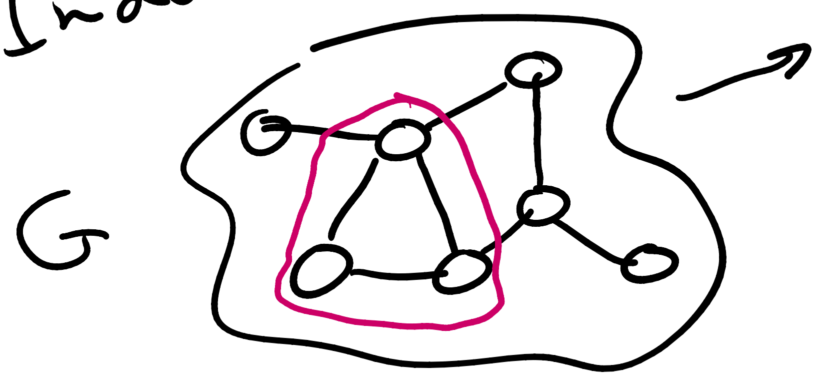
Eccentricity - $\epsilon(v)$ = max $d(u,v)$ over all $u \in V(G)$

radius - $R(G)$ = minimum $\epsilon(v)$ $\forall v \in V(G)$

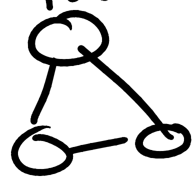
center of G = the induced subgraph of

center of G = the induced subgraph of vertices that have minimum eccentricity

Induced subgraph



induced



non-induced