7.1 Graceful Labeling

A **graceful labeling** of a graph is a labeling of all n vertices of a graph with unique labels from $0 \ldots m$, such that each of the m edges has a unique value computed as the difference between the labels of its endpoints. A graph is **graceful** if it has a graceful labeling.

Ringel-Kotzig Conjecture: all trees are graceful. This is unproven, however, certain subsets of of trees have been proven to be. These include paths and **caterpillar graphs**. Caterpillar graphs are trees in which a single path is incident to or contains every edge in the graph.