Lecture 17 - epidemiology

Tuesday, March 23, 2021 2:28 PM

Epidemiolog

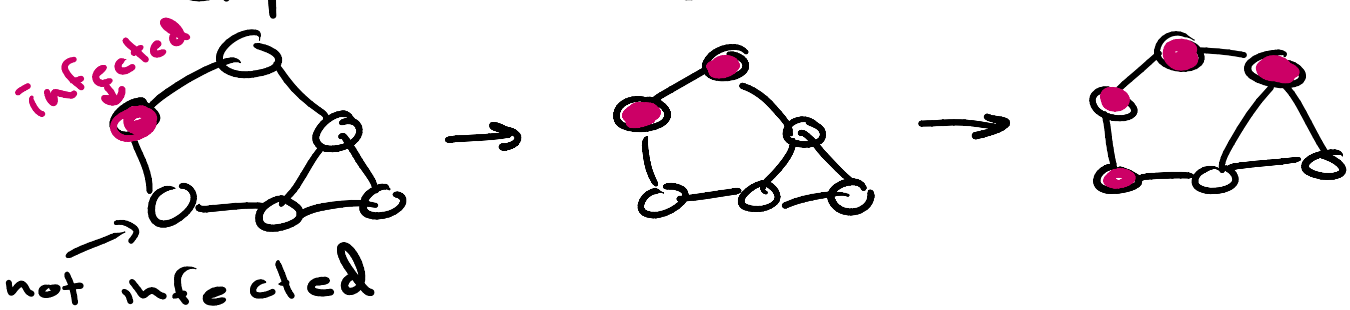
- What is this stuff?

→ study of disease patterns in a population

→ how does a disease spread, etc.

How is it relevant to graph theory?

- Most models are implicity defined on some underlying contact graph
- Simulations can be run on some explicit contact



- also: covid thing

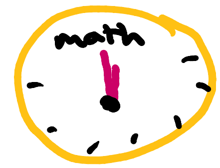
How do these graphs look?

- Models → homogenous (Erdös-Rényi)

→ heterogenous
(Chung-Lu)

- Simulations (agent based)
  - arbitrary complexity
  - geographic / spatial considerations
  - work, school, etc.
  - movement patterns
  - Even more detailed
    → satellite imagry + census data = pop. density

Math O'clock



- mathematical models for the spread of a disease

Classic model: compartmental model

- population is separated into "compartments"
- spread is captured via changes in the population in compartment

in the population ...

Classic of the classics: SIR

S: susceptible, can be infected

I: infectious, can spread to those in S

R: removed, immune and non-contagious

variations $\to$ SIS, SEIR

Model Dynamics of SIR

How does the population move through $S \to I \to R$ $\quad S \to I$

$$\frac{dS}{dt} = \text{change in } S \text{ over time}$$

$$\frac{dI}{dt} = \text{change in } I$$

$$\frac{dR}{dt} = \text{change in } R$$

Parameters affecting the model

- Population N (often divided out)
- contact rate → assume Erdös-Rényi-like homogeneous network
- probability of transmission

$$\rightarrow \beta = \frac{\cancel{\text{contacts}}}{\text{time}} \times \frac{\text{prob. trans.}}{\cancel{\text{contact}}}$$

→ in a given time, number of transmission

→ how many get infected per time

- duration of infection = T

$\rightarrow \gamma = \frac{1}{T}$ → rate of people who recover

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Now let's get definin'

$$\frac{dS}{dt} = -\frac{\beta I S}{N}$$ ← possible $I \leftrightarrow S$ interactions

how many will a infectious infect

how many will a infectious infect

rate of recovery

$$\frac{dI}{dt} = \frac{\beta I S}{N} - \underbrace{\gamma I}_{\text{\# of infectious who recover}}$$

$$\frac{dR}{dt} = \gamma I$$

Model limitations and simplifications

- N = fixed, births & deaths ignored
- ignore reinfectibility (SIS)
- ignore exposed but not infected (SEIR)
- homogenous mixing
  in reality → contacts might follow power-law distribution
- assumes behaviors are static
- etc.

Note:

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$

← population is constant

$$S(t) + I(t) + R(t) = N$$

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- dynamics only depend on β, γ

↳ R_0 = new infections resulting from a single infection

$$R_0 = \beta T = \frac{\beta}{\gamma}$$

R_0 = basic reproductive number

$R(t) = R = R_{eff}$ = reproductive number at some time t

obviously $R(t) \leq R_0$

Generally: remove N and consider our dynamics via ratios

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lil' $s = S/N$

$$\frac{ds}{dt} = -\beta i s \qquad s(0) \geq 0 \qquad \text{IVP}$$

$$\frac{di}{dt} = \beta i s - \gamma i \qquad i(0) \geq 0$$

$r(t) = 1 - s(t) - i(t)$

Let's consider the limit behavior of this system, i.e, $t \to \infty$

What can that tell us?

Total infected: $r(\infty)$

$$\sim \underbrace{S(0) - S(\infty)}$$

What else do we care about?

$\max_t i(t)$ = peak infected over all $t = 0 \ldots \infty$

Let's get a "nice" solution combining all of our $i(0), i(\infty), s(0), s(\infty)$

- take $\frac{di}{dt} \Big/ \frac{ds}{dt}$

$$\Rightarrow \frac{di}{ds} = -1 + \frac{\gamma}{\beta s} = -1 + \frac{1}{R_0 s}$$

do some math stuff....

do some math stu.....

$$i(\infty) - i(0) = -\left(s(\infty) - s(0)\right) + \frac{\ln\left(\frac{s(\infty)}{s(0)}\right)}{R_0}$$

Let's assume $i(\infty) = 0$ ← given

$i(0) \approx 0$

$s(0) \approx 1$

$$0 = -s(\infty) - -1 + \frac{\ln\left(\frac{s(\infty)}{1}\right)}{R_0}$$

$$\boxed{0 = 1 - s(\infty) + \frac{\ln(s(\infty))}{R_0}}$$

→ proportion of population infected is solely a function of R_0

To get $s(\infty)$ and therefore get $r(\infty) = 1 - s(\infty)$, find the roots

Via our example with $R_0 = 2$

→ $s(\infty) = 0.2$ → $r(\infty) = 0.8$

$\rightarrow s(\infty) = 0.2 \rightarrow r(\infty) = 0.8$

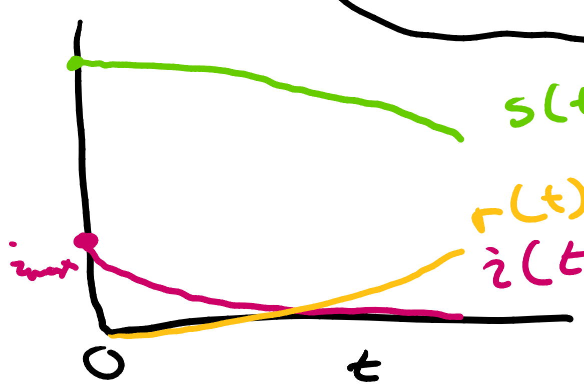
What about $\max_t i(t)$?

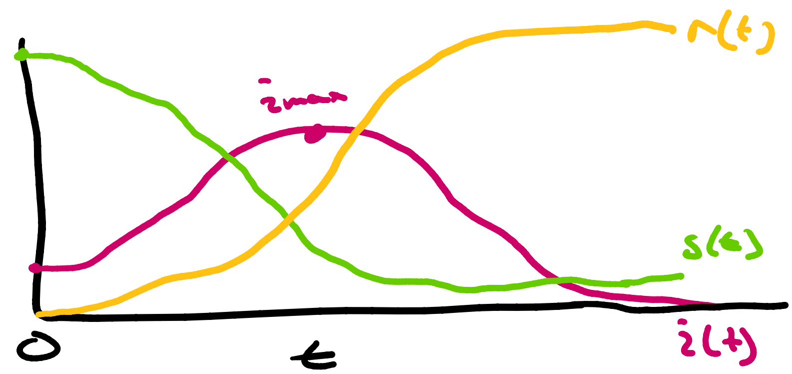
Note: depends on R_0

if $R_0 s(0) \leq 1$ $\quad i_{max} \approx i(0)$

else $R_0 s(0) > 1$

$i(t)$ increases first, then decreases once the growth of the disease switches to decay





what is that i_{max}?

$$i_{max} = i(0) + s(0) - \frac{1}{R_0} - \frac{\ln(R_0 s(0))}{R_0}$$

assuming $i(0) \approx 0$
$\qquad\qquad s(0) \approx 1$

$i \quad 1 \quad 1 \quad \ln(R_0)$

$$\boxed{i_{max} = 1 - \frac{1}{R_0} - \frac{\ln(R_0)}{R_0}}$$

via our computational intelligence

→ if $R_0 = 2$, $i_{max} = 0.15$

Finally → how can we estimate R_0?

Once again, assuming $s(0) = 1$
$i(0) = 0$
$i(\infty) = 0$

Some math stuff...

$$s(\infty) - 1 = \frac{\ln(s(\infty))}{R_0}$$

$$R_0 = \frac{\ln(s(\infty))}{s(\infty) - 1}$$

In reality, the susceptible population is not $s(0) = 1$

$$\rightarrow s(\infty) - s(0) = \frac{\ln\left(\frac{s(\infty)}{s(0)}\right)}{R_0}$$

$$\frac{\qquad}{R_0}$$

$$R_0 = \frac{\ln\left(\frac{S(\infty)}{S(0)}\right)}{S(\infty) - S(0)}$$

$$\boxed{R_0 = \frac{\ln\left(\frac{S(0)}{S(\infty)}\right)}{S(0) - S(\infty)}}$$