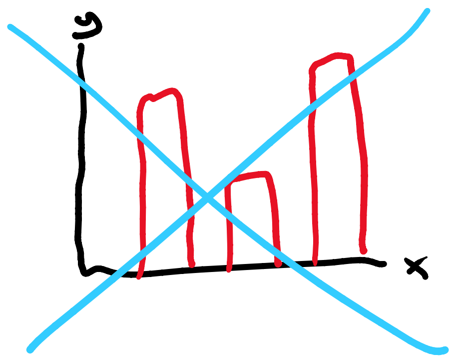
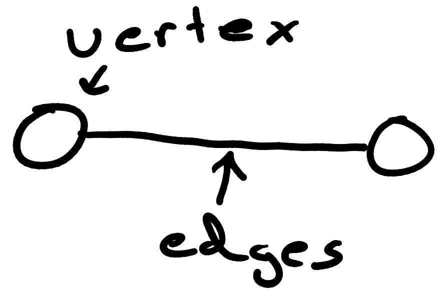
Lecture 1

Sunday, January 9, 2022 11:31 PM

What is a graph?





Real-world graph

social networks

Bob — Joe

Karen

Road networks

Schen.

Troy

Albany

Info. nets.

website A

website B

Properties of Real-world graphs

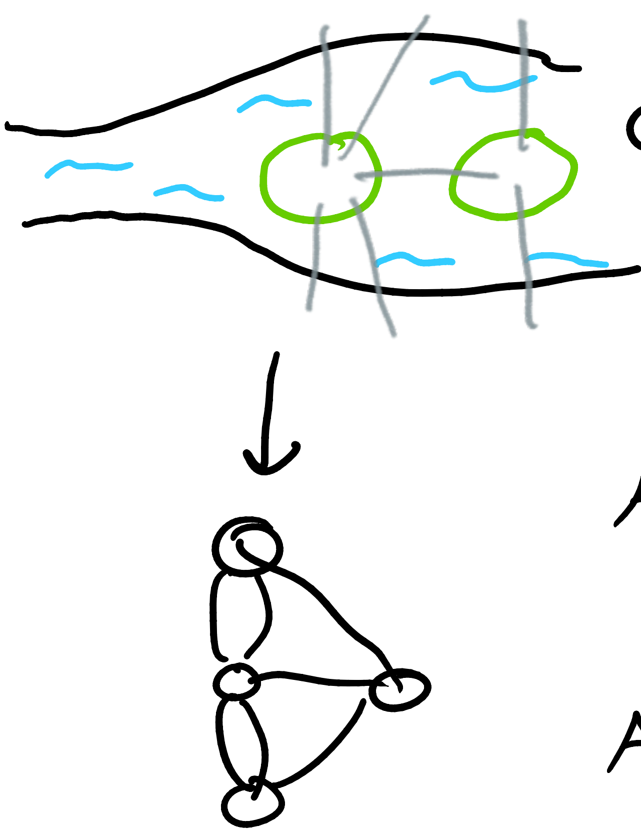
- Sparsity → $|E| << |V|^2$
- Skew → # low degree verts >> # high degree verts
- Hubs → large degree or otherwise important vertices

important vertices

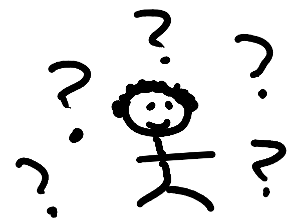
- Irregularity → not necessarily physically constrained
- Small-world → avg. shortest paths length is small
  "6 degrees of Kevin Bacon"

History of Graph Theory

Königsberg



Euler :



Q: Can I start at one point, traverse each bridge exactly once, and return to my start?

A: Invent graph theory
"O.G. of G.T."

Ap2: Define properties of a graph for an

of a graph for an
Euler Tour

Basic Definitions

Graph G: tuple of vertices $V(G)$
and eges $E(G)$

$$G = \{V(G), E(G)\}$$

for each $e \in E(G)$, $e \to (u, v)$
$u, v \in V(G)$

Terminology:

u, v are <u>endpoints</u> of e

e <u>joins</u> u and v

u, v are incident with e

e is <u>incident</u> on u, v

u, v are <u>adjacent</u> to each other

u, v are <u>neighbors</u>

$u \in N(v)$, $v \in N(u)$

neighborhood
of u

neighborhood of v

The **degree** of v is its number of incident edges

$|N(v)| = d(v) \leftarrow$ degree of v

(only in simple graphs)

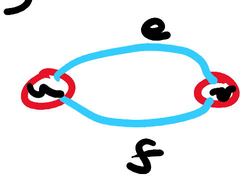
order of $G = |V(G)| = n$

size of $G = |E(G)| = m$

Multi-edges are edges in G that have the same endpoints

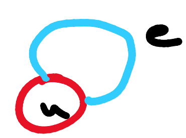
e.g. $e = (u, v)$, $f = (u, v)$

e, f are multi-edges



a self-loop in G is an edge that connects the same vertex to itself

e.g. $e = (u, u)$



a **simple** graph has no multi-edges or self-loops

a **loopy** graph has self-loops

... graph has multi-edges

a multi-graph has multi-edges

If $|V|$ and $|E| < \infty$

$\rightarrow$ G is finite

If $|V|$ and $|E| = 0$ $\quad V = \emptyset$
empty set

$\rightarrow$ G is null

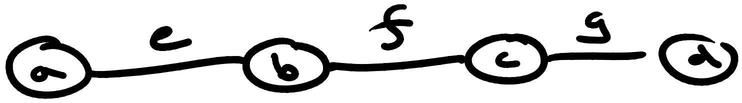
If $|V| = 1$ and $|E| = 0$

$\rightarrow$ G is trivial

If $|V| \geq 1$ and $|E| = 0$

$\rightarrow$ G is empty

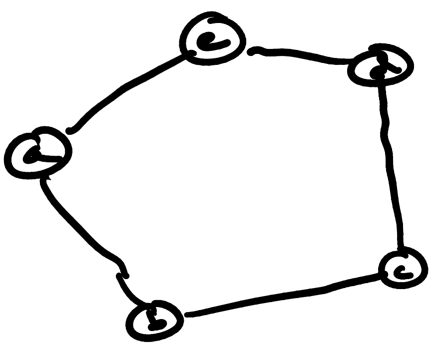
Path: a simple graph whose vertices can be listed s.t. any two vertices
(such that)
are adjacent iff they are
(if and only if)
consecutive in the list

P: 

$P: \{a, b, c, d\}$

$P: \{a, e, b, f, c, g, d\}$

<u>cycle</u>: a simple graph with an equal number of vertizes and edges whose vertices can be placed in a circle and two vertices are adjacent iff they appear consequitively along that circle

C: 

$C: \{a, b, c, d, e, a\}$

Note: can also be called a <u>closed path</u>

Tree: a simple connected graph with no cycles

for all   in

<u>Connected</u> graph G: $\forall u, v \in V(G)$:

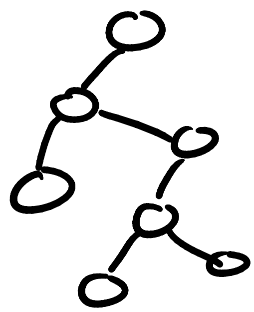
<u>Connected</u> graph G: $\forall\, u,v \in V(G)$:

$\exists\, u,v$-path

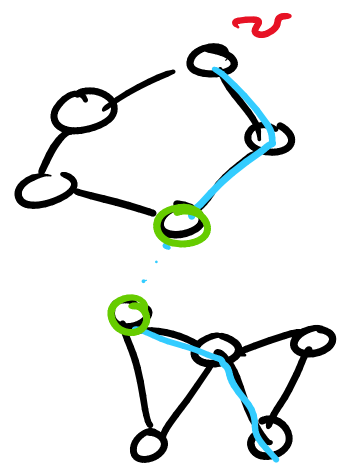
$\exists$ ← there exists

<u>u,v-path:</u> a path from u to v

Tree T:

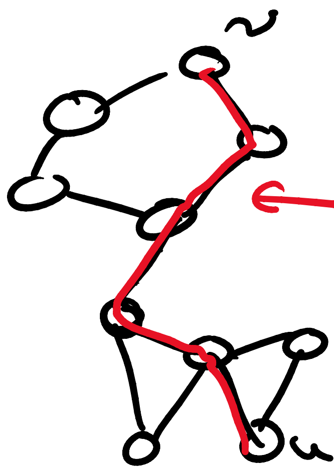


G:



disconnected

G:



← u,v-path

v,u-path

connected