

Page Rank

- "A modern classic"
 - Gittens
- a centrality algorithm
 - ↳ importance

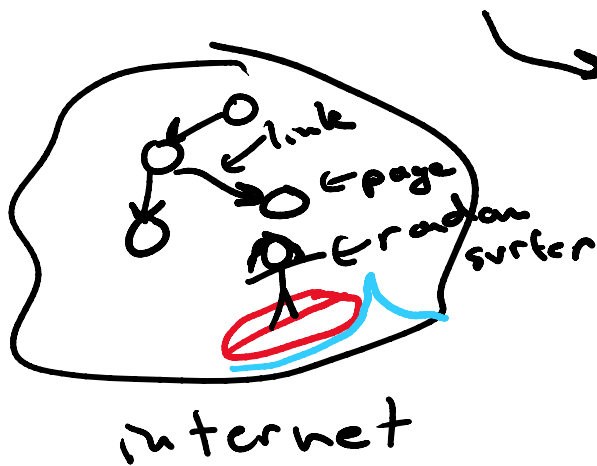
BITD: a lot of internet,
but no good way to
find stuff

Google: we'll incorporate
"trust" into our search

↳ you're trusted if trusted
pages link to you

$\Rightarrow \text{Page Rank} = \text{trust}$

Random surfer model



performing a random walk

↳ random traversal of edges via a walk

PageRank: probability that our surfer is at vertex v at some point in time

Issue: what about $d^+(v) = 0$ ^{← sink}
or $d^-(v) = 0$

↳ randomly jump from sinks to anywhere else in graph ^{← source}

Graph Algorithmic model

→ Vertex centric computation

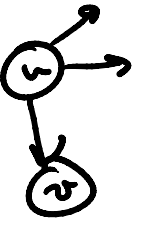
initialize $P(v) = \frac{1}{|V(G)|}$ for all v



initialize $P^{(0)} = \frac{1}{|V(G)|}$

we iterate

$$P^{(t)}(v) = \sum_{u \in N^-(v)} \frac{P^{(t-1)}(u)}{d^+(u)}$$



Linear Algebraic model

consider our adjacency matrix A

consider the diagonal degree matrix D

- D has nonzeros only along diagonal

- $D_{ii} = \text{out degree of vertex } i$

$$= \sum_j A_{ij}$$

- $D_{ij} = 0$ for $i \neq j$

define transition probability matrix M'

$M'_{ij} = \text{probability of transition from } i \text{ to } j$

$$M' = D^{-1}A$$

← This defines transitional probs. following out edges

$$M = (D^{-1}A)^T \leftarrow \text{in edges probs.}$$

↳ How to calculate PRs using this

initialize $p_0(v) = \frac{1}{|V(G)|}$

update $p_{i+1} = M p_i$

eventually $p_{\infty} = M p_{\infty}$
← power-iteration

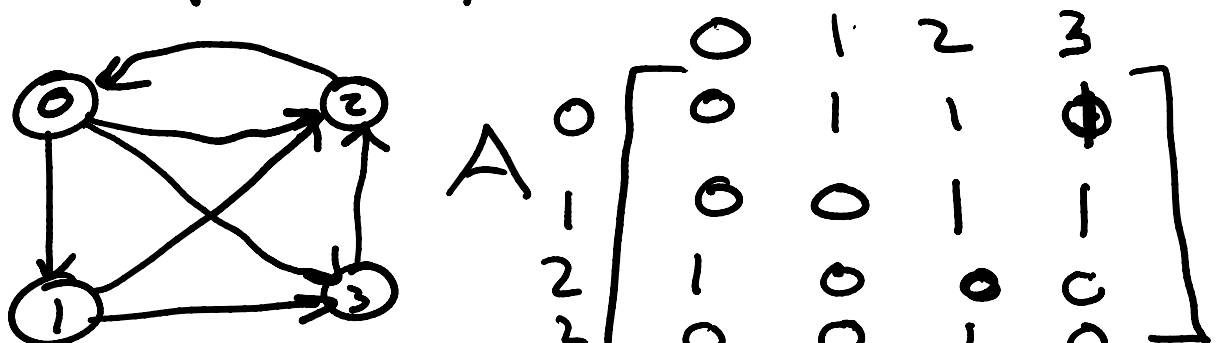
recall for matrix A

$$Ax = \lambda x$$

↑ ↖ eigenvalue
eigenvector

→ PageRanks are just the eigenvector of M that corresponds to eigenvalue 1

Example computation





$$\begin{array}{l} 2 \\ 3 \end{array} \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$D = \begin{bmatrix} 2 & \dots & 0 \\ \vdots & 2 & \vdots \\ 0 & \dots & 1 \end{bmatrix} \quad D^{-1} = \begin{bmatrix} 1/2 & & \\ & 1/2 & \\ & & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ & & & \end{bmatrix}$$

Competition network

vertices = competitors

edges = competitions

orient: add direction to
some edge

For PageRank:

- We can orient edges to winners

- PR "flow" to winners

- PR "flows" to winners
- ↳ Gives us a ranking of competitors