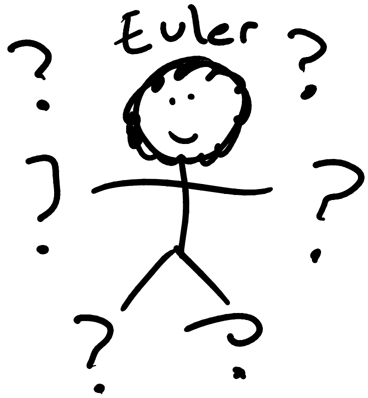
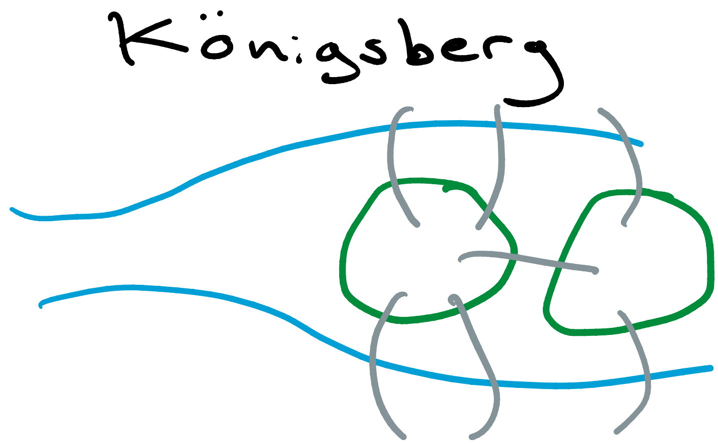
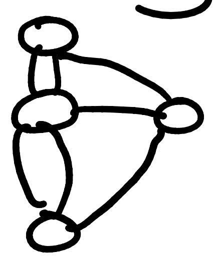
History of graph theory





Euler: Can I start at one location, traverse all bridges exactly once, and return to my starting location?

Answer: inventing graph theory

Graph: 

Real Answer: No
(Euler Tour)

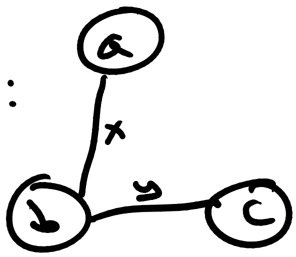
Graphs can be considered a tuple of vertices and edges a tuple of vertices and edges

$$G = \{V(G), E(G)\}$$

$V(G)$: vertices of G

$E(G)$: edges of G

G:



$V(G) = \{a, b, c\}$

$E(G) = \{x = (a,b), y = (b,c)\}$

$|V(G)| = 3$

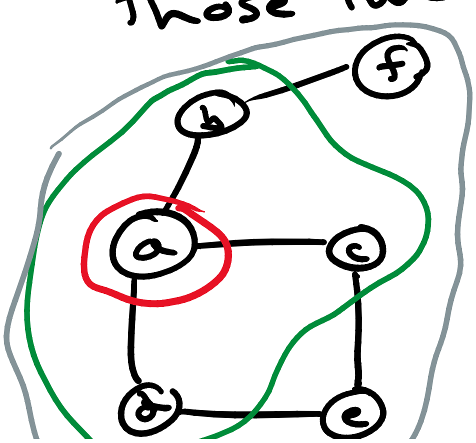
cardinality

An edge in G has two __endpoints__

that edge is __incident__ on those two endpoints

those two endpoints are __adjacent__

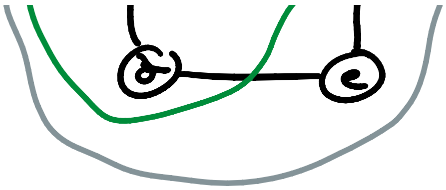
those two endpoints are __neighbors__



1-hop neighborhood

$N(a)$

2-hop neighborhood



2-hop neighborhood

The degree of some vertex v is the number of incident edges on v

$d(v) =$ degree of v

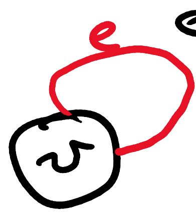
$d(v) = |N(v)|$ in simple graphs

simple graph: has no self loops or multi-edges

loopy graph: has self loops

multi-graph: has multi-edges

self loop: an edge with both end points as a single vertex



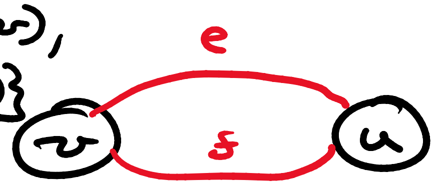
$\leftarrow$ note: $d(v) = 2$

multi-edge: one of multiple edges incident on the same two vertices

$E(v) = \{e = (u, v),$ ... multi-edges

two vertices

$E(G) = \{e = (u, v), f = (u, v)\}$



e, f are multi-edges

(aka net)

Hypergraphs: can have an edge connecting multiple vertices (aka pins)

Graph order and size

order $= |V(G)| =$ number of vertices

size $= |E(G)| =$ number of edges

if $|V(G)| = 0$ and $|E(G)| = 0$

$\rightarrow$ null graph

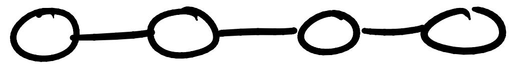
if $|V(G)| = 1$ and $|E(G)| = 0$

$\rightarrow$ trivial graph

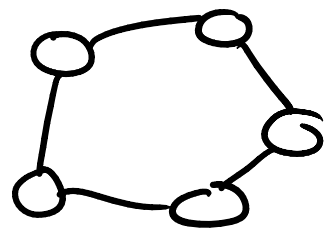
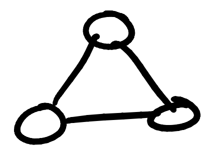
if $|V(G)| \geq 1$ and $|E(G)| = 0$

$\rightarrow$ empty graph

Basic graph configurations

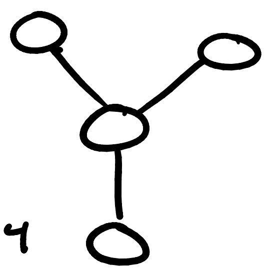
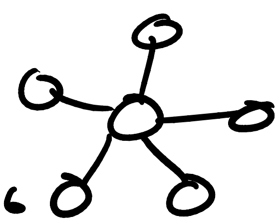
Path graph: 

P_4 ← path graph of order 4

Cycle graph:  

C_5 C_3

aka a <u>triangle</u>

star graphs:  

S_4 S_6

aka a claw

tree graph: a <u>connected</u> and acyclic undirected simple graph

Connected graph G:

$\forall u, v \in V(G) : \exists\ u,v\text{-path}$

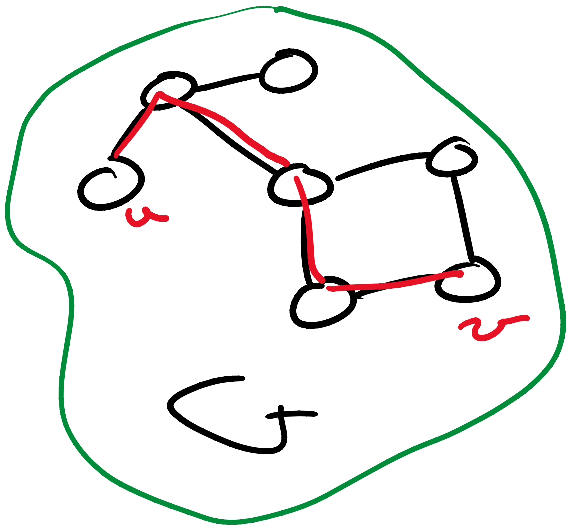
$\forall$ ← forall

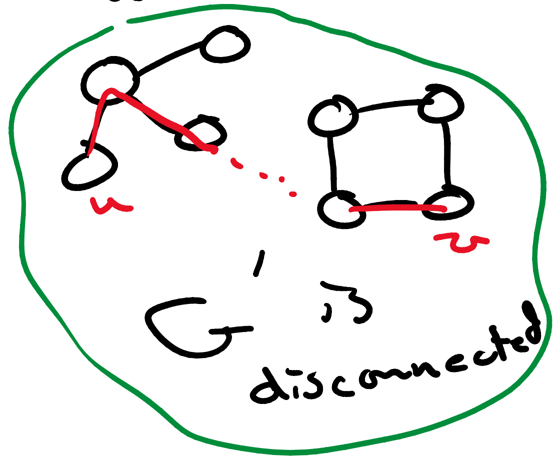
$\in$ ← in

$\exists$ ← exists

u,v-path → a path that starts at u and ends at v

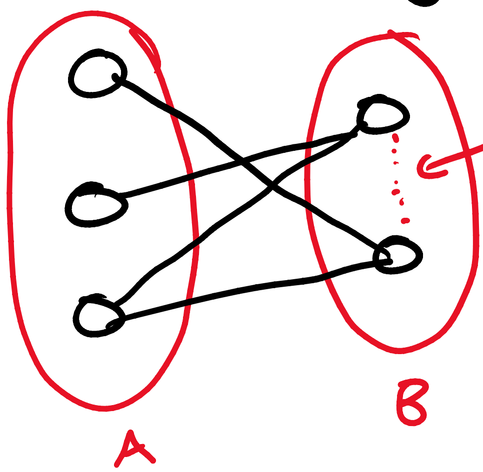
a path that starts at u and ends at v





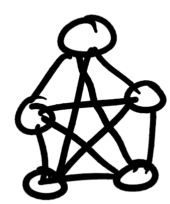
bipartite graph: a graph whose vertex set can be separated into two vertex disjoint sets, s.t. between any two vertices in a given set, there are no edges connecting them

s.t. = such that

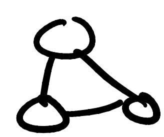


no edges among vertices in the same set

Complete graph: aka a clique



K_5



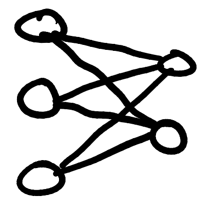
K_3

A graph on same number of vertices

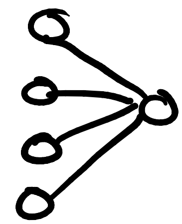
A graph on same number of vertices with edge between all pairs of vertices

Complete bipartite graph:
aka a biclique

A bipartite graph with edges among all pairs of vertices in two bipartite sets



$K_{3,2}$



$K_{4,1}$

subgraph H of graph G:

$$V(H) \subseteq V(G)$$

$$E(H) \subseteq E(G)$$

subset