12.1 Strong Connectivity of Directed Graphs

Earlier, we introduced the concepts of connectivity for undirected graphs and weak connectivity for directed graphs. An additional connectivity concept for digraphs is that of **strong connectivity**. The definition of strong connectivity is similar to regular connectivity in undirected graphs: for any u, v in a strongly connected component, there exists a directed u, v-path from u to v. Recall that weak connectivity of a directed graph is equivalent to connectivity of its underlying graph, where the **underlying graph** of a digraph is the undirected representation created by removing directionality from the directed edges.

12.2 Vertex Connectivity of Undirected Graph

We're going to now somewhat generalize the concept of connectedness for undirected graphs in terms of network robustness. Essentially, given a graph, we may want to answer the question of how many vertices or edges must be removed in order to disconnect the graph; i.e., break it up into multiple components.

Formally, for a connected graph G, a set of vertices $S \subseteq V(G)$ is a **separating set** if subgraph $G - S$ has more than one component or is only a single vertex. The set S is also called a **vertex separator** or a **vertex cut**. The **connectivity** of G, $\kappa(G)$, is the minimum size of any $S \subseteq V(G)$ such that $G - S$ is disconnected or has a single vertex; such an S would be called a **minimum separator**. We say that G is k-**connected** if $\kappa(G) \geq k$.

12.3 Edge Connectivity

We have similar concepts for edges. For a connected graph G, a set of edges $F \subseteq E(G)$ is a **disconnecting set** if $G - F$ has more than one component. If $G - F$ has two components, F is also called an **edge cut**. The **edge-connectivity** if G, $\kappa'(G)$, is the minimum size of any $F \subseteq E(G)$ such that $G - F$ is disconnected; such an F would be called a **minimum cut**. A **bond** is a *minimal* non-empty edge cut; note that a bond is not necessarily a minimum cut. We say that G is k-**edge-connected** if $\kappa'(G) \geq k$. In a couple classes, we'll talk about how one might find a minimum cut in an arbitrary graph.

For a simple graph, we can show that $\kappa(G) \leq \kappa'(G) \leq \delta(G)$.