

A few people haven't selected papers yet  
Submitty gradeable for presentation submissions  
HW1 released this week, discuss on Thursday

Plan:

- Quick Review
  - Small-world graphs
  - Shortests paths and diameter
  - Social networks and triadic closure
  - Homophily
  - Dynamic and Temporal graphs
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## Review

- connectivity:  $k$ -connectivity  
can vertex  $u$  reach vertex  $v$ ?

- Network measures

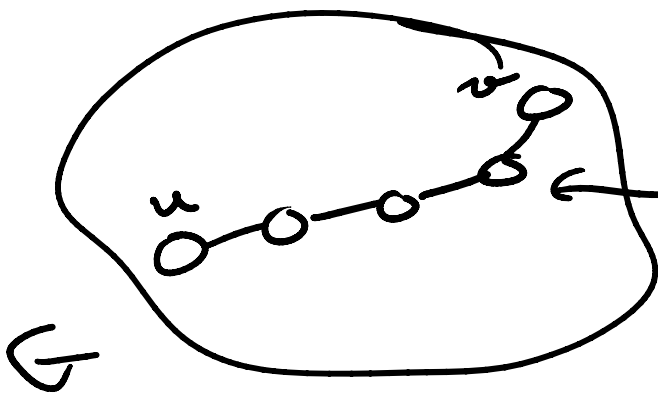
Irregularity: degree distributions  
cluster sizes, connected  
component sizes

Pseudo-power-law:  $P(k) \sim k^{-\alpha}$   
why?

Small worldedness

small worldness

→ low avg. shortest path lengths



shortest  $u, v$ -path is fewest number of edges to traverse to get from  $u$  to  $v$

How "small world" a graph:

Obvious measure: what's the average shortest path length

\* over all possible  $u, v$ -pairs

$\frac{\text{Sum}(\text{all pairs shortest paths})}{|\text{all pairs}|}$

Issue:  $O(n^2)$  possible pairs

if  $|V(G)| = n > 1 \text{ million}$

↳ gets "tough" computationally

↳ we use approximation algos

So: we use approximation algos

e.g.: sample a subset of  
 $u, v$  pairs

Why do we care: small worldedness  
of a graph affect things  
like information diffusion,

idea on a social network  
disease spread on  
a contact network

Also: diameter

↳ largest possible shortest  
path on same graph

How to solve: all pairs shortest  
paths

(very slow)

However: we can approximate  
using BFS

using BFS

\* sampling  $u, v$  won't work here,  
since we'd need to select  
the  $u, v$  with longest  
shortest path

Algorithm:

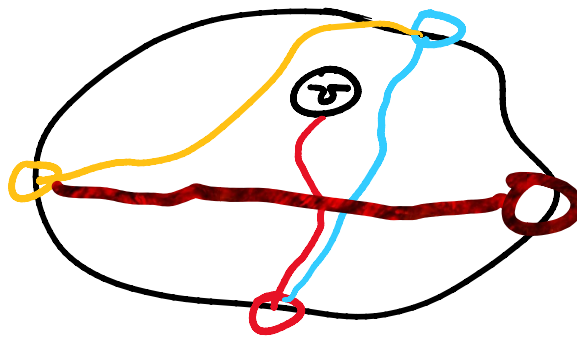
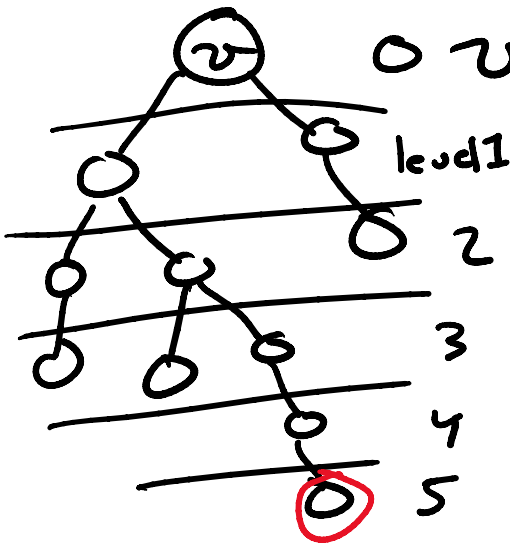
select some root  $r$

For some # iter: ←

BFS( $v$ )

$v$  = some random vertex  
furthest from our root

keep going until our  
estimate stops increasing



Why?

why do the empirical properties



Why do the empirical properties we measure arise in real-world networks?

Our focus: social networks  
skewed, irregular, small-world,  
small diameter

Social Networks:

Human interaction networks

vertices: humans (or animals)

edges: friendships, interactions,  
communication

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Triadic closure

Story time w/ Slota



For sharing photos on my phone,

Google gave me 4 default options:

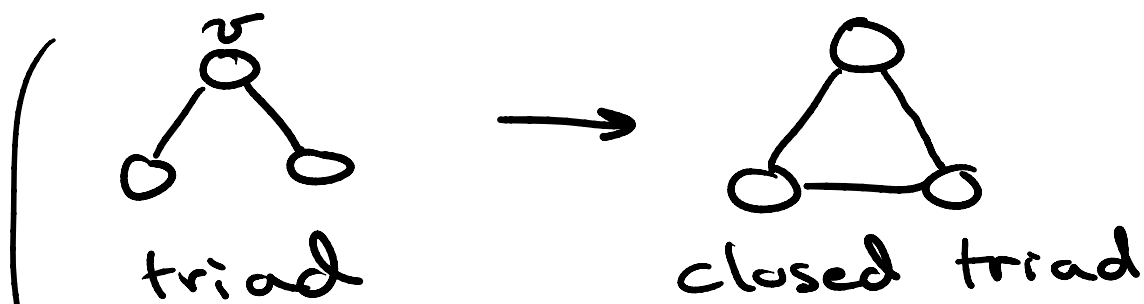
Google gave me 4 default options:

1. my wife → communicate regularly
2. my friend's wife → I don't communicate with at all
3. my ex-girlfriend → haven't communicated with in years
4. my mom → same as 1

Speculation:

why are options 2, 3 shown?

Triadic closure



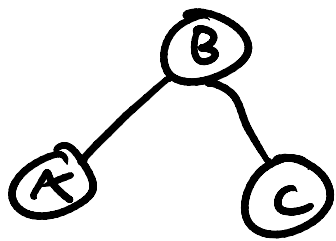
→ triads close at a frequency higher than randomly expected

Why do triads close?

Why do triads close:

Note: triadic closure is a main driver of social network growth over time

Consider: A and B are friends



B and C are friends

A and C are not friends

→ there is a higher-than-random chance A, C become friends

Why:

Opportunity: B spends time with A and C separately, but likely could spend time with A, C together

Trust: B trusts A and C

A and C trust B

Over time: A and C might trust each other

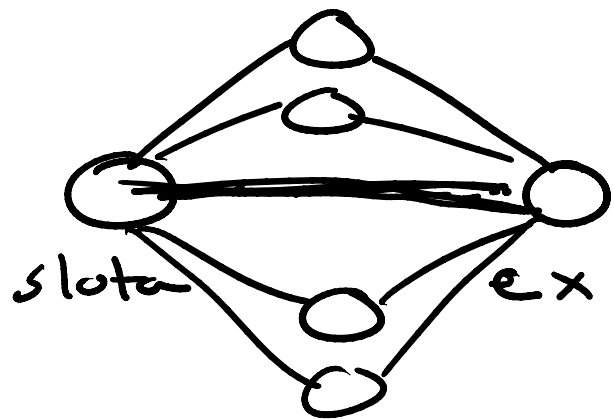
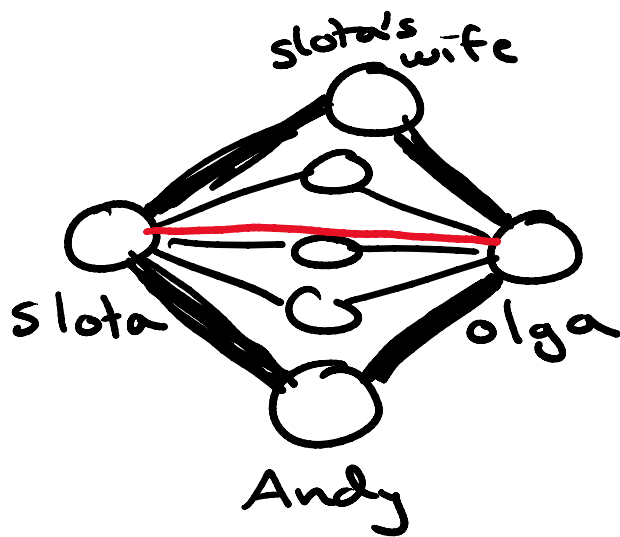
trust each other

Incentive: B might want to close the triad

Result: triads close at a higher than random rate

AND

They close faster proportionally to the number of common neighbors



Consequences of triad closure:

- Large clustering

- small world

- small world
- small diameter

→ clustering coefficient will increase over time

$$\text{Clustering coefficient} = \frac{\# \text{ closed triads}}{\text{all possible triads}}$$

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Similar concept: Homophily

Homophily: "birds of a feather flock together"  
"like attracts like"

OR: similar people become friends

- Selection: we seek out similar people as friends
- Influence: we become more similar to our friends over time

How this drives social

How this drives social network growth:

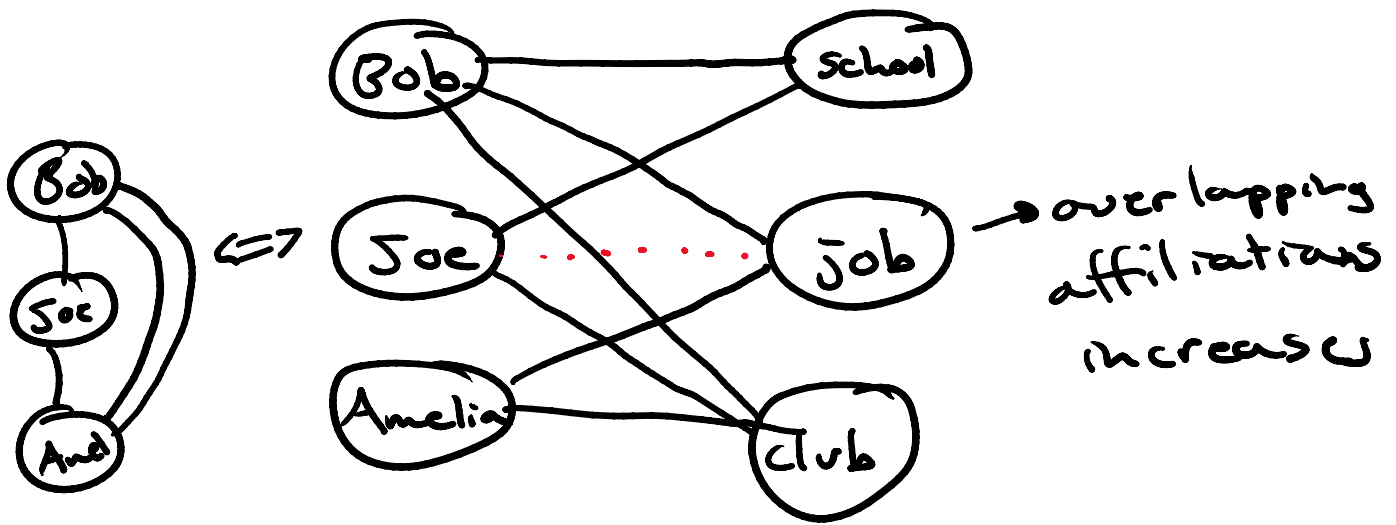
\* results in dense clusters with people who are "similar"

Why might online social networks care about this:

Easy to infer <sup>(unknown)</sup> properties of individuals within a topological cluster

Expanding triadic closure:

Affiliation networks



Same basic idea, if not "triads"

same basic idea, ...  
→ mutual affiliations will grow over time, proportional to existing mutual affiliations

Takeaway: we also observe an increase in strength of connections over

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Dynamic and temporal networks  
dynamic: changes over time  
temporal: we have timesteps associated with creation of vertices/edges

Experiment: (aka gettin' mihih')

- we expect triads to close over time
- First, we measure at some  $t_0$  the number of open triads
- we then after some  $\Delta t$

→ we then after some time

we measure how many have closed

→ then we look at this measure  
in terms of the number of  
mutual neighbors

i.e., strength of triad

Q: is there the empirical  
evidence of triadic closure?