Graph Theory Final Practice Problems

Use these as study aids in conjunction with notes, homeworks, and weekly problems..

1. Use the concept of ear decompositions to prove that for connected but **not** biconnected graph G, where $C_n \notin G : n =$ even, all blocks B_i of G are isomorphic to some $C_n : n =$ odd or K_2.

2. Graph G is **not** perfect, is connected, has $|V(G)| \geq 5$, and $\chi(G) = \omega(G)$. Draw a possible G.

3. $\forall C_n, C_m \subseteq G$where$n, m =$ odd : $\exists v \in C_n, C_m$. I.e., all odd cycles in G pairwise share a common vertex. Prove $\chi(G) \leq 5$.

4. Consider minimal non-planar graph G and graph $H = G - e$. Prove or disprove that we can guarantee $\exists e \in E(G)$ such that H is a maximal planar graph.

5. For each of the following, give tight bounds (upper and/or lower) on the possible connectivity $\kappa(G)$ **and** edge connectivity $\kappa'(G)$ of graph G. Note that strict equality is the tightest possible bound. (5 pts each)

   (a) G has a closed-ear decomposition but does not have an open-ear decomposition. G also has a minimum degree of $\delta(G) = 4$.

   (b) G is biconnected, has more than one maximal biconnected block, and one block is isomorphic to K_4.

6. For each of the following, give tight bounds (upper and/or lower) on the possible chromatic number $\chi(G)$ of graph G. Note that strict equality is the tightest possible bound. (5 pts each)

   (a) G has chromatic polynomial $\chi(G, k) = k(k-1)^4 - k(k-1)^3 + k(k-1)(k-2)$.

   (b) G is biconnected, has more than one maximal biconnected block, and one block has K_4 as an induced subgraph.

7. Prove that the vertices of an outerplanar graph can be partitioned into two sets such that the subgraph induced by each set forms a disjoint union of paths.

8. For each of the following, prove that the described G must be Hamiltonian or give a counter-example.

   (a) G is Eulerian.

   (b) $L(G)$ is Eulerian.

   (c) $\bar{G}$ is not Hamiltonian.

   (d) The closure of G is Hamiltonian.

   (e) $G = L(H)$ where H is Hamiltonian.