Graph Theory Final: Stuff to Know

General Info

- Exam will be 11:30am on Tuesday April 30th in LOW 4050.
- Exam will be closed notes, closed book, closed neighbor.
- **One** 8.5x11 crib sheet is allowed (written or printed).
- Material will cover Chapters 1-7 of the book, with particular emphasis on Chapters 4-7.
- Know everything in the online notes that's in bold.
- Questions will require direct knowledge of the definitions and graph properties we discussed, applying algorithms and knowledge to problems, and working through proofs.

The following material is only a guide. Regardless of what is listed below, everything discussed in class and in the online notes can appear on the test unless explicitly stated otherwise.

Chapter 4

1. Vertex connectivity - separating sets; cut vertices; minimum separators
2. Edge Connectivity - disconnecting sets; edge cut; edge connectivity of digraphs
3. Biconnectivity - blocks; articulation vertices; block-cutpoint graphs
4. 2-connectivity and k-connectivity - Whitney's connectivity theorem; open and closed-ear decompositions; Menger's Theorem
5. Network Flow - Fork-Fulkerson/Edmonds-Karp algorithms; f-augmenting paths; Max-flow Min-cut Theorem

Chapter 5

1. Vertex coloring - basic definitions; chromatic number; greedy algorithm

2. Coloring bounds - all bounds we talked about; color-criticality; perfect graphs; Mycielski's Construction; Turán Graphs

3. Counting colorings - chromatic polynomial, general form and form for cliques and trees; recurrence relation using edge contraction

4. Chordal graphs - chords and chordless cycles; simplicial elimination ordering; relation to perfect graphs

Chapter 6

1. Graph planarity - basic definitions; crossings; drawings/embeddings; faces and lengths of faces; dual graphs; outerplanarity; maximal planar/minimal non-planar graphs; triangulations

2. Planarity conditions - Euler's formula and resultant inequalities; Kuratowski subgraphs and Kuratowski's Theorem (along with results of subproofs)

3. Coloring of planar graphs - four and five color theorems

Chapter 7

1. Line graphs - definition; relation between problems on G and problems on $L(G)$; conditions for the existence of H such that $G = L(H)$; forbidden subgraphs (don't need to know all of them specifically, but just the properties that they have)

2. Edge coloring - basic definitions; bounds; relation between vertex coloring of $L(G)$ and edge coloring of G

3. Hamiltonian cycles and paths - necessary conditions; sufficient conditions

Random Graphs

1. Graph Models - Erdos-Renyi; Configuration Model; Chung-Lu

2. Analysis - using expected degree, attachment probabilities, and other model parameters to estimate graph properties