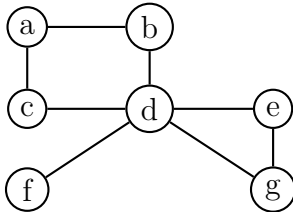


Graph Theory Homework 3

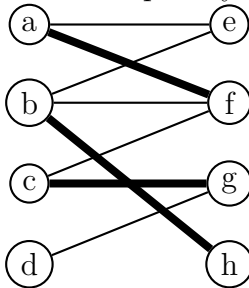
Due: 23 Feb 2024 at midnight EST as a PDF on Submittity

v1.1: Last Updated February 19, 2024

1. True or False: Tree T has at most one unique perfect matching. Prove your response.
2. Consider maximum match M on G . Prove that every maximal match M' has cardinality bounded by $|M'| \geq \frac{|M|}{2}$.
3. Prove that the complement of any vertex cover on a simple undirected graph is an independent set.
4. Consider graph G where $\forall v \in V(G) : d(v) = k$, $|V(G)|$ is even, and G remains connected after the deletion of any $(k - 2)$ edges. Prove that G has a perfect match.
5. Consider the below graph.
 - (a) Provide an edge cover F and vertex cover C for the below graph.
 - (b) Prove whether it possible to draw a perfect match M , such that $F = M$.
 - (c) Provide the complement of C , and show that it is an independent set.



6. Demonstrate a single iteration of our M -augmenting paths algorithm for the bipartite graph below to increase the size of the match M given in bold on the graph below. Explicitly show your steps.



7. For each of the following values of k , construct a k -regular graph that does not have a perfect match. (v1.1) *If that is not possible, prove why not.*
 - (a) $k = 1$
 - (b) $k = 2$
 - (c) $k = 3$