Graph Theory Homework 4

Due: 22 Mar 2024 at midnight EST as a PDF on Submitty

v1.1: Last Updated March 17, 2024

1. Prove or disprove: if $v \in V(G)$ is a cut vertex in some simple graph G, then some $e = (u, v) : u \in N(v)$ is correspondingly a cut edge in G.

2. Prove that $\kappa(G) = \kappa'(G)$ when G is a simple graph with $\Delta(G) \leq 3$.

3. Use Menger's theorem to prove that $\kappa(G) = \kappa'(G)$ when $\delta(G) = 3 = \Delta(G)$ for some arbitrary graph G.

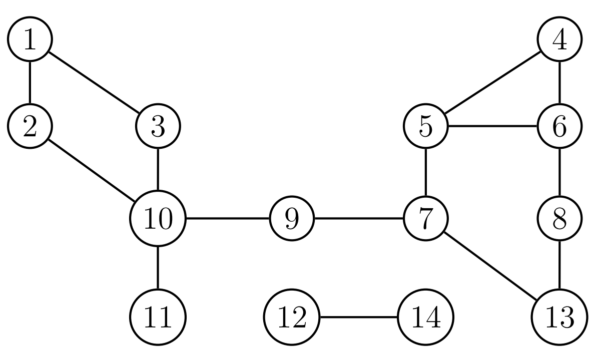
4. G is a k-connected graph with $|V(G)| =$ even that does not have any $K_{1,k+1}$ biclique as an induced subgraph. Prove that G has a perfect match.

5. E and F are two different edge cuts where $E \cap F \geq 0$. (**v1.1**) E and F are also defined in terms of vertex partitions $E = [S, \bar{S}]$ and $F = [T, \bar{T}]$. Prove that the $G = E \Delta F$ is also an edge cut.

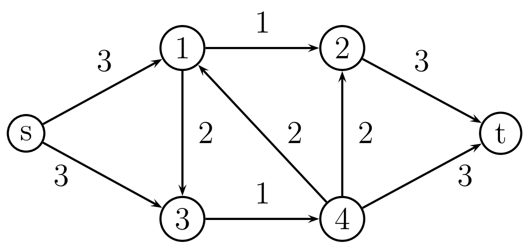
6. Consider bipartite graph B. Use Menger's Theorem prove that the size of a maximum matching is equal to the size of a minimum vertex cover on B.

7. Consider adding a new u, v-path P to some 2-connected graph G, where $u, v \in V(G)$. Formally demonstrate that for any $w \in V(P)$, there exists two internally disjoint paths to any $x \in V(G)$. For this proof, you cannot use the results of Whitney's or Menger's Theorems.

8. Draw a block-cutpoint graph by identifying the articulation vertices (cut vertices) and biconnected components (blocks) in the graph below.



9. In the below graph, give an integer flow of maximum value and prove that this flow is maximum by giving a cut with the same value.



10. Use the concept of network flows to prove that the size of a maximum matching is equal to the size of a minimum vertex cover on bipartite graph B.