Graph Theory Homework 5

Due: 29 March 2024 at midnight EST as a PDF on Submitty

v1.0: Last Updated March 19, 2024

1. Prove that the greedy coloring algorithm has an ordering for any graph that will produce an optimal coloring.

2. Draw a non-clique graph G of at least 6 vertices where the chromatic number of G is equal to upper bound given by Brooks' Theorem.

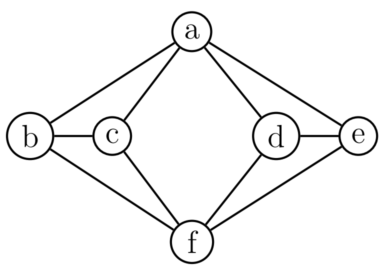
3. Prove that every triangle-free n-vertex graph has a chromatic number of at most $2\sqrt{n}$.

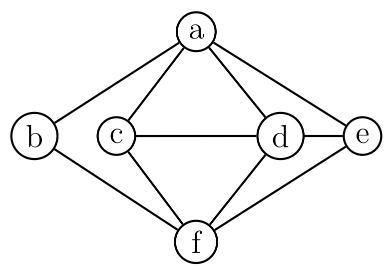
4. Prove that $\chi(G) = \omega(G)$ when the complement of G is bipartite.

5. Prove that all chordal graphs are perfect.

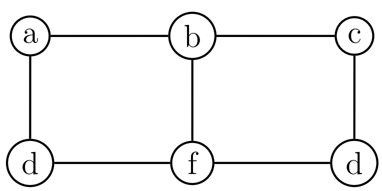
6. G is a 4-critical graph having a separating set S of size 4. Prove that a subgraph induced on G with S has at most 4 edges.

7. Prove that the chromatic polynomials of the below 2 graphs are equal.





8. Consider the below G for the following problems.



   (a) Draw G' created via Mycielski's Construction on G. What is $\omega(G)$, $\omega(G')$, $\chi(G)$ and $\chi(G')$?

   (b) Use the Fundamental Reduction Theorem to give the chromatic polynomial $\chi(G; k)$ of G. Use the polynomial to determine the chromatic number of G.

   (c) Does the above G have a simplicial elimination ordering? Provide one if possible; if not possible, prove why not. What does that say about whether or not G is perfect?