Graph Theory Homework 6

Due: 12 April 2024 at midnight EST as a PDF on Submitty

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1. Prove that all outerplanar graphs are 3-colorable.

2. Consider graph G' with $\Delta(G') \leq 3$. Prove that graph G contains a subdivision of G' if and only if G has some subgraph that can produce G' solely from edge contraction operations.

3. Consider isomorphic graphs G and G'. There existis some planar embeddings of G and G' such that the dual graphs G^* and G'^* are non-isomorphic. Find such a G and G', draw their embeddings, and prove the duals are non-isomorphic.

4. What is the maximum number of edges possible on some outerplanar graph G on $n = |V(G)|$ vertices? Prove your response.

5. A **straight-line embedding** of planar G is a planar embedding where all edges are drawn as straight lines, instead of curves. Prove that all simple planar graphs have a straight-line embedding.

6. Use the 4-coloring theorem to prove that all planar graphs decompose into two bipartite graphs.

7. For each of the following, prove that G must be planar or construct a counter-example:

   (a) G is 4-colorable.

   (b) G does not contain K_5 or $K_{3,3}$ as subgraphs.

   (c) The largest n for which G contains a K_n subdivision is $n = 4$.

   (d) G has no chordless cycles and has chromatic number $\chi(G) = 3$. *Hint: Consider whether G can contain a Kuratowski subgraph, given the above.*