Graph Theory Homework 7
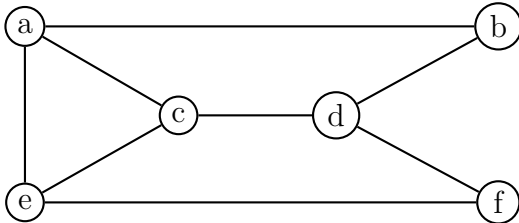Due: 19 April 2024 at midnight EST as a PDF on Submitty
v1.0: Last Updated April 8, 2024

1. Consider the below graph.

   (a) Give an optimal edge coloring of G. What is $\chi'(G)$?

   (b) Draw $L(G)$.

   (c) Does there exist some H for which $L(H) = G$? Provide such an H if possible or prove why such an H does not exist.

2. Consider simple graph G. Prove that G is isomorphic to $L(G)$ iff G is 2-regular.

3. Prove that for complete biclique $K_{i,j}$ that $\Delta(K_{i,j}) = \chi'(K_{i,j})$.

4. Consider simple bipartite graph G. Prove that there is a $\Delta(G)$-regular simple graph containing G.

5. G is a k-regular graph with a cut vertex. Prove that $\chi'(G) > \Delta(G)$.

6. Prove that $L(G)$ is Hamiltonian iff G has a closed trail that visits at least one endpoint of each edge.

7. 2-connected graph G has at least 6 vertices. The closure of G is not a clique. Provide a G that is Hamiltonian. Provide some other G that is non-Hamiltonian.

8. Consider a $G(n, p)$ graph with $n = 100$ and $p = 0.1$. How many unique K_3 subgraphs are expected in the graph? A subgraph S is unique if a set $V(S)$ defined by it's constituent vertices is unique. *Hint: Consider how many different ways you can select 3 vertices and what the probability is that a triangle exists between them.*