Graph Theory Midterm Practice Problems

Use these as study aids in conjunction with notes, homeworks, and weekly problems..

1. Consider the following in-degree and out-degree sequences for some hypothetical directed graph D. These sequences are not in any particular vertex order, so $S^+(1)$ and $S^-(1)$ don't necessarily refer to the same vertex. Is an Eulerian circuit possible on D? Justify your response.
$S^+ = \{2, 4, 4, 2, 2, 6, 2, 8, 3\}, S^- = \{3, 2, 6, 2, 2, 4, 4, 2, 8\}$

2. Prove that connected G contains a cycle iff $|V(G)| \leq |E(G)|$.

3. Prove or disprove: Havel-Hakimi can generate all possible graph configurations.

4. Draw and gracefully label a connected graph of at least 4 vertices.

5. Consider the following enumerative questions for undirected graphs, where loopy graphs and multi-graphs are proper supersets of simple graphs and we're considering a vertex set of cardinality n:

   (a) How many possible loopy graphs are there?

   (b) How many possible loopy multigraphs, with a maximum number of multi-edges of 2?

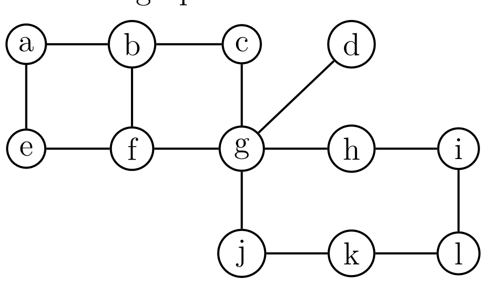
6. G has a unique weight for each edge. Prove that G has a unique minimum spanning tree.

7. Prove or disprove: Every tree with an even number of vertices has a perfect matching.

8. Consider graph G, and prove that $\forall v \in G : G - v$ has a perfect match iff $|V(G)|$ is odd and $o(G - S) \leq |S| : \forall S \subseteq V(G)$.

9. Prove that every 3-regular graph without a cut edge has a perfect match.

10. Consider graph G below. We'll consider a few questions about G.



   (a) Identify a maximum match on G. Prove that this match is optimal.

   (b) Identify a minimum vertex cover on G. Prove that this vertex cover is optimal.

   (c) Identify a minimum dominating set on G. Prove that this dominating set is optimal.