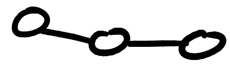
Weekly Problems 2 - Solution

Tuesday, January 23, 2024 7:00 PM

1 a) Basis $P(|E|=2) =$  $\Rightarrow$ trivial decomp. into one P_3

b) Assume we have $P(n)$, $n>2$ which is a simple even connected graph

c) We construct $P(k) = P(n) - P_3$, where P_3 is arbitrarily selected

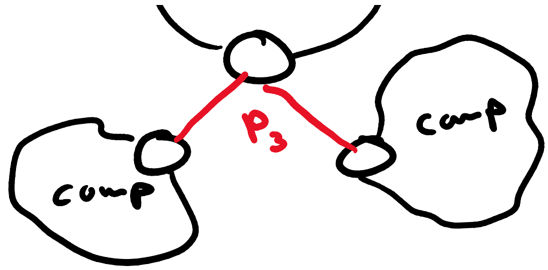
only delete edges

Note: we can also specifically select some P_3 in some configuration

d) Because of our arbitrary selection of P_3, $P(k)$ might not be connected and the components of $P(k)$ might be odd

$\rightarrow$ However, we can still modify our construction to be able to use our I.H. (inductive hypothesis)

Note: we can have at most 3 components in $P(k)$



Let's consider the possibilities

Note: parity %

Case 1: $P(k)$ is a single even component

Case 2: $P(k)$ is two even components

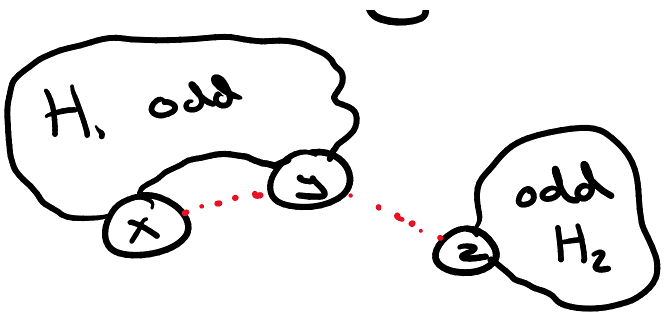
Case 3: $P(k)$ is three even components

Case 4: $P(k)$ is two odd components

Case 5: $P(k)$ is two odd and one even component

e) Case 1,2,3: I.H. on components of $P(k)$ gives us valid decompositions for $P(k)$, we combine them with removed P_3 to get decomposition on $P(n)$

Case 4: w.l.o.g. we have the following structure

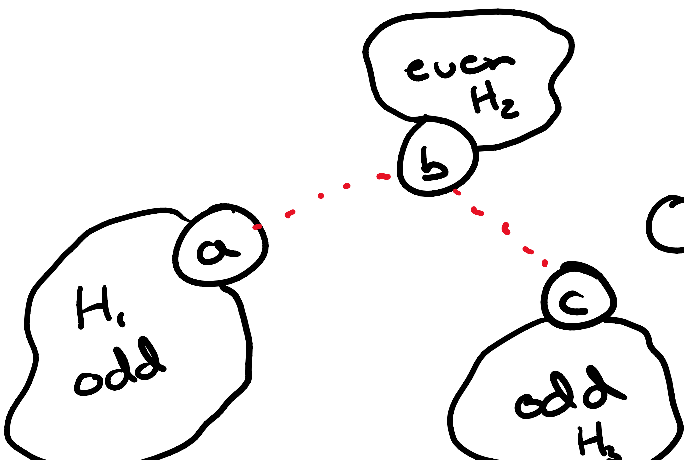


Note: $H_1 + (x,y)$ is even

$H_2 + (y,z)$ is even

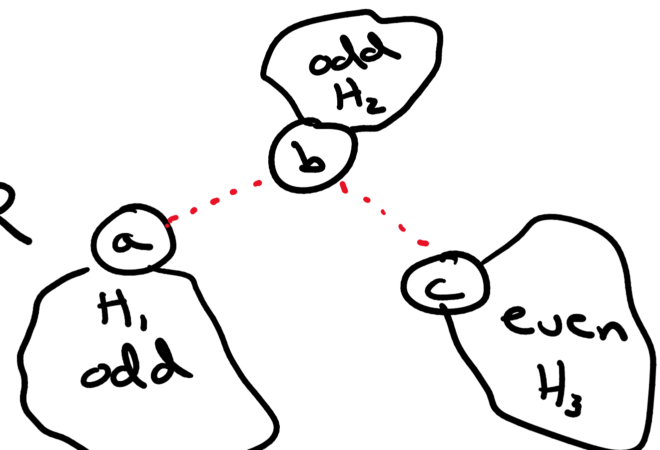
We use I.H. on each of the above and combine decompositions to get our decomposition on $P(n)$

Case 5: We similarly have the following



case 5.1

OR



case 5.2

We do I.H. on:

$H_1 + (a,b)$
H_2
$H_3 + (b,c)$

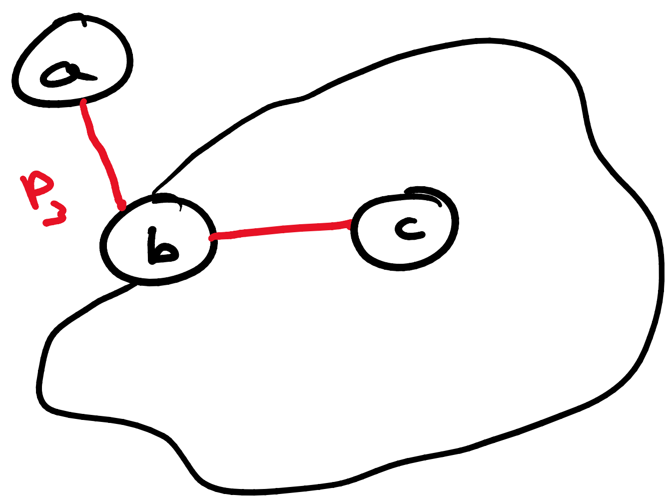
$H_1 + (a,b)$
$H_2 + (b,c)$
H_3

$H_3 + (b,c)$ $\qquad$ H_3

We combine together to get our decomp on $P(n)$ $\square$

Alternate constructions:

Can we select an edge incident on a degree-1 vertex?



What do components look like if we delete above P_3?

What if there are no degree-1 vertices? (remember from class)

What do the possible configurations look like if we remove edges from a cycle?