Graph Theory Weekly Problems 2
Due: 19 Jan 2024 at Midnight EST as a PDF on Submitty
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1. Consider simple connected graph G where $|E(G)|$ is even. Use induction to prove that $\exists D = \{P_3, P_3, \ldots, P_3\}$, where D is a decomposition of G and P_3 is the path graph of length 2.

   (a) First, determine an appropriate basis. Your basis must be in the same class $\mathbb{C}$ specified for the general graph $G \to \mathbb{C} = \{$simple, connected, even number of edges$\}$. Generally, we want our basis to be the smallest possible graph in $\mathbb{C}$.

   (b) Note that this problem is straightforward to prove via strong induction. To do so, we will next consider some general $G \in \mathbb{C}$, where $|E(G)|$ is greater than our basis.

   (c) To proceed, we need to figure out some *construction* on G. This construction must be able to be applied to all possible $G \in \mathbb{C}$, and it must result in some $G' \in \mathbb{C}$ where $|E(G')| < |E(G)|$.

   (d) When we have our G', we can use our induction hypothesis to assume that our proof statement holds. I.e., there exists a decomposition of G' that contains only length-2 paths.

   (e) To finish the proof, we 'undo' our construction and demonstrate how the proof statement (there exists a P_3 decomposition) also holds on G.