Weekly Problems 3 - Solutions

Sunday, January 28, 2024 4:13 PM

1. $Z = \{2, 2\} \rightarrow$ Not graphic

In a simple graph on n vertices, the maximum degree is $n-1$, hence a degree-2 vertex can't exist in a 2-vertex graph $\square$

Alternatively: Havel-Hakimi

$E = \{1, 2, 3, 4\} \rightarrow$ Not graphic

Same as above, as degree-4 vertex can't exist in a 4-vertex graph OR Havel-Hakimi

$P = \{1, 2, 2, 3, 3, 2\}$

$\sum_{i \in P} i = 13 \rightarrow$ Not graphic, as degree sum can't be odd OR H-H

$Y = \{1, 2, 3, 2, 2\}$

Using Havel-Hakimi

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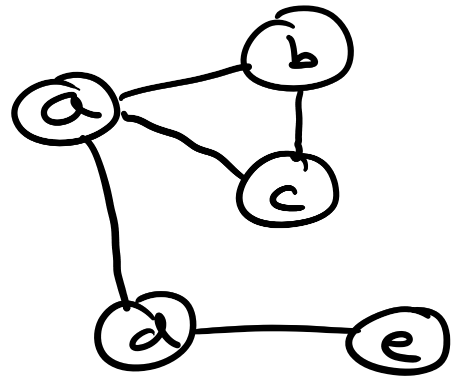
$$Y = \{\overset{a}{3}, \underset{-1}{\overset{b}{2}}, \underset{-1}{\overset{c}{2}}, \underset{-1}{\overset{d}{2}}, \overset{e}{1}\}$$

$$Y' = \{\overset{b}{1}\ \underset{-1}{\overset{c}{1}}\ \overset{d}{1}\ \overset{e}{1}\}$$

$$Y'' = \{\overset{d}{1}\ \underset{-1}{\overset{e}{1}}\ 0\}$$

$$Y''' = \{0\ 0\} \rightarrow \text{Yes graphic}$$



② We note that all degrees are even in S

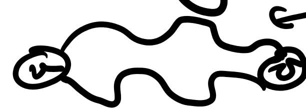
Hence, a graph G realized from S has properties → {connected, $\forall v \in V: d(v) = \text{even}$}

These properties characterize an Eulerian graph

→ $\exists$ a closed trail containing all edges in the graph

Hence, the exists two edge-disjoint paths from any u to any v

← Euler Tour

paths from any u to any v
within the graph  ← Euler Tour

→ deleting any edge will only disconnect at most one such path

$\Rightarrow \forall e \in E$: $G-e$ has at least one u,v-path $\forall u,v \in V$, so no edge can be a cut edge $\square$