

Graph Theory Weekly Problems 5

Due: 9 Feb 2024 at midnight EST as a PDF on Submitty

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1. You and your crew of Dirty Dave, Bad Boi Brian, Carlitos, and Amelia (aka D-squared, B-cubed, C-prime, and Amelia) compete with each other in a Pokémon Trading Card Game™ tournament, with the following outcomes:

- You defeat Carlitos and Bad Boi Brian
- Amelia defeats Carlitos and Dirty Dave
- Bad Boi Brian defeats Amelia
- Dirty Dave defeats you
- Carlitos defeats Dirty Dave

Because you and Amelia have the same number of victories, you both agree that obviously the overall winner should be determined as the one with the highest PageRank calculated on a graph modeling the competition.

- (a) First, create this graphical model where each competitor is a vertex and an edge exists between any two competitors that competed. *Orient* the graph so that each edge points to the victor.
- (b) Compute the transition probability matrix M as $M = (D^{-1}A)^T$, where A is the adjacency matrix of the graph and D is a diagonal degree matrix.
- (c) Initialize PageRanks equally among competitors and calculate a single iteration using the simple algebraic formulation. Who wins?