Weekly Problem 7 - Solutions

Thursday, February 22, 2024 5:22 PM

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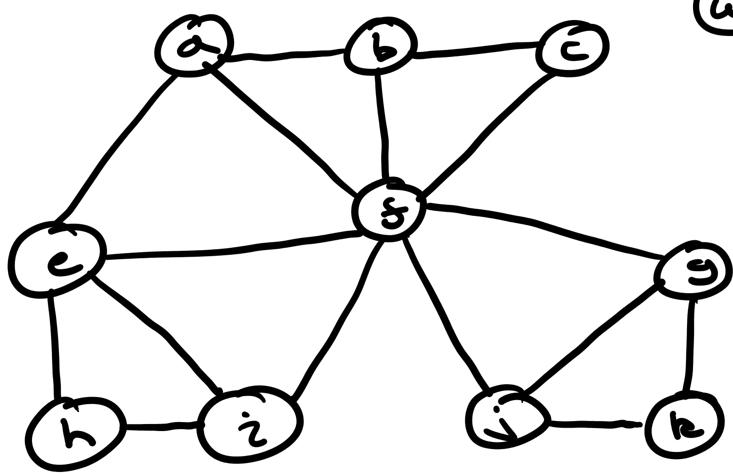

(where's d?)

Closed ear decomposition:

$$P_0 = \{(f,i), (i,h), (h,e), (e,a), (a,b), (b,c), (c,f)\}$$

$P_1 = (e,i)$

$P_2 = (e,f)$

$P_3 = (f,a)$

$P_4 = (f,b)$

$P_5 = \{(f,j), (j,k), (k,g), (g,f)\}$

$P_6 = (j,g)$

Open ear decomposition $\boxed{\text{not possible}}$ due to cut vertex f

This implies that for the above G:

$$K'(G) \geq 2 \quad . \quad K(G) = 1$$

$\kappa'(G) \geq 2$, $\kappa(G) = 1$

closed ear decomp. exists

no open ear decomp, but G is still connected

(2) a) is useless for us (classic Slota trick)

b) we know $\delta(G)$ bounds both $\kappa(G)$ and $\kappa'(G)$ above

c) We proved that this is an equivalent statement to saying G is at least 2-connected

d) We noted that this implies G is at least 2-edge-connected

$$\Rightarrow \underbrace{\kappa(G)}_{k} = \underbrace{\kappa'(G)}_{k'} = 2 \quad \square$$

(Note, we can also say that G is 1-connected and 1-edge-connected, but the above is sufficient)

sufficient)