Weekly Problems 8 - Solutions

Sunday, March 17, 2024 5:08 PM

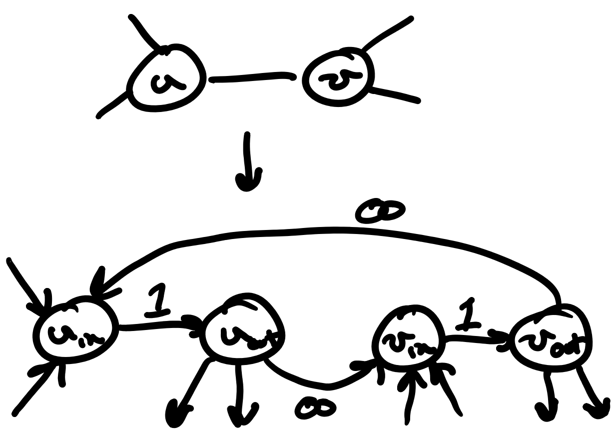
We transform G into a flow network F

For any $x \in V(G) \to$ source $s \in V(F)$

$y \in V(G) \to$ sink $t \in V(F)$

For all $e=(u,v) \in E(G)$ (undirected) $\to \{(u,v),(v,u)\} \in E(F)$ (directed)

For all $v \in V(G) \to \{v_{in}, v_{out}\} \in V(F)$, $(v_{in}, v_{out}) \in E(F)$



v_{in} gets all $N^-(v)$

v_{out} gets all $N^+(v)$

(v_{in}, v_{out}) get weight 1

all other edges get a weight of ∞

We then compute a max s,t-flow on F

$\to$ This equivalently gives us same minimum cut

Note: only the unit (v_{in}, v_{out}) edges will be part of the cut

$\to$ This gives us an equivalent

→ This gives us an equivalent vertex cut on G

Note 2: We can follow a unit of flow from s to t on F

→ only a single unit of flow can go into any v_{in}, and it must flow into v_{out}

→ The above holds for any unit of flow along an s,t-path

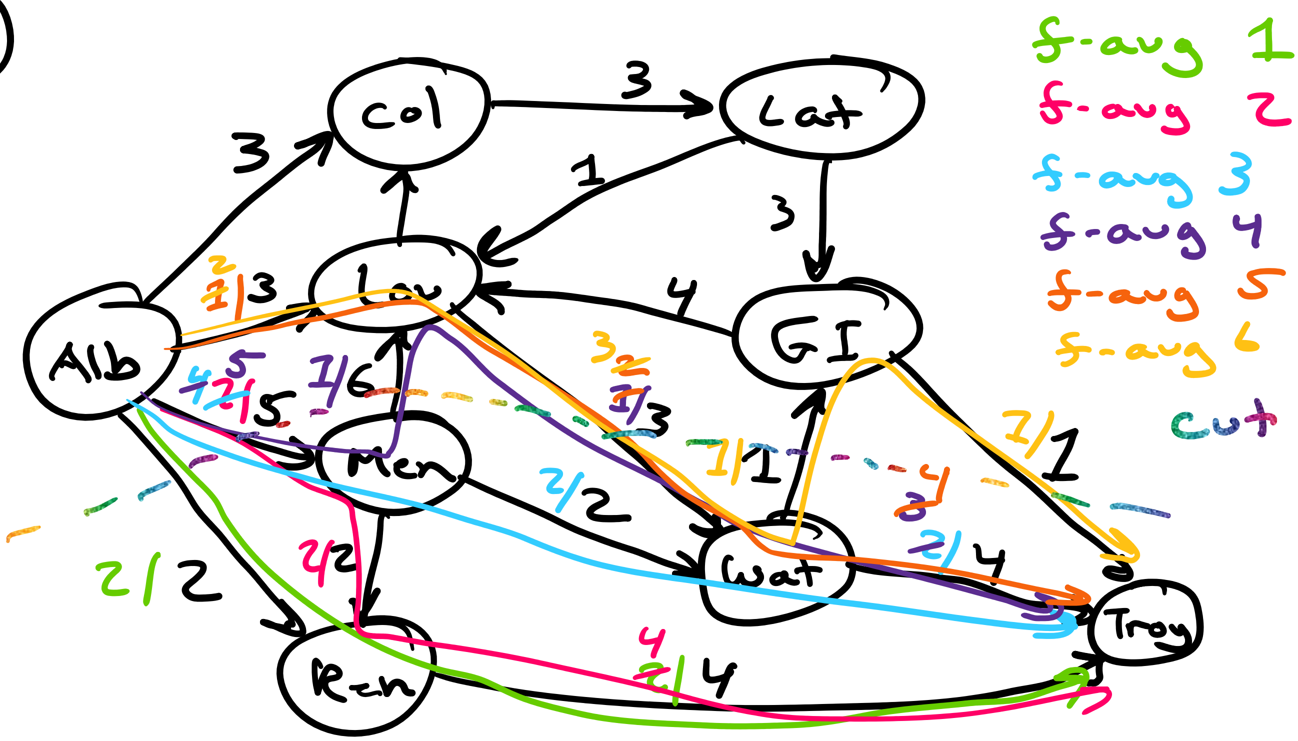
→ Hence, these s,t-paths are internally disjoint on both G and F

⟹ so we have $\lambda(s,t)$ paths and $K(s,t)$ min cut/max flow on F, where $\lambda(s,t) = K(s,t)$

equivalently

$\lambda(x, y) = K(x, y)$ on G $\square$

(2)



Max flow = 9 = min cut

Albany = {Alb, Col, Lat, Lou, GI}

Troy = {Men, Ren, Wat, Troy}

Note: The above cut is not unique