WP11 - Solutions

Monday, April 8, 2024 11:16 AM

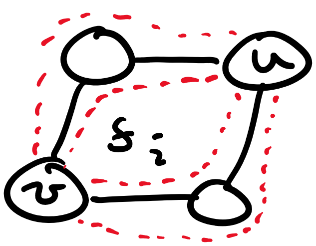
① Consider if there was a maximal planar graph which is not a triangulation

→ $\exists f_i$, a face with a length of 4 or greater

Consider non-adjacent $u, v \in V(f_i)$

→ we will always be able to add an edge (u,v) while remaining planar

→ simply draw (u,v) by following the boundary of f_i



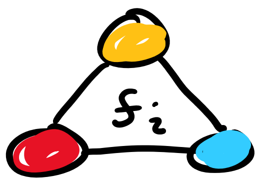
→ possible drawings of (u,v), depending on if f_i is the outer face

× × **contradiction** × × ×

$\Rightarrow$ as we can always add such a

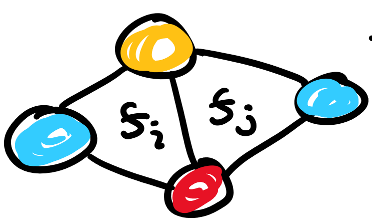
$\Rightarrow$ as we can always add such a (u,v) to any $f_i : \ell(f_i) \geq 4$, a maximal planar G must be a triangulation $\square$

(2) Consider the vertex colorings of any face of G



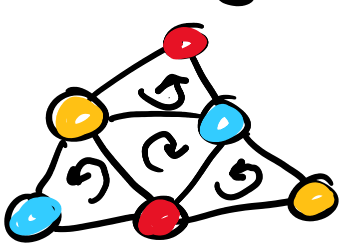
$\rightarrow$ we note they must all have the same 3 colors

Consider two adjacent faces



$\rightarrow$ we note there is a dependence via the shared edge

Consider an arbitrary number of adjacent faces



$\rightarrow$ given some color order {eg., $\rightarrow$ $\rightarrow$ }, each face follows it clockwise or counter-clockwise or counter-clockwise

→ No two adjacent faces have the same order, so any closed walk on the dual graph must be even, hence G^* is bipartite

Now consider $(G^*)^*$

→ we know there exists a way to draw $(G^*)^*$ s.t. $G \cong (G^*)^*$

→ we know that every face of G^* must be even

=> this implies $G \cong (G^*)^*$ will have even degrees $\forall v \in V(G)$ and it is therefore Eulerian $\square$