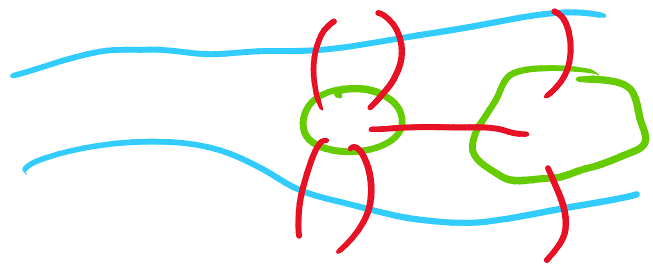
Intro and Basic Definitions

Monday, January 8, 2024 8:55 AM

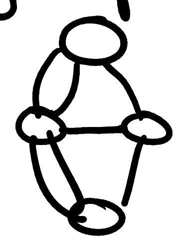
History of Graph Theory

Bridges of Königsberg



Euler: Can I start at one location, traverse all bridges exactly once, and return to my original location?

Answer: Invent graph theory

Graph: 

Real Answer: No (Euler Tour)

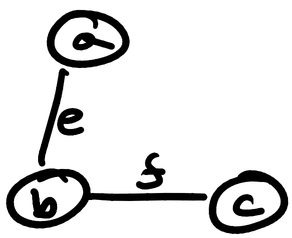
=> Common theme = real graphs motivating theoretic study motivating theoretic study

Our basic definitions

A <u>graph</u> is a tuple of vertices and edges

$$G = \{V(G), E(G)\}$$

$\uparrow$ vertices of G $\qquad$ $\uparrow$ edges of G

G: 

a set

$V(G) = \{a, b, c\}$

$E(G) = \{e = (a,b), f = (b,c)\}$

$|V(G)| = 3$

$\uparrow$ cardinality of set $V(G)$

More terminology

An edge in G has two <u>endpoints</u>

(vertices)

endpoints

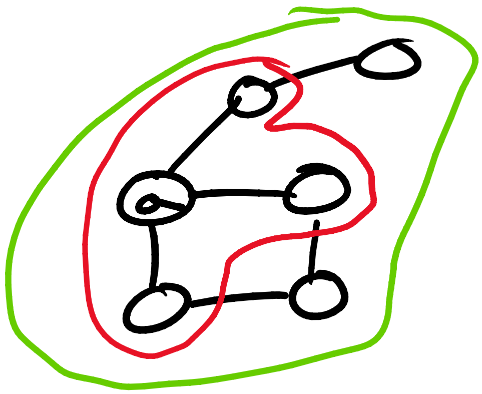
edge

edge

That edge is <u>incident</u> on those endpoints

Those two endpoints are <u>adjacent</u>

Those two endpoints are <u>neighbors</u>



1-hop neighborhood of a

2-hop neighborhood of a

The <u>degree</u> of some vertex v is the number of times v is an endpoint of an edge

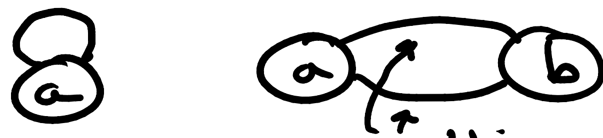
$d(v)$ = degree of v

$d(v) = |N(v)|$

(depends on <u>simple</u> vs. <u>multi</u> graphs and whether $N(v)$ is equivalent to v's adjacency list)

Graph class: a set of all possible graphs having same property or properties

Simple graph: has no self loops or multi edges



self loop

multi-edge

loopy graphs: can have self loops

multi-graphs: can have multi-edges

loopy multi-graphs: can have self loops and multi-edges

Graph order and size

$$\text{order} = |V(G)| = n$$
$$\text{size} = |E(G)| = m$$

if $|V| = 0$ and $|E| = 0$

→ <u>null</u> graph

if $|V| = 1$ and $|E| = 0$

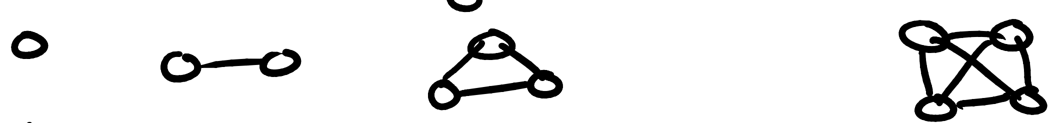
→ <u>trivial</u> graph

if $|V| \geq 1$ and $|E| = 0$

→ <u>empty</u> graph

Basic graph configurations

clique K_n: graph on n vertices where all vertices are adjacent to one another



K_1 K_2 K_3 = triangle K_4

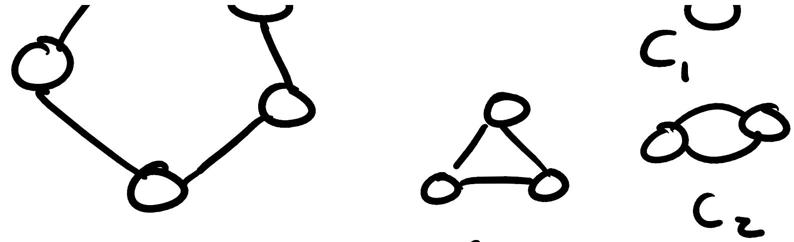
path P_n:

P_4 P_3

cycle C_n:

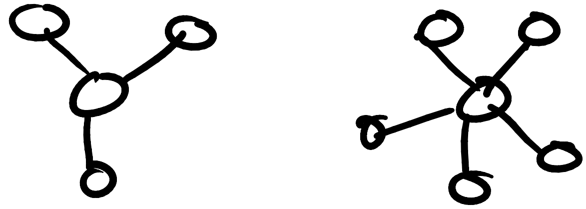
C_1

cycle C_n:



C_5 C_3

star graphs:



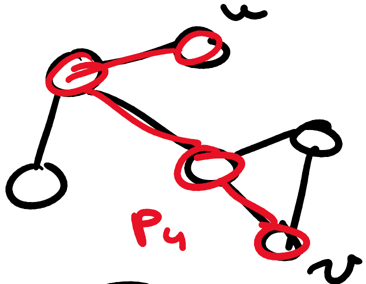
S_4 = a claw S_6

tree graphs: a connected and acyclic undirected simple graph

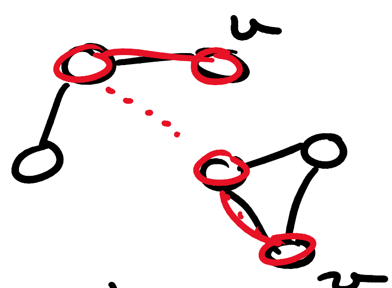
connected graph G:

$\forall u, v \in V(G) : \exists\ u, v$- path

for all — in — there exists — path from u to v



G



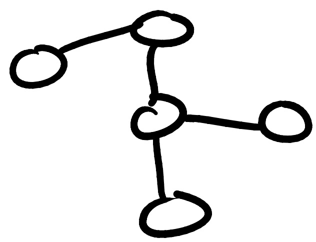
G'

= disconnected

G is connected

G' is disconnected

acyclic graph: contains no cycle graph



T is acyclic

undirected: edges have no associated direction

undirected

directed