

Scalable Generation of Graphs for Benchmarking HPC Community-Detection Algorithms

George M. Slota¹ Jonathan Berry² Simon D. Hammond² Stephen L. Olivier² Cynthia Phillips² Siva Rajamanickam²

¹Rensselaer Polytechnic Institute, ²Sandia National Labs

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

୬ ବ (ବ 1 / 31

Highlights: Adapted BTER

Primary results of this work:

- <u>Novel</u>: First scalable approach for HPC community detection benchmarking via "engineered solutions"
 We also develop a novel approach for *scaling* degree and clustering coefficient distributions
- <u>Realistic</u>: We utilize *real-world* graph distributions with a BTER implementation¹ and *edge-skipping*²
- <u>Fast</u>: 1 trillion edges/minute on current supercomputers
 - Orders-of-magnitude faster than state-of-the-art

We call our approach "Adapted BTER"

¹[Kolda et al., 2014]

²[Miller and Hagberg, 2011]

• We have some real-world interaction network (e.g., Facebook)



• We have some real-world interaction network (e.g., Facebook)



- We have some real-world interaction network (e.g., Facebook)
- Community detection: identifying *clusters* within the network



- We have some real-world interaction network (e.g., Facebook)
- Community detection: identifying *clusters* within the network
- Why: communities are often homogeneous (like-attracts-like) so we can often infer information about community members.



What is community detection?

- We have some real-world interaction network (e.g., Facebook)
- Community detection: identifying *clusters* within the network
- Why: communities are often homogeneous (like-attracts-like) so we can often infer information about community members.



How do we evaluate algorithm solution quality? Community Detection Algorithms: Evaluation

Given some community detection algorithm, how can we determine the quality of its output?

- Ideally: Evaluate on real-world datasets with "known" communities
 - Very few such datasets exists, none at $\ensuremath{\mathsf{HPC}}\xspace/\ensuremath{\mathsf{real}}\xspace$ social network scale
- Small scale: Generate synthetic networks with an "approximate engineered solution" (EAS) as communities

 Until this current work, infeasible to generate and evaluate at a large scale
- Large scale: Calculate some global measurement such as modularity (how well-clustered is the solution versus random expectation)
 - For modularity in particular, this approach is rather flawed

- Modularity suffers from a "resolution limit"
 - Small well-defined communities can not be individually resolved
 - Example: maximizing modularity on ring of cliques
 - cliques converge into single communities against intuition



- Modularity suffers from a "resolution limit"
 - Small well-defined communities can not be individually resolved
 - Example: maximizing modularity on ring of cliques
 - cliques converge into single communities against intuition



- Modularity suffers from a "resolution limit"
 - Small well-defined communities can not be individually resolved
 - Example: maximizing modularity on ring of cliques
 - cliques converge into single communities against intuition



- Modularity suffers from a "resolution limit"
 - Small well-defined communities can not be individually resolved
 - Example: maximizing modularity on ring of cliques
 - cliques converge into single communities against intuition
- Real-world networks scale to billions of vertices
 - Yet human "community" sizes tend to be relatively constant
- Takeaway: higher modularity != higher solution quality



A 3 3 5

Generate a synthetic network with some set of engineered "communities"



- Generate a synthetic network with some set of engineered "communities"
- Include a *mixing parameter* μ that controls the ratio of inter- to intra-community edges: $\mu \approx \frac{\text{inter-comm. edges}}{\text{total edges}}$
 - Effectively, this determines how well-defined the communities are



- Generate a synthetic network with some set of engineered "communities"
- Include a *mixing parameter* μ that controls the ratio of inter- to intra-community edges: $\mu \approx \frac{\text{inter-comm. edges}}{\text{total edges}}$
 - Effectively, this determines how well-defined the communities are



- Generate a synthetic network with some set of engineered "communities"
- Include a *mixing parameter* μ that controls the ratio of inter- to intra-community edges: $\mu \approx \frac{\text{inter-comm. edges}}{\text{total edges}}$
 - Effectively, this determines how well-defined the communities are



- Generate a synthetic network with some set of engineered "communities"
- Include a *mixing parameter* μ that controls the ratio of inter- to intra-community edges: $\mu \approx \frac{\text{inter-comm. edges}}{\text{total edges}}$
 - Effectively, this determines how well-defined the communities are
- Evaluate how well an algorithm's output matches the defined solution
 - Commonly utilize Normalized Mutual Information (NMI)



- Generate a synthetic network with some set of engineered "communities"
- Include a *mixing parameter* μ that controls the ratio of inter- to intra-community edges: $\mu \approx \frac{\text{inter-comm. edges}}{\text{total edges}}$
 - Effectively, this determines how well-defined the communities are
- Evaluate how well an algorithm's output matches the defined solution
 - Commonly utilize Normalized Mutual Information (NMI)
- \blacksquare Compare how well algorithms perform as you increase edge mixing via μ



6/31

Current State-of-the-Art: LFR

For benchmark graph generation with engineered solutions

"Lancichinetti–Fortunato–Radicchi" (LFR)³:

- With >1600 citations, this is a de facto standard
- Generates approximate solution to test against
 Uses tunable parameter for community coherence: μ
- \blacksquare Limited scalability: best implementation takes ${\sim}17 hrs$ to generate ${\sim}10B~edges^4$
 - Original code takes hours for million+ edge graphs

Our Goal: Develop methods to evaluate algorithms at HPC scale against an "engineered approximate solution."

³[Lancichinetti et al., 2008] ⁴[Hamann et al., 2018]

A-BTER: Adapted BTER Our full approach for HPC-scale benchmark generation and evaluation

Input: Real or synthetic degree and clustering coefficient distributions

- Optional step: Scale the input degree and clustering coefficient distributions
- 2 Solve linear program to (further) shift clustering coefficient distributions to match some target μ_g
- **3** Pass new distributions to an efficient edge-skipping based BTER implementation
- Run community detection algorithm on generated graph, evaluate versus "engineered approximate solution" (EAS)

Output: A measure of algorithm solution quality

A-BTER: Adapted BTER Our full approach for HPC-scale benchmark generation and evaluation

Input: Real or synthetic degree and clustering coefficient distributions

- **1** Optional step: Scale the input degree and clustering coefficient distributions
- 2 Solve linear program to (further) shift clustering coefficient distribution to match some target μ_g
- **3** Pass new distributions to an efficient edge-skipping based BTER implementation
- Run community detection algorithm on generated graph, evaluate versus "engineered approximate solution" (EAS)

Output: A measure of algorithm solution quality

Degree Distribution: How many vertices in the graph with each degree?



- Degree Distribution: How many vertices in the graph with each degree?
- Clustering Coefficient Distribution: What is the average clustering coefficient for each unique degree?



- Degree Distribution: How many vertices in the graph with each degree?
- Clustering Coefficient Distribution: What is the average clustering coefficient for each unique degree?
- Clustering Coefficient: Fraction of my friends that are friends with each other



イロト 不同下 イヨト イヨト

- Degree Distribution: How many vertices in the graph with each degree?
- Clustering Coefficient Distribution: What is the average clustering coefficient for each unique degree?
- Clustering Coefficient: Fraction of my friends that are friends with each other



- Degree Distribution: How many vertices in the graph with each degree?
- Clustering Coefficient Distribution: What is the average clustering coefficient for each unique degree?
- Clustering Coefficient: Fraction of my friends that are friends with each other



- Degree Distribution: How many vertices in the graph with each degree?
- Clustering Coefficient Distribution: What is the average clustering coefficient for each unique degree?
- Clustering Coefficient: Fraction of my friends that are friends with each other



Distribution Scaling How to generate a 4× Twitter A-BTER graph

Consider some input degree distribution (e.g., Twitter)



How to generate a $4\times$ Twitter A-BTER graph

- Consider some input degree distribution (e.g., Twitter)
- We can interpolate and smooth this distribution to create a probability curve for degrees



How to generate a $4 \times$ Twitter A-BTER graph

- Consider some input degree distribution (e.g., Twitter)
- We can interpolate and smooth this distribution to create a probability curve for degrees
- Then scale and shift this curve to analytically match some new # vertices and # edges



How to generate a $4 \times$ Twitter A-BTER graph

- Consider some input degree distribution (e.g., Twitter)
- We can interpolate and smooth this distribution to create a probability curve for degrees
- Then scale and shift this curve to analytically match some new # vertices and # edges
- \blacksquare Finally, we randomly sample using this curve "new # vertices" times to create the new distribution



Comparison to real-world graph growth

- Our approach can often closely match real-world growth for degree and CC distributions
- We compare against uk-2005 and uk-2007 real world degree (left) and CC coefficients (right) as crawled from LAW⁵
- We scale from scaling uk-2005 to the N, M of uk-2007
 - Compare against graph growing EvoGraph [Park and Kim, 2018]



A-BTER: Adapted BTER Our full approach for HPC-scale benchmark generation and evaluation

Input: Real or synthetic degree and clustering coefficient distributions

- Optional step: Scale the input degree and clustering coefficient distributions
- 2 Solve linear program to (further) shift clustering coefficient distributions to match some target μ_g
- **3** Pass new distributions to an efficient edge-skipping based BTER implementation
- Run community detection algorithm on generated graph, evaluate versus "engineered approximate solution" (EAS)

Output: A measure of algorithm solution quality

Linear Program - This paper has math! Shifting the native μ of a graph's CC distribution

Minimally shift the input clustering coefficient (CC) distribution such that the output graph has a desired goal μ_g :

		$\mu_g = rac{1}{N}\sum_d^D rac{d_{inter}}{d}$	definition of mixing parameter
minimize	$\sum_{\substack{d\\ =}}^{D}$	$ \hat{p}_d - p_d $	minimize shift in CC distribution
subject to	$\sum_{i=1}^{D}$	$n_d \hat{p}_d = n(1 - \mu_g)$	achieve target mixing parameter
	d	$\begin{array}{l} 0 \leq \hat{p}_d \leq 1 \\ \hat{p}_d - p_d \geq \hat{p}_{d+1} - p_{d+1} \end{array}$	keep CC curve smooth and feasible
output	$\hat{c}_d = \hat{p}$	$ \hat{p}_d - \hat{p}_{d+1} \le 0.01$	

- p_d is G(n,p) probabilities per degree from CC distribution c_d , $p_d = \sqrt[3]{c_d}$
- \hat{p}_d is output probabilities to get new CC distribution \hat{c}_d , $\hat{c}_d = \hat{p}_d^3$
- n_d is degree distribution: n vertices of d degree D is unique degrees
- d_{inter} is expected number of inter-community edges for vertex of degree d
- N is number of vertices in graph, M is number of edges

Shifting the CC distribution CC distribution output running our LP on Twitter

We can shift the CC distribution while still retaining *interesting* properties. We show the CC distributions for various μ on Twitter. Labeled as "Twitter_ μ ".



Generation Accuracy Final outputs in terms of μ – TLDR: It works!

Our LP and CC shifting procedure has good accuracy in terms of achieving some target mixing parameter when combined with BTER. On the right, we compare to LFR itself by using distributions output by that generator.



Solution Time Solve time for our LP isn't a deal-breaker

While LP solve times in general can be slow, we have several things working in our favor:

- We only require non-zeros in the degree distribution D to be variables in the LP
 - Generally, $|D| \ll d_{max} \ll M$
- We can utilize a binning strategy, where we group x vertices in degree order into a bin represented by a single variable in the LP
- Our most difficult test case (Twitter, ~20K nonzeros) takes only a second to solve

A-BTER: Adapted BTER Our full approach for HPC-scale benchmark generation and evaluation

Input: Real or synthetic degree and clustering coefficient distributions

- Optional step: Scale the input degree and clustering coefficient distributions
- 2 Solve linear program to (further) shift clustering coefficient distributions to match some target μ_g
- **3** Pass new distributions to an efficient edge-skipping based BTER implementation
- Run community detection algorithm on generated graph, evaluate versus "engineered approximate solution" (EAS)

Output: A measure of algorithm solution quality

Step 0: Input degree (n_d) and clustering coefficient (c_d) distributions



- Step 0: Input degree (n_d) and clustering coefficient (c_d) distributions
- Step 1: With ordered degree sequence, group d+1 vertices v of degree d(v) >= d into affinity blocks



- Step 0: Input degree (n_d) and clustering coefficient (c_d) distributions
- Step 1: With ordered degree sequence, group d + 1 vertices v of degree d(v) >= d into affinity blocks



- Step 0: Input degree (n_d) and clustering coefficient (c_d) distributions
- Step 1: With ordered degree sequence, group d+1 vertices v of degree d(v) >= d into affinity blocks
- Step 2: Use Erdös-Rényi probability $p_d = \sqrt[3]{c_d}$ to create intra-block edges via G(n,p) process



- Step 0: Input degree (n_d) and clustering coefficient (c_d) distributions
- Step 1: With ordered degree sequence, group d+1 vertices v of degree d(v) >= d into affinity blocks
- Step 2: Use Erdös-Rényi probability $p_d = \sqrt[3]{c_d}$ to create intra-block edges via G(n,p) process
- Step 3: Create inter-block edges via Chung-Lu process



Our Implementation - For Community Detection How we adapt BTER for community detection benchmarking

Adapting BTER:

- We wrap the BTER process to generate benchmark graphs
- Treat affinity blocks as EAS communities
- The LP shifting from the prior steps results in some goal mixing parameter μ_g based on EAS assignments
- We utilize the *edge-skipping* technique for very efficient generation

For efficient Erdős-Rényi, Chung-Lu, and BTER graph generation

 \blacksquare Consider the undirected simple Erdős-Rényi G(n,p) model as an adjacency matrix



For efficient Erdős-Rényi, Chung-Lu, and BTER graph generation

- \blacksquare Consider the undirected simple Erdős-Rényi G(n,p) model as an adjacency matrix
- \blacksquare Nonzeros (i.e., edges) appear in upper triangle at index i,j with probability p



For efficient Erdős-Rényi, Chung-Lu, and BTER graph generation

- \blacksquare Consider the undirected simple Erdős-Rényi G(n,p) model as an adjacency matrix
- \blacksquare Nonzeros (i.e., edges) appear in upper triangle at index i,j with probability p
- Instead of flipping a coin for all indices, we can instead sample $\sim m = p \times \frac{(n-1)(n-2)}{2}$ skip lengths



For efficient Erdős-Rényi, Chung-Lu, and ${}_{\rm BTER}$ graph generation

- \blacksquare Consider the undirected simple Erdős-Rényi G(n,p) model as an adjacency matrix
- \blacksquare Nonzeros (i.e., edges) appear in upper triangle at index i,j with probability p
- Instead of flipping a coin for all indices, we can instead sample ${\sim}m=p\times\frac{(n-1)(n-2)}{2}$ skip lengths
- We traverse through an ordered space of unique possible edges, moving by each samples skip length and outputting the edge we land on



For efficient Erdős-Rényi, Chung-Lu, and ${}_{\rm BTER}$ graph generation

- \blacksquare Consider the undirected simple Erdős-Rényi G(n,p) model as an adjacency matrix
- \blacksquare Nonzeros (i.e., edges) appear in upper triangle at index i,j with probability p
- Instead of flipping a coin for all indices, we can instead sample ${\sim}m=p\times\frac{(n-1)(n-2)}{2}$ skip lengths
- We traverse through an ordered space of unique possible edges, moving by each samples skip length and outputting the edge we land on



For efficient Erdős-Rényi, Chung-Lu, and ${}_{\rm BTER}$ graph generation

- \blacksquare Consider the undirected simple Erdős-Rényi G(n,p) model as an adjacency matrix
- \blacksquare Nonzeros (i.e., edges) appear in upper triangle at index i,j with probability p
- Instead of flipping a coin for all indices, we can instead sample ${\sim}m=p\times \frac{(n-1)(n-2)}{2}$ skip lengths
- We traverse through an ordered space of unique possible edges, moving by each samples skip length and outputting the edge we land on



Edge-skipping For Erdős-Rényi, Chung-Lu, and BTER graph generation

- Edge-skipping is provably equivalent to flipping a coin for each possible edge
- Recall: BTER creates Erdős-Rényi blocks and layers a Chung-Lu graph for inter-block edges
 - For Chung-Lu generation, we use edge skipping to create bipartite graphs with attachment probability $p_{i,j} = \frac{d_i \times d_j}{2m}$ for each unique degree pair (d_i, d_j)
 - Combining all of these bipartite graphs gives us the full Chung-Lu graph
 - Combined with the Erdős-Rényi blocks, we have our BTER graph!
- We can parallelize block and Chung-Lu generation with MPI and OpenMP to get a parallel time of $O(\frac{M}{P} + |D|)$

Edge Generation – Experimental setup for scaling Non-graph SC people: you can start paying attention again

Test Systems:

- Mutrino 96× KNL nodes with 68 cores, 96 GB DDR, 16 GB MCDRAM
- Trinity 9800× KNL nodes with 68 cores, 96 GB DDR, and 16 GB MCDRAM
- Astra 2500× ARM nodes with 56 cores and 128 GB DDR

Network	N	M	d_{avg}	d_{max}	C_{avg}	C _{max}	Source
LiveJournal Wikilinks RMAT $_{26}$ Twitter Friendster	2.1 M 1.9 M 63 M 39 M 40 M	25 M 21 M 1.1 B 1.4 B 1.8 B	24 21 33 73 90	2.0 K 8.6 K 6.7 K 56 K 5.2 K	0.27 0.12 0.00 0.07 0.13	0.39 0.18 0.00 0.49 0.33	SNAP Koblenz GTGraph Max Planck Inst. SNAP
uk-2007	81 M	3.3 B	80	82 K	0.78	0.99	LAW

Test Graphs:

We pre-process the distributions such that the minimum degree is 5 and maximum is $\sqrt{n} \log(n)$. This pre-processing is not necessary for our methods to work, but it enables more defined community boundaries.

Strong Scaling Strong scalability of our edge-skipping BTER generator

- Strong scaling on Mutrino (KNL)
 - Generating 9 test graphs from $\mu=0.1\dots0.9$
- Average speedup is 5.8× across 16 nodes



Terascale Scaling Scalability of our edge-skipping BTER generator

- Friendster scaled from 2× to 512× and generated on on Trinity (KNL) and Astra (ARM)
- The largest test utilizes 512 nodes of each system and generates a 15 terabyte edge list of 925 billion edges

Scale	m	n	d_{max}	Memory	T _{KNL}	T _{ARM}
1 imes	1.8 B	40 M	5.2 K	29 GB	33 s	22 s
$4 \times$	7.2 B	93 M	10 K	115 GB	35 s	28 s
$16 \times$	29 B	260 M	15 K	459 GB	35 s	29 s
$64 \times$	115 B	786 M	20 K	1.8 TB	55 s	32 s
$256 \times$	464 B	2.5 B	26 K	7.4 TB	102 s	69 s
$512 \times$	925 B	4.6 B	30 K	15 TB	134 s	76 s

We generate edges at a rate of almost 1T per minute!

Matching Input Distributions Output quality in terms of distribution matching

Our edge-skipping BTER generator also closely matches the input degree and clustering coefficient distributions.



26/31

Exact A–BTER

Matching Input Distributions Output quality in terms of distribution matching

Our edge-skipping BTER generator also closely matches the input degree and clustering coefficient distributions.



Exact A–BTER

26 / 31

A-BTER: Adapted BTER

Our full approach for HPC-scale benchmark generation and evaluation

Input: Real or synthetic degree and clustering coefficient distributions

- Optional step: Scale the input degree and clustering coefficient distributions
- 2 Solve linear program to (further) shift clustering coefficient distributions to match some target μ_g
- **3** Pass new distributions to an efficient edge-skipping based BTER implementation
- Run community detection algorithm on generated graph, evaluate versus "engineered approximate solution" (EAS)

Output: A measure of algorithm solution quality

Using our Benchmark Generator

Benchmarking algorithms in practice

- We develop a parallel evaluation algorithm for Normalized Mutual Information
 - Optimal work and parallel time complexity
 - ${\cal O}(n)$ and ${\cal O}(1),$ respectively
 - Recent literature has claimed ${\cal O}(n^2)$ work to compute
- We use it with A-BTER to benchmark parallel Louvain⁶ and Label Propagation⁷ community detection algorithms

 And compare our benchmark performance vs. LFR
- We also benchmark Label Propagation at the very very large scale
 - And compare its output quality as we strong scale

⁶[Ghosh et al., 2018] ⁷[Slota et al., 2016]

Benchmark Comparison to LFR

The conclusions drawn between benchmarks are the same

- We compare benchmark outputs from A-BTER (top) and LFR (bottom)
 Read labels as (num vertices)_(average degree)
- We generate A-BTER graphs using degree and CC distributions from LFR
- We note similar observations in terms of performance of Louvain vs. Label Propagation – Takeaway: Louvain > Label Propagation



--- Louvain --- LabelPropagation

Massive Scale Evaluation

Comparing algorithm output up to $\sim 0.5T$ edges

-- 16 -- 32 -- 64

- \blacksquare We run Label Propagation on $16\times$ (left) and $256\times$ (right) Friendster on various node counts
- We note running Label Propagation in distributed memory initially has a large effect on solution quality, though further strong scaling has minimal impact

→ 512 → 1024 → 2048

First benchmark evaluation of community detection algorithms against Engineered Approximate Solution at HPC scale





- Our approach can output graphs for community detection order-of-magnitudes faster than commonly-used generators, e.g., LFR
- Our approach can output graphs with more realistic degree and CC distributions than commonly-used generators
- We can scale up degree and CC distributions to generate terascale benchmarks
- Future Work: Develop generation methods for hierarchical or overlapping communities

If you propose a new community detection algorithm, you now have to use our code Code: www.github.com/HPCGraphAnalysis/SAGE

www.gmslota.com, slotag@rpi.edu

Bibliography I

- Sayan Ghosh, Mahantesh Halappanavar, Antonino Tumeo, Ananth Kalyanaraman, Hao Lu, Daniel Chavarria-Miranda, Arif Khan, and Assefaw Gebremedhin. Distributed louvain algorithm for graph community detection. In 2018 IEEE International Parallel and Distributed Processing Symposium (IPDPS), pages 885–895. IEEE, 2018.
- Michael Hamann, Ulrich Meyer, Manuel Penschuck, Hung Tran, and Dorothea Wagner. I/o-efficient generation of massive graphs following the LFR benchmark. J. Exp. Algorithmics, 23(1):2.5:1–2.5:33, August 2018. ISSN 1084-6654. doi: 10.1145/3230743. URL http://doi.acm.org/10.1145/3230743.
- Tamara G. Kolda, Ali Pinar, Todd Plantenga, and C Seshadhri. A scalable generative graph model with community structure. SIAM Journal on Scientific Computing, 36(5):C424–C452, 2014.
- Andrea Lancichinetti, Santo Fortunato, and Filippo Radicchi. Benchmark graphs for testing community detection algorithms. *Physical Review E*, 78(4):1–5, October 2008. ISSN 1539-3755. doi: 10.1103/PhysRevE.78.046110. URL http://link.aps.org/doi/10.1103/PhysRevE.78.046110.
- Joel C Miller and Aric Hagberg. Efficient generation of networks with given expected degrees. In International Workshop on Algorithms and Models for the Web-Graph, pages 115–126. Springer, 2011.
- Himchan Park and Min-Soo Kim. Evograph: an effective and efficient graph upscaling method for preserving graph properties. In Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, pages 2051–2059. ACM, 2018.
- G. M. Slota, S. Rajamanickam, and K. Madduri. A case study of complex graph analysis in distributed memory: Implementation and optimization. In International Parallel & Distributed Processing Symposium (IPDPS), 2016.