PuLP: Scalable Multi-Objective Multi-Constraint Partitioning for Small-World Networks

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Highlights

- We present PuLP, a multi-constraint multi-objective partitioner designed for small-world graphs
- Shared-memory parallelism
- PuLP demonstrates an average speedup of 14.5$\times$ relative to state-of-the-art partitioners
- PuLP requires 8-39$\times$ less memory than state-of-the-art partitioners
- PuLP produces partitions with comparable or better quality than state-of-the-art partitioners for small-world graphs

Overview

- PuLP: Partitioning Using Label Propagation
  - Overview
    - Graph partitioning formulation
    - Label propagation
    - Using label propagation for partitioning
  - PuLP Algorithm
    - Degree-weighted label prop
    - Label propagation for balancing constraints and minimizing objectives
    - Label propagation for iterative refinement
  - Results
    - Performance comparisons with other partitioners
    - Partitioning quality with different objectives

- **Graph Partitioning**: Given a graph $G(V, E)$ and p processes or tasks, assign each task a p-way disjoint subset of vertices and their incident edges from G
  - Balance constraints – (weighted) vertices per part, (weighted) edges per part
  - Quality metrics – edge cut, communication volume, maximal per-part edge cut
- We consider:
  - Balancing edges **and** vertices per part
  - Minimizing edge cut (EC) **and** maximal per-part edge cut (EC_{max})

Overview

Partitioning - Objectives and Constraints

- Lots of graph algorithms follow a certain iterative model
  - BFS, SSSP, FASCIA subgraph counting (Slota and Madduri 2014)
  - computation, synchronization, communication, synchronization, computation, etc.
- Computational load: proportional to vertices and edges per-part
- Communication load: proportional to total edge cut and max per-part cut
- We want to minimize the maximal time among tasks for each comp/comm stage

Overview

Partitioning - Balance Constraints

- Balance vertices and edges:

$$(1-\epsilon_l)\,\frac{|V|}{p} \quad \le |V(\pi_i)| \quad \le (1+\epsilon_u)\,\frac{|V|}{p} \tag{1}$$

$$|E(\pi_i)| \quad \le (1+\eta_u)\,\frac{|E|}{p} \tag{2}$$

- ϵ_l and ϵ_u: lower and upper vertex imbalance ratios
- η_u: upper edge imbalance ratio
- $V(\pi_i)$: set of vertices in part π_i
- $E(\pi_i)$: set of edges with both endpoints in part π_i

- Given a partition Π, the set of *cut edges* $(C(G,\Pi))$ and cut edge per partition $(C(G,\pi_k))$ are

$$C(G,\Pi) = \{\{(u,v) \in E\} \mid \Pi(u) \neq \Pi(v)\} \quad (3)$$
$$C(G,\pi_k) = \{\{(u,v) \in C(G,\Pi)\} \mid (u \in \pi_k \vee v \in \pi_k)\} \quad (4)$$

- Our partitioning problem is then to minimize total edge cut EC and max per-part edge cut EC_{max}:

$$EC(G,\Pi) = |C(G,\Pi)| \quad (5)$$
$$EC_{max}(G,\Pi) = \max_k |C(G,\pi_k)| \quad (6)$$

- (Par)METIS (Karypis et al.), PT-SCOTCH (Pellegrini et al.), Chaco (Hendrickson et al.), etc.
- Multilevel methods:
  - *Coarsen* the input graph in several iterative steps
  - At coarsest level, partition graph via local methods following balance constraints and quality objectives
  - Iteratively *uncoarsen* graph, refine partitioning
- **Problem 1**: Designed for traditional HPC scientific problems (e.g. meshes) – limited balance constraints and quality objectives
- **Problem 2**: Multilevel approach – high memory requirements, can run slowly and lack scalability

- **Label propagation**: randomly initialize a graph with some p labels, iteratively assign to each vertex the maximal per-label count over all neighbors to generate clusters (Raghavan et al. 2007)
  - Clustering algorithm - dense clusters hold same label
  - Fast - each iteration in $O(n+m)$, usually fixed iteration count (doesn't necessarily converge)
  - Naïvely parallel - only per-vertex label updates
  - *Observation*: Possible applications for large-scale small-world graph partitioning

Overview

Partitioning - "Big Data" Approaches

- Methods designed for small-world graphs (e.g. social networks and web graphs)
- Exploit label propagation/clustering for partitioning:
  - Multilevel methods - use label propagation to coarsen graph (Wang et al. 2014, Meyerhenke et al. 2014)
  - Single level methods - use label propagation to directly create partitioning (Ugander and Backstrom 2013, Vaquero et al. 2013)
- **Problem 1**: Multilevel methods still can lack scalability, might also require running traditional partitioner at coarsest level
- **Problem 2**: Single level methods can produce sub-optimal partition quality

PuLP : **P**artitioning **U**sing **L**abel **P**ropagation

- Utilize label propagation for:
  - Vertex balanced partitions, minimize edge cut (PuLP)
  - Vertex and edge balanced partitions, minimize edge cut (PuLP-M)
  - Vertex and edge balanced partitions, minimize edge cut and maximal per-part edge cut (PuLP-MM)
  - Any combination of the above - multi objective, multi constraint

- PuLP-MM Algorithm
  - Constraint 1: balance vertices, Constraint 2: balance edges
  - Objective 1: minimize edge cut, Objective 2: minimize per-partition edge cut
  - Pseudocode gives default iteration counts

Initialize p random partitions
Execute 3 iterations degree-weighted label propagation (LP)
for $k_1 = 1$ iterations **do**
    for $k_2 = 3$ iterations **do**
        Balance partitions with 5 LP iterations to satisfy constraint 1
        Refine partitions with 10 FM iterations to minimize objective 1
    for $k_3 = 3$ iterations **do**
        Balance partitions with 2 LP iterations to satisfy constraint 2
          and minimize objective 2 with 5 FM iterations
        Refine partitions with 10 FM iterations to minimize objective 1

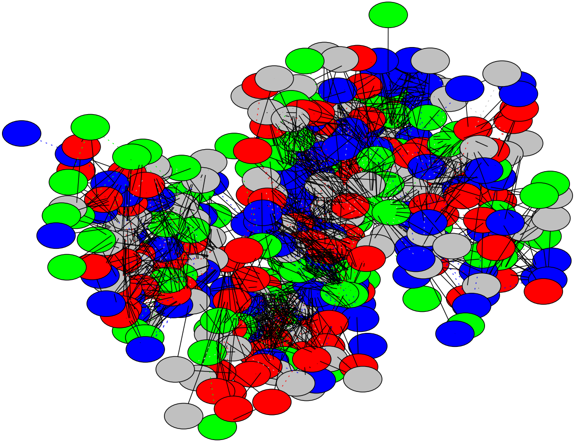
Algorithms

Primary Algorithm Overview

Initialize p random partitions
Execute degree-weighted label propagation (LP)
for k_1 iterations **do**
    for k_2 iterations **do**
        Balance partitions with LP to satisfy vertex constraint
        Refine partitions with FM to minimize edge cut
    for k_3 iterations **do**
        Balance partitions with LP to satisfy edge constraint and minimize max per-part cut
        Refine partitions with FM to minimize edge cut

Algorithms
Primary Algorithm Overview

Randomly initialize p partitions ($p = 4$)



Network shown is the Infectious network dataset from KONECT (http://konect.uni-koblenz.de/)

- After random initialization, we then perform label propagation to create partitions
- **Initial Observations**:
  - Partitions are unbalanced, for high p, some partitions end up empty
  - Edge cut is good, but can be better
- **PuLP Solutions**:
  - Impose loose balance constraints, explicitly refine later
  - Degree weightings - cluster around high degree vertices, let low degree vertices form boundary between partitions

Initialize p random partitions
Execute degree-weighted label propagation (LP)
for k_1 iterations **do**
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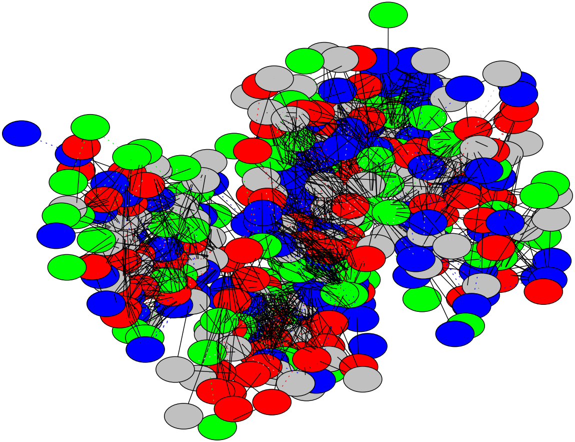
Part assignment after random initialization.



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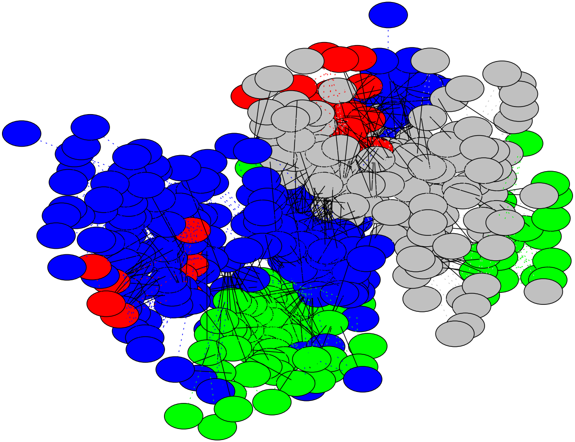
Part assignment after degree-weighted label propagation.



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Algorithms
Primary Algorithm Overview

- After label propagation, we balance vertices among partitions and minimize edge cut (baseline PuLP ends here)
- **Observations**:
  - Partitions are still unbalanced in terms of edges
  - Edge cut is good, max per-part cut isn't necessarily
- **PuLP-M and PuLP-MM Solutions**:
  - Maintain vertex balance while explicitly balancing edges
  - Alternate between minimizing total edge cut and max per-part cut (for PuLP-MM, PuLP-M only minimizes total edge cut)

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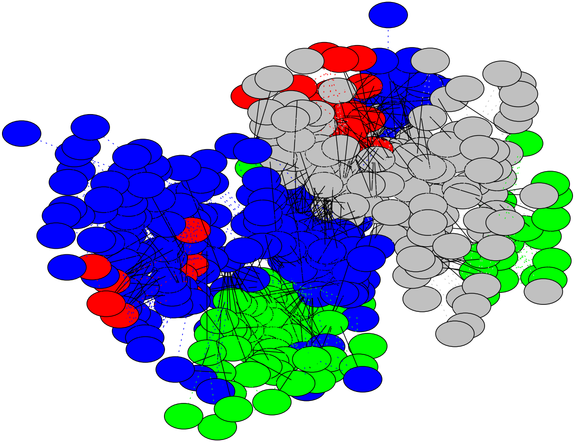
Part assignment after degree-weighted label propagation.



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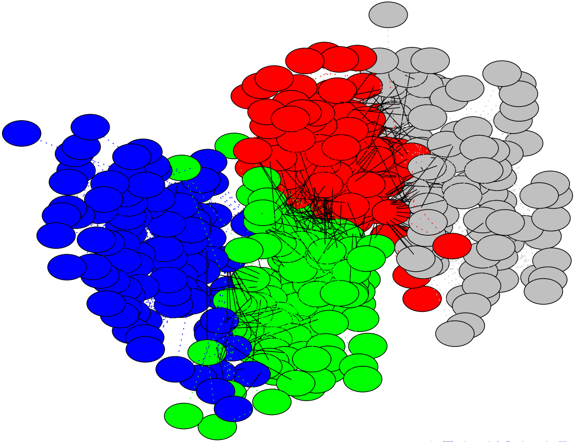
Algorithms
Primary Algorithm Overview

Part assignment after balancing for vertices and minimizing edge cut.



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Algorithms
Primary Algorithm Overview

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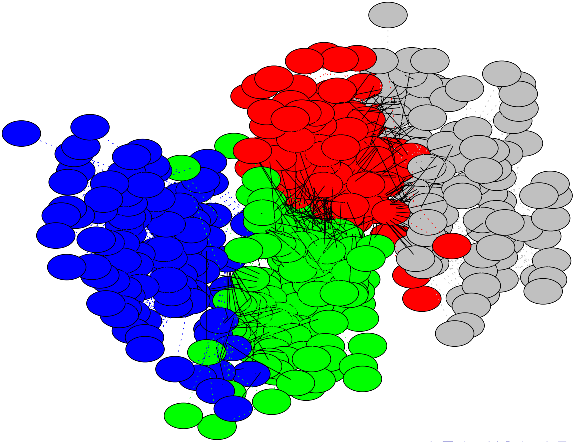

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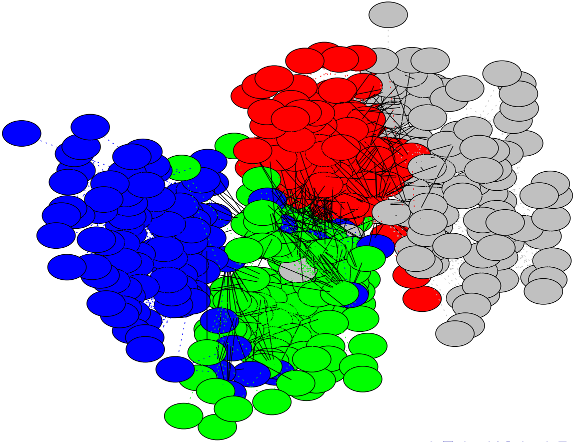
Part assignment after balancing for edges and minimizing total edge cut and max per-part edge cut



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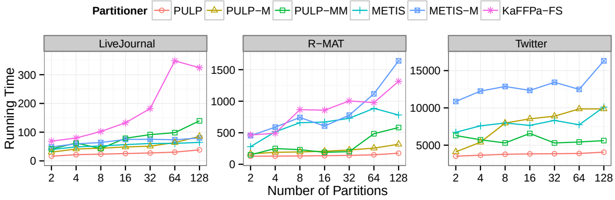
Results
Test Environment and Graphs

- Test system: *Compton*
  - Intel Xeon E5-2670 (Sandy Bridge), dual-socket, 16 cores, 64 GB memory.
- Test graphs:
  - LAW graphs from UF Sparse Matrix, SNAP, MPI, Koblenz
  - Real (one R-MAT), small-world, 60 K–70 M vertices, 275 K–2 B edges
- Test Algorithms:
  - **METIS** - single constraint single objective
  - **METIS-M** - multi constraint single objective
  - **ParMETIS** - METIS-M running in parallel
  - **KaFFPa** - single constraint single objective
  - **PuLP** - single constraint single objective
  - **PuLP-M** - multi constraint single objective
  - **PuLP-MM** - multi constraint multi objective
- Metrics: 2–128 partitions, serial and parallel running times, memory utilization, edge cut, max per-partition edge cut

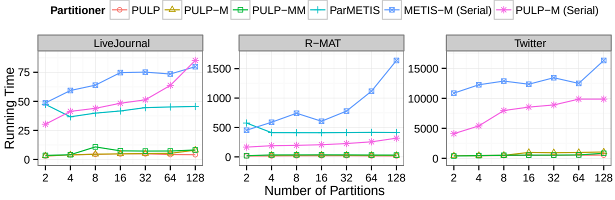
Results

Running Times - Serial (top), Parallel (bottom)

- In serial, PULP-MM runs 1.7× faster (geometric mean) than next fastest



- In parallel, PULP-MM runs 14.5× faster (geometric mean) than next fastest (ParMETIS times are fastest of 1 to 256 cores)



Results

Memory utilization for 128 partitions

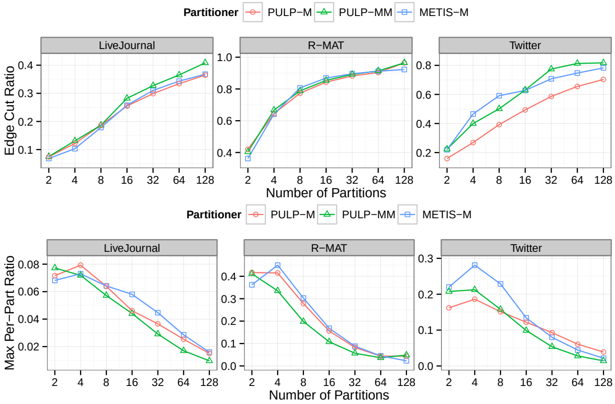
- PuLP utilizes minimal memory, $O(n)$, 8-39× less than other partitioners
- Savings are mostly from avoiding a multilevel approach

Network	Memory Utilization				**Improv.**
	METIS-M	KaFFPa	PuLP-MM	Graph Size	
LiveJournal	7.2 GB	5.0 GB	0.44 GB	0.33 GB	21×
Orkut	21 GB	13 GB	0.99 GB	0.88 GB	23×
R-MAT	42 GB	-	1.2 GB	1.02 GB	35×
DBpedia	46 GB	-	2.8 GB	1.6 GB	28×
WikiLinks	103 GB	42 GB	5.3 GB	4.1 GB	25×
sk-2005	121 GB	-	16 GB	13.7 GB	8×
Twitter	487 GB	-	14 GB	12.2 GB	39×

Results

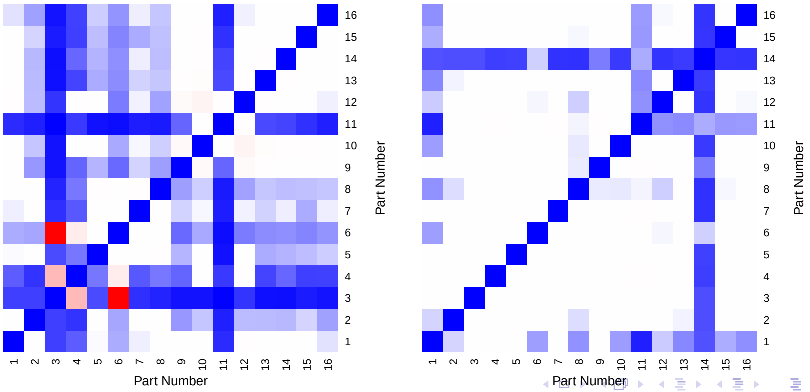
Performance - Edge Cut and Edge Cut Max

- PuLP-M produces better edge cut than METIS-M over most graphs
- PuLP-MM produces better max edge cut than METIS-M over most graphs



Results
Balanced communication

- uk-2005 graph from LAW, METIS-M (left) vs. PuLP-MM (right)
- Blue: low comm; White: avg comm; Red: High comm
- PuLP reduces max inter-part communication requirements and balances total communication load through all tasks



Future Work

- Explore techniques for avoiding local minima, such as simulated annealing, etc.
- Further parallelization in distributed environment for massive-scale graphs
- Demonstrate performance of PuLP partitions with graph analytics
- Explore tradeoff and interactions in various parameters and iteration counts

Conclusions

- We presented PULP, a multi-constraint multi-objective partitioner designed for small-world graphs
- Shared-memory parallelism
- PULP demonstrates an average speedup of 14.5$\times$ relative to state-of-the-art partitioners
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- **Questions?**

Acknowledgments

- DOE Office of Science through the FASTMath SciDAC Institute
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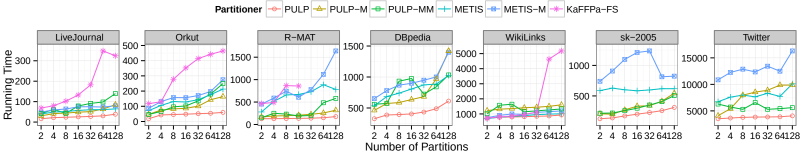
- Used NERSC hardware for generating partitionings - supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231

- Backup slides

Results

Running Times - Serial (top), Parallel (bottom)

- PuLP faster than others over most tests in serial
- In parallel, PuLP always faster than other



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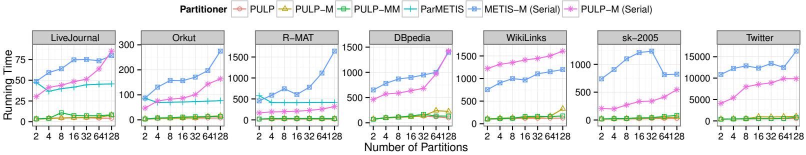

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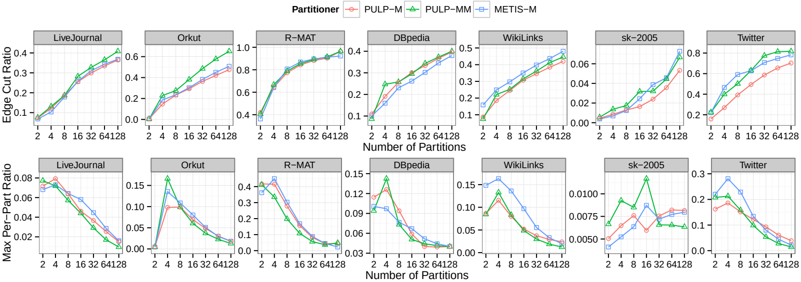

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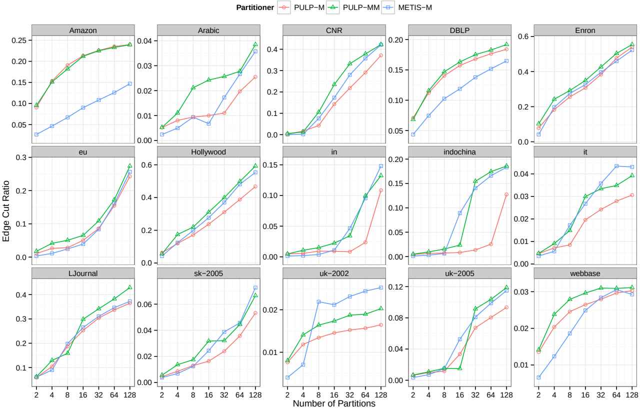

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