

**CSci 4968 and 6270
Computational Vision,
Fall Semester, 2009-2010
Homework 5**

Due: Thursday, November 12, 2009 at the start of class

This assignment is worth 90 points toward your homework grade. It is a partial replacement for the in-class test. You are to work on these questions on your own and you may use only your notes and the Szeliski text. Please turn in either a hand-written or printed copy of your solutions to me at the start of class on the due date. Assignments given to me after 12:05 pm on November 12 will incur a rapidly increasing late penalty! Do NOT email me your solutions!

In all cases, you must justify your answers to these questions, but these justifications should be brief. None of these questions requires extremely long answers. Clearly state any assumptions that you feel you must make.

Problems

1. **(10 points)** What is the effect of different types of smoothing kernels on (a) a one-pixel wide line of brighter pixels against a darker background, and (b) a 90 degree corner of brighter pixels against a darker background. Consider Gaussian filters, bilateral filters and median filters. On this problem, I am looking for qualitative descriptions rather than mathematical formulii.
2. **(6 points)** Why is it important that values in a smoothing kernel values sum to 1? What should be the sum of the values in a differentiation kernel?
3. **(10 points)** Under the assumption that a morphological structuring element K contains the $(0, 0)$ location, show that
 - (a) For dilation, $B \oplus K \subset (B \oplus K) \oplus K$
 - (b) For opening, $(B \bullet K) \bullet K = B \bullet K$where B is the set of pixels having value 1 in a binary image.
4. **(6 points)** What do opening and closing with disk structuring elements do to 90 degree corners in a binary image?
5. **(6 points)** Using the parametric form of the line, show how to compute the minimum distance of a point to a line. This does not require use of Lagrange multipliers.

6. **(12 points)** Suppose we are fitting a least-squares line to a set of edgel points. Describe, mathematically, an estimation technique that would weight the points by their gradient magnitude. Next, how might you incorporate the gradient direction into the least-squares estimation?
7. **(12 points)** Outline a Hough transform technique to find circles in images. Consider both the parameter space and the voting technique. How would you use edgel gradient information to improve the Hough transform?
8. **(10 points)** Prove algebraically that a line in 3d projects onto a line in the image under perspective projection using the 3x4 projection matrix M . Hint: use a parametric form for the line and then, after projecting, eliminate the parameter. If you can not get this to work, try solving the problem for the simple model of perspective projection:

$$u = \frac{fx}{z} \quad \text{and} \quad v = \frac{fy}{z}.$$

9. **(16 points)** Suppose two images, I_1 and I_2 , are taken by a camera that is known to be translating, but not rotating. Also assume that the two intrinsic camera matrices are

$$\mathbf{K}_1 = \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{K}_2 = \begin{pmatrix} f_2 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- (a) Give the essential matrix \mathbf{E} corresponding to this case.
- (b) When the fundamental matrix \mathbf{F} is known, what can be inferred from it about f_1 and f_2 ?
- (c) When a camera is only translating, there is one image location (x_0, y_0) , called the focus of expansion, that is stationary. At every other point, (x, y) , the line through (x, y) in the direction of the correct local motion estimate, (t_x, t_y) includes (x_0, y_0) . (Note that these t_x and t_y values are not the same as the first 2 components of the camera translation!) Assume a local motion estimate (t_x, t_y) is available on a regularly-sampled set of image locations (for example at every 5th pixel), and assume these estimates are obtained using the techniques presented in the Lecture 18 notes. Describe a technique to estimate the location of (x_0, y_0) .