

CSci 4968 and 6270  
Computational Vision  
Lecture 11  
Cameras and Calibration, Part 2

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## Cameras and Calibration — Part 2

### Resetting the stage

- What we have covered:
  - Perspective projections
  - Transformations
  - Extrinsic and intrinsic parameters
- What we need to cover:
  - Real lenses and lens distortions
  - Camera (scene) motion, special cases: rotation about optical center, and images of planar surfaces
  - General case of two images — intro to stereo geometry
  - Calibration

## Lens Effects

When we consider real lenses, a number of issues arise:

- Defocus blurring
- Vignetting and other intensity distortions
- Lens distortions

We will consider each of these briefly, and the latter in a bit more detail.

## Radial Lens Distortion

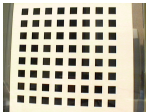


Image from Microsoft Research

- Shift radially away from the center (barrel distortion) or toward the center (pin-cushion).
- Quadratic or quartic model, applied after division but before decentering

## Radial Lens Distortion

- Let  $(u_c, v_c)$  be the centered coordinates and compute  $r = \sqrt{u_c^2 + v_c^2}$ .

- Then the distorted (still centered) coordinates are

$$\begin{aligned}u'_c &= u_c(1 + \kappa_1 r^2 + \kappa_2 r^4) \\v'_c &= v_c(1 + \kappa_1 r^2 + \kappa_2 r^4)\end{aligned}$$

Sometimes  $\kappa_2 = 0$ , so that only a quadratic model is used.

- $(u'_c, v'_c)$  is decentered and (perhaps) scaled to obtain the actual pixel coordinates.
- Unfortunately, this model is not algebraically invertible, so either approximations or numerical-inverse methods must be used when going from image pixels back to world coordinates.

## Multiple Images

- We now extend our understanding of a single camera to multiple images and "cameras".
  - Each new image, even when taken with the same physical camera, is modeled using a distinct camera matrix because its extrinsic parameters, its intrinsic parameter, or both may have changed.
- As part of our goal of building image mosaics, we need to understand the relationship between different images and cameras.
  - This tells us when mosaics can and can not be build.
- We will look at
  - Two special cases
  - The two-view general case

## Multiple Images: Rotation About the Image Center

- For convenience, we place the world coordinate system origin at the optical center.
- Doing this, each "camera" (without considering radial lens distortion) can be written

$$\mathbf{M}_i = \mathbf{K}_i (\mathbf{R}_i \quad \mathbf{0})$$

- Also, any pixel in image coordinates correspondences to a line in the world of the form

$$\mathbf{p}(r) = \hat{\mathbf{d}}r$$

where  $\hat{\mathbf{d}}$  is a unit direction vector.

## Rotation About the Image Center (continued)

- Combining these two, we will show in class that pixel location  $\hat{\mathbf{u}}_1$  in image  $I_1$  maps onto pixel location

$$\hat{\mathbf{u}}_2 = \mathbf{K}_2 \mathbf{R}_2 \mathbf{R}_1^{-1} \mathbf{K}_1^{-1} \hat{\mathbf{u}}_1$$

in image  $I_2$ .

- This is independent of the distance of points from the camera!
- $\mathbf{H} = \mathbf{K}_2 \mathbf{R}_2 \mathbf{R}_1^{-1} \mathbf{K}_1^{-1}$  is a  $3 \times 3$  homography matrix known as a *conjugate rotation*.
- We will soon consider methods to estimate the parameters of  $\mathbf{H}$  (not the individual  $\mathbf{K}$ 's and  $\mathbf{R}$ 's).

## Multiple Images of a Planar Surface

A slightly more involved proof, which we will not go into shows...

- Suppose we are taking images of a planar surface
  - a wall, the earth, a billboard,
  - almost anything when the distance from the camera is small relative to the distance variation
- Then, even if we move the camera (or cameras) in arbitrary ways, a pixel location  $\tilde{\mathbf{u}}_1$  in image  $I_1$  maps onto pixel location  $\tilde{\mathbf{u}}_2$  in  $I_2$  as

$$\tilde{\mathbf{u}}_2 = \mathbf{H}\tilde{\mathbf{u}}_1$$

where  $\mathbf{H}$  is a  $3 \times 3$  homography.

- We do not need to know all the parameters of the plane, only that the plane exists!

## When Can We Build a Montage?

- We now have the two cases: rotation about the optical center, and images of planar surface.
- In each case, the images are related by a planar homography.
- The depths of the world coordinate points need not be known in order to map between images — only the  $3 \times 3$  matrix  $\mathbf{H}$  must be known.
- The goal of estimation for building the mosaic is to estimate the parameters of this  $\mathbf{H}$  matrix.
- In some cases, affine and even similarity transformations can serve as a good approximation to  $\mathbf{H}$ .

## General Case of Two Images

When the optical centers of the cameras do not coincide...

- The backprojection of a point  $p_1$  in  $I_1$  is a line of possible points in the world.
- When this line is "projected" down into image  $I_2$ , it forms a line in  $I_2$ .  
This line is called the "epipolar line"
- More generally, this epipolar line in  $I_2$  corresponds to another line, back in  $I_1$ , that contains  $p_1$ .
- Of course, we must know something about the two cameras in order to determine these lines!

## The Fundamental Matrix

- For each pair of images  $I_1$  and  $I_2$ , there exists a special  $3 \times 3$  matrix  $\mathbf{F}$  called the *fundamental matrix*
- If  $\tilde{\mathbf{x}}_1$  is the homogeneous coordinate vector for a point from  $I_1$ , then the epipolar line in  $I_2$  has (implicit form) parameters

$$\mathbf{a}_2 = \mathbf{F}\tilde{\mathbf{x}}_1.$$

- This fundamental matrix may be computed from the camera matrices or estimated when there are at least 7 known matching points between the images.
- This matrix has rank 2 (not 3).

## Two View Summary

### Only two cases for relating the image projection of a points

- Point coordinates are  $\bar{x}_1$  in  $I_1$  and  $\bar{x}_2$  in  $I_2$ .
- When the cameras have the same optic center or the image points are of a planar surface, then there is a  $3 \times 3$  homography matrix  $\mathbf{H}$  such that

$$\bar{x}_2 = \mathbf{H}\bar{x}_1$$

- Otherwise, there is a  $3 \times 3$ , rank-2 matrix  $\mathbf{F}$  such that

$$\bar{x}_2^T \mathbf{F} \bar{x}_1 = 0.$$

- In the first case,  $\mathbf{H}$  **determines** the matching point location for  $\bar{x}_1$ .
- In the second case,  $\mathbf{F}$  **constrains** the matching point location for  $\bar{x}_1$ .

## Estimation Problems

### We now have three matrices for which we would like to know the parameters

- $3 \times 4$  camera matrix  $\mathbf{M}$  (plus radial lens distortion terms) for a single camera
- $3 \times 3$  homography matrix  $\mathbf{H}$  for two important special cases of two cameras
- $3 \times 3$  fundamental matrix  $\mathbf{F}$  for the general case

## Overview of Solving these Estimation Problems

### Two primary steps:

1. Establish "correspondences"
  - Between features in two images (for  $\mathbf{H}$  or for  $\mathbf{F}$ )
  - Between points on a known "calibration target" and points in an image.
2. Estimate parameters based on these correspondences:
  - Use the defining equations for  $\mathbf{H}$ ,  $\mathbf{F}$  and  $\mathbf{M}$  to write a least-squares objective function.
- In the next set of notes, we will focus on estimation of  $\mathbf{H}$ , the easiest of these.

## Correspondences for Camera Calibration: Brief Overview



Image from [cmp.felk.cvut.cz](http://cmp.felk.cvut.cz)

- A calibration target is created, usually a three-dimensional object, that has a known (machined) shape and precisely measured markings.
- The target is placed in a fixed position, so the markings have known positions in 3D.
- Extract features in images of the target, e.g.
  - ▶ Edges (with subpixel accuracy) and then fit lines
  - ▶ Intersect lines to form corners.
- Use corners and knowledge of target to establish 3d-to-2d correspondences.

## Project Idea

### Simpler Calibration

Over the past decade researchers have proposed a variety of simpler calibration methods, based on less-complex targets. These targets, often patterned, planar surfaces, are then imaged several times, but not at precise positions. These are used to estimate the intrinsic parameters of the camera, including the distortion terms. Study several of these methods, implement one or two of them, and evaluate the results. This is a good project for more mathematically-inclined students.

## Summary

- Lens distortion as a quadratic or quartic function of the image coordinates.
- The effect of camera movement or using two cameras:
  - Homography matrix  $\mathbf{H}$  for mapping between images taken of a planar surface or taken by a rotating (not translating) camera.
  - Fundamental matrix  $\mathbf{F}$  for establishing epipolar lines in the general case.
- Estimation problems:
  - Establish correspondences and derive constraints to estimate  $\mathbf{H}$ ,  $\mathbf{F}$  and  $\mathbf{M}$ .