Statistical and Learning Techniques in Computer Vision
Lecture 8: Belief Propagation

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1 Overview

In the next several classes we will discuss more recent methods for estimation using Markov Random Fields. At the same time we will begin to look more closely at examples and applications from the computer vision literature. This means the developed lecture notes will be less detailed, but papers from the literature will be distributed. Here is an outline:

- **Belief propagation:** We will discuss the basic algorithm and its generalization. Both [FPC00] and [YFW03] give good introductions. We will not discuss the application details of the former immediately, but rather will do so in a week or two. In [YFW03] you may skip Sections 3 and 4 for now. Our primary initial focus once we have discussed the basic idea of belief propagation (BP) will be a particular application in stereo using [SZS03]. Then we will cover an efficient algorithm redesign in [FH04].
- **Graph cuts:** Again we will focus on stereo, using [BVZ01] to explain the graph-cut algorithm and its application. We will also summarize some results from the literature on what problems can and can not be solved using graph-cut approximation algorithms.
- We will end our discussion of MRF inference techniques using a recent survey paper [FJ05]. The advanced material in [YFW03] will also help here.

Our subsequent discussions of MRFs and graphical models will focus on parmater learning, generalizations to what are known as *conditional* or *discriminative random fields*, and a number of other interesting applications in computer vision and graphics.

2 Outline of Discussion of the BP Formulation

- Thinking again about a graphical model:
  - The image data / evidence becomes the "visible" nodes or variables.
  - The hidden nodes/variables are the pixel values of the reconstructed image.
  - There is a *discrete set* of possible values (labels) at each hidden node.
  - The goal is to estimate the optimum values of the hidden variables.

  - Contrast this with the EM formulation for Gaussian mixture models.
  - In the general case we have an NP-Hard optimization problem.

- We will write down the joint posterior and factorize.

- From there we will develop max-sum (for minimizing the mean-square error —- MMSE) and max-product (for MAP estimation) update rules, and show that they converge in one iteration through the network to the correct MMSE and MAP estimates.

- Despite the fact that these update rules do not have similar convergence properties in graphical models with cycles, evidence over the past 5-10 years shows that they generally converge to good approximate solutions. Using message passing / belief propagation in graphical models with cycles is called *loopy belief propagation*.

- We will analysis the cost in time and in memory for MRFs constructed or image analysis problems.

- Theoretical analysis has shown ties between belief propagation and various energy formulations in statistical mechanics [YFW03] [YFW05]. This has led to better understanding of the convergence properties of belief propagation as well as generalized and more effective algorithm. We will defer discussion of these methods for a week or so, and will not go into nearly the detail of these papers, especially [YFW05].

3 Stereo Matching Using Belief Propagation

We will examine in detail [SZS03], one of the first algorithms that used BP to solve the stereo matching problem. Here is an outline of our discussion.

- Challenges of stereo: noise, textureless region, discontinuities and occlusions.

- Label model:
  - Labeling at each pixel is its disparity or that it is occluded
  - Discontinuities between pixels (on the dual grid) form "line processes" to cut off interpixel connections.
  - Together these form the desired sets D, O, L.

- The prior is a function $P(O, D, L)$

- The data term is a matching cost based on the sample-invariant technique of Birchfield and Tomasi.

- Once they form the posterior probability function, they convert it to a simpler form that no longer needs the explicit labels O and L:

  - The line process is converted to an outlier process on the grid connections, eliminating the need for explicit line process labels. The same is true of the occlusion process.
  - The approximation algorithm for estimation is loopy belief propagation.

- Some efficiency improvements are obtained by looking for unique peaks in messages — corresponding to message from locations where there is strong evidence for a particular disparity estimate.
- The algorithm also incorporates constraints based on an oversegmentation of the image.
- Results are presented on some standard stereo image pairs.
- A multiview stereo algorithm is briefly presented. Note that there are other multivew algorithms in the literature.
- The paper ends with an extensive discussion of the effectivenes of the MAP formulation, the BP algorithm, the use of segments and multiview stereo.

4 Efficient Belief Propagation for Early Vision

This paper from CVPR 2004 by Felzenszwalb and Huttechlocher attempts to further reduce the computation time to make BP algorithms feasible, both in terms of computation time and memory space.

- Loopy BP is presented in terms of an energy formulation and a min-product message passing and update rule. Would this have worked for the sum-product form of BP?
- The first innovation is a method to reduce the message computation time from quadratic to linear for many vision applications. This depends on a specific form to the neighbor compatibility function that allows only a small range of labels to be considered in generating each entry in a message.
- The second is a memory-savings scheme that depends on realizing that the MRF graph is bipartite.
- The third innovation is a multiscale formulation of BP.
- Results are presented for stereo, for optical flow computation and for image restoration.

References

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