Frontiers of Network Science
Fall 2022

Class 14 Robustness
(Chapter 8 in Textbook)

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based on slides by
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Apply the Malloy-Reed Criteria to an Erdos-Renyi Network

A giant cluster exists if each node is connected to at least two other nodes.

$$\kappa \equiv \frac{<k^2>}{<k>} = 2$$

κ>2: a giant cluster exists;

κ<2: many disconnected clusters;

$$<k> = <k>$$

$$<k^2> = <k>(1+<k>)$$

$$\sigma_k = (<k^2> - <k>^2)^{1/2} = <k>^{1/2}$$

$$\kappa \equiv \frac{<k^2>}{<k>} = \frac{<k>(1+<k>)}{<k>} = 1+<k> = 2$$

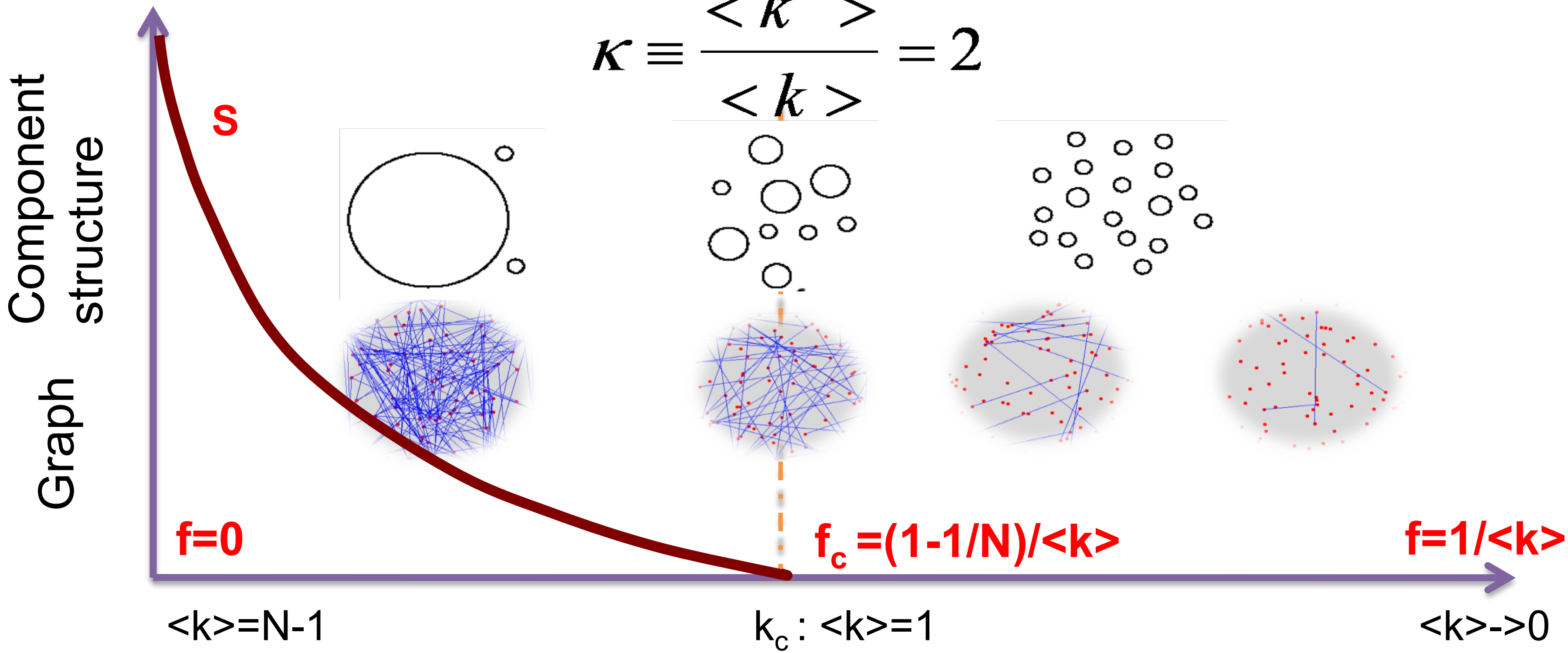
$$<k> = 1$$

Malloy-Reed; Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

RANDOM NETWORK:
DAMAGE IS MODELED AS AN INVERSE PERCOLATION PROCESS

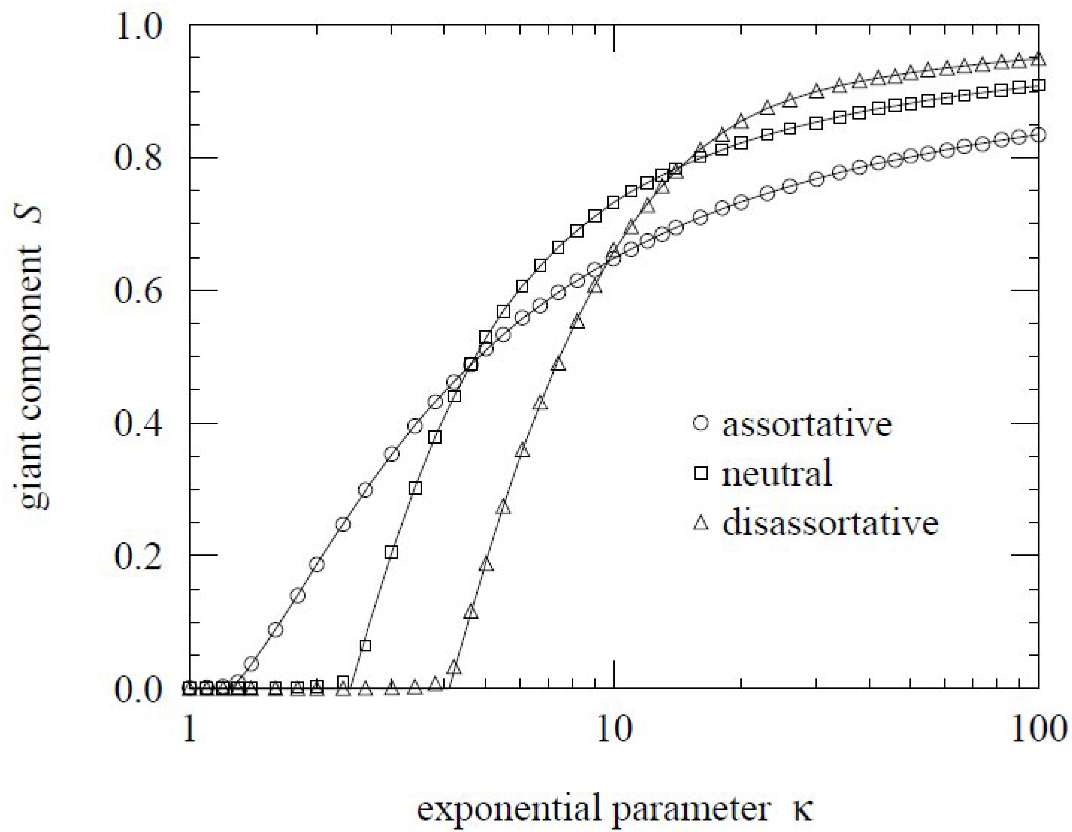
f = fraction of removed nodes

$$\kappa \equiv \frac{<k^2>}{<k>} = 2$$



(Inverse percolation phase transition)

FINAL REMARKS: EFFECT OF ASSORTATIVE MIXING: PERCOLATION



M. E. J. Newman, Phys. Rev. Lett. **89,** 208701 (2002)

BREAKDOWN THRESHOLD FOR ARBITRARY P(k)

Problem: What are the consequences of removing a fraction *f* of all nodes?

At what threshold f_c will the network fall apart (no giant component)?

Random node removal changes

the degree of individual nodes [k → k' ≤k]

the degree distribution [P(k) → P'(k')]

A node with degree *k* will loose some links and become a node with degree *k'* with probability:

$$\binom{k}{k'} f^{k-k'} (1-f)^{k'} \qquad k' \leq k$$

Remove k-k' links, each with probability f

Leave k' links untouched, each with probability 1-f

The prob. that we had a *k* degree node was *P(k)*, so the probability that we will have a new node with degree *k'*:

$$P'(k') = \sum_{k=k'}^{\infty} P(k) \binom{k}{k'} f^{k-k'} (1-f)^{k'}$$

Let us asume that we know <k> and <k^2> for the original degree distribution P(k)
→ calculate <k'> , <k'^{2}> for the new degree distribution P'(k').

Cohen et al., *Phys. Rev. Lett*. **85,** 4626 (2000).

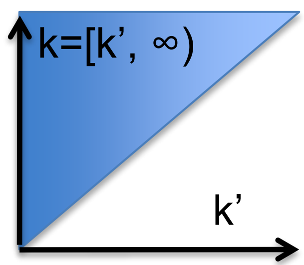
BREAKDOWN THRESHOLD FOR ARBITRARY P(k)

$$P'(k') = \sum_{k=k'}^{\infty} P(k) \binom{k}{k'} f^{k-k'} (1-f)^{k'}$$

Degree distribution after we removed *f* fraction of nodes.

$$< k' >_f = \sum_{k'=0}^{\infty} k' P'(k') = \sum_{k'=0}^{\infty} k' \sum_{k=k'}^{\infty} P(k) \frac{k!}{k'!(k-k')!} f^{k-k'} (1-f)^{k'} = \underline{\sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} P(k) \frac{k(k-1)!}{(k'-1)!(k-k')!} f^{k-k'} (1-f)^{k'-1} (1-f)}$$

The sum is done over the triangle shown in the right, so we can replace it with



$$\sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} = \sum_{k=0}^{\infty} \sum_{k'=0}^{k}$$

$$< k' >_f = \underline{\sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} P(k) \frac{k(k-1)!}{(k'-1)!(k-k')!} f^{k-k'} (1-f)^{k'-1} (1-f)} = \sum_{k=0}^{\infty} (1-f) k P(k) \sum_{k'=0}^{k} \frac{(k-1)!}{(k'-1)!(k-k')!} f^{k-k'} (1-f)^{k'-1} =$$

$$\sum_{k=0}^{\infty} (1-f) k P(k) \sum_{k'=0}^{k} \binom{k-1}{k'-1} f^{k-k'} (1-f)^{k'-1} = \sum_{k=0}^{\infty} (1-f) k P(k) = \boxed{(1-f) < k >}$$

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

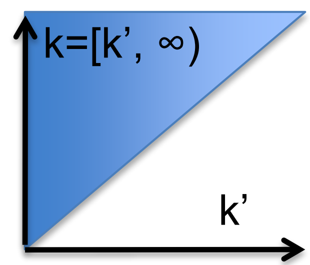
BREAKDOWN THRESHOLD FOR ARBITRARY P(k)

$$P'(k') = \sum_{k=k'}^{\infty} P(k) \binom{k}{k'} f^{k-k'} (1-f)^{k'}$$

Degree distribution after we removed f fraction of nodes.

$$< k'^2 >_f = < k'(k'-1) - k' >_f = \sum_{k'=0}^{\infty} k'(k'-1) P'(k') - < k' >_f$$

The sum is done over the triangle shown in the right, i.e. we can replace it with



$$\sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} = \sum_{k=0}^{\infty} \sum_{k'=0}^{k}$$

$$< k'(1-k') >_f = \sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} P(k) \frac{k(k-1)(k-2)!}{(k'-2)!(k-k')!} f^{k-k'} (1-f)^{k-2'} (1-f)^2 = \sum_{k=0}^{\infty} (1-f)^2 k(k-1) P(k) \sum_{k'=0}^{k} \frac{(k-2)!}{(k'-2)!(k-k')!} f^{k-k'} (1-f)^{k'-2} =$$

$$\sum_{k=0}^{\infty} (1-f)^2 k(k-1) P(k) \sum_{k'=0}^{k} \binom{k-2}{k'-2} f^{k-k'} (1-f)^{k'-2} = \sum_{k=0}^{\infty} (1-f)^2 k(k-1) P(k) = (1-f)^2 < k(k-1) >$$

$$< k'^2 >_f = < k'(k'-1) - k' >_f = (1-f)^2 (< k^2 > - < k >) - (1-f) < k > = \underline{(1-f)^2 < k^2 > + f(1-f) < k >}$$

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

BREAKDOWN THRESHOLD FOR ARBITRARY P(k)

Robustness: we remove a fraction *f* of the nodes.

At what threshold f_c will the network fall apart (no giant component)?

Random node removal changes

the degree of individuals nodes [k → k' ≤k)

the degree distribution [P(k) → P'(k')]

$$<k'>_f = (1-f)<k>$$

$$<k'^2>_f = (1-f)^2<k^2> + f(1-f)<k>$$

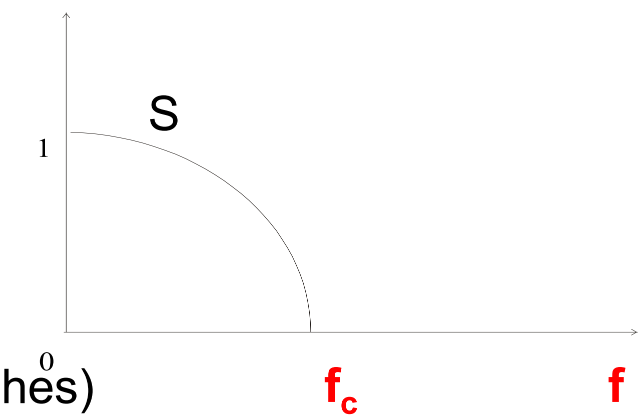
$$\kappa \equiv \frac{<k'^2>_f}{<k'>_f} = 2$$

κ>2: a giant cluster exists

κ<2: many disconnected clusters

Breakdown threshold:

$$\boxed{f_c = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}}$$



$f<f_c$: the network is still connected (there is a giant cluster)

$f>f_c$: the network becomes disconnected (giant cluster vanishes)

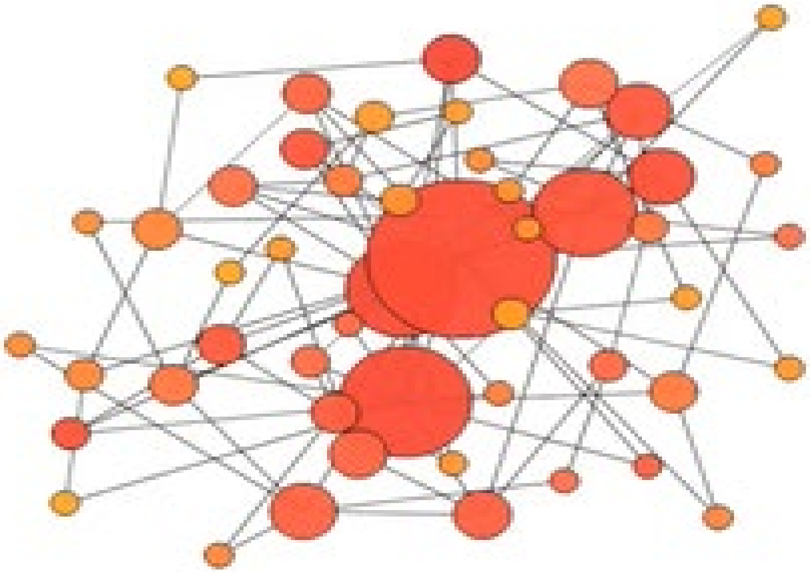
Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

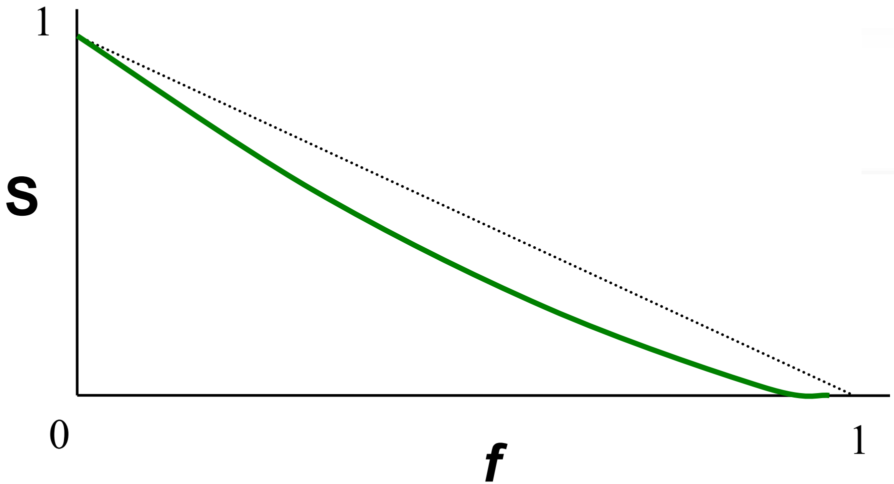
ROBUSTNESS OF SCALE-FREE NETWORKS

Scale-free networks do not appear to break apart under random failures.

Reason: the hubs.

The likelihood of removing a hub is small.





Albert, Jeong, Barabási, *Nature* **406** 378 (2000)

ROBUSTNESS OF SCALE-FREE NETWORKS

Scale-free networks do not appear to break apart under random failures. Why is that?

$$f_c = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$$

$$K_{max} = K_{min} N^{\frac{1}{\gamma-1}}$$

$$<k^m> = (\gamma-1) K_{min}^{\gamma-1} \int_{K_{min}}^{K_{max}} k^{m-\gamma} dk = \frac{(\gamma-1)}{(m-\gamma+1)} K_{min}^{\gamma-1} \left[k^{m-\gamma+1} \right]_{K_{min}}^{K_{max}}$$

$$<k^m> = \frac{(\gamma-1)}{(m-\gamma+1)} K_{min}^{\gamma-1} \left[K_{max}{}^{m-\gamma+1} - K_{min}{}^{m-\gamma+1} \right]$$

$$\frac{<k^2>}{<k>} = \frac{(2-\gamma)}{(3-\gamma)} \frac{K_{max}{}^{3-\gamma} - K_{min}{}^{3-\gamma}}{K_{max}{}^{2-\gamma} - K_{min}{}^{2-\gamma}}$$

$$\gamma > 3: \quad \kappa = \frac{(2-\gamma)}{(3-\gamma)} \frac{K_{max}{}^{3-\gamma} - K_{min}{}^{3-\gamma}}{K_{max}{}^{2-\gamma} - K_{min}{}^{2-\gamma}} = \left| \frac{2-\gamma}{3-\gamma} \right| K_{min}$$

$$3 > \gamma > 2: \quad \kappa = \frac{(2-\gamma)}{(3-\gamma)} \frac{K_{max}{}^{3-\gamma} - K_{min}{}^{3-\gamma}}{K_{max}{}^{2-\gamma} - K_{min}{}^{2-\gamma}} = \left| \frac{2-\gamma}{3-\gamma} \right| K_{max}{}^{3-\gamma} K_{min}^{\gamma-2}$$

$$2 > \gamma > 1: \quad \kappa = \frac{(2-\gamma)}{(3-\gamma)} \frac{K_{max}{}^{3-\gamma} - K_{min}{}^{3-\gamma}}{K_{max}{}^{2-\gamma} - K_{min}{}^{2-\gamma}} = \left| \frac{2-\gamma}{3-\gamma} \right| K_{max}$$

ROBUSTNESS OF SCALE-FREE NETWORKS

$$f_c = 1 - \frac{1}{\kappa - 1} \qquad \kappa = \frac{<k^2>}{<k>} = \left|\frac{2-\gamma}{3-\gamma}\right| \begin{cases} K_{\min} & \gamma > 3 \\ K_{\max}{}^{3-\gamma} K_{\min}^{\gamma-2} & 3 > \gamma > 2 \\ K_{\max} & 2 > \gamma > 1 \end{cases}$$

$$K_{\max} = K_{\min} N^{\frac{1}{\gamma-1}}$$

γ>3: κ is finite, so the network will break apart at a finite f_c that depens on K_{min}

γ<3: κ diverges in the N→ ∞ limit, so f_c → 1 !!!

for an infinite system one needs to remove all the nodes to break the system.

For a finite system, there is a finite but large f_c that scales with the system size as: $\kappa \cong 1 - CN^{-\frac{3-\gamma}{\gamma-1}}$

Internet: Router level map, N=228,263; γ=2.1±0.1; *κ=28* → ***f_c=0.962***

AS level map, N= 11,164; γ=2.1±0.1; *κ=264* → ***f_c=0.996***

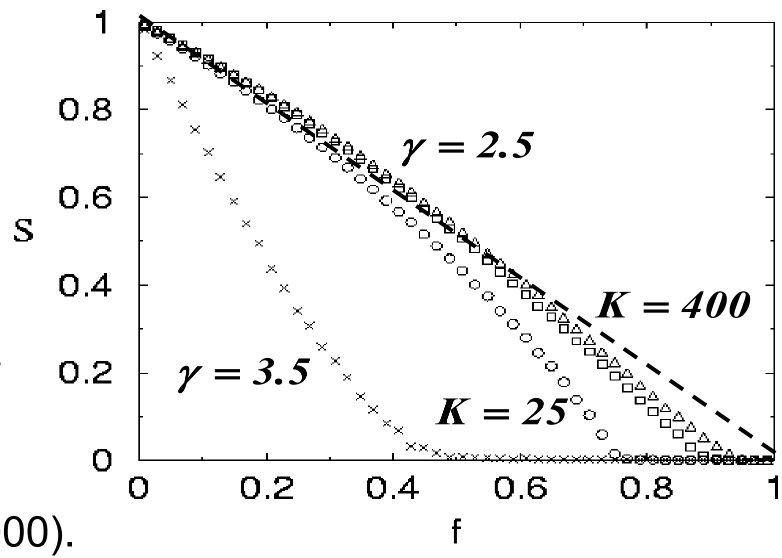
NUMERICAL EVIDENCE

Scale-free random graph with

$$P(k) = Ak^{-\gamma}, \text{ with } k = m, \ldots K$$

$$f_c = 1 - \frac{1}{\frac{\gamma-2}{\gamma-3}m - 1} \quad \text{if } \gamma > 3$$

$$f_c = 1 - \frac{1}{\frac{\gamma-2}{3-\gamma}m^{\gamma-2}K^{3-\gamma} - 1} \quad \text{if } 2 < \gamma < 3$$



Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

Infinite scale-free networks with $\gamma < 3$ do not break down under random node failures.

SIZE OF THE GIANT COMPONENT DURING RANDOM DAMAGE –WITHOUT PROOF-

S: size of the giant component, f fraction of randomly removed nodes, not damage for $f<f_c$

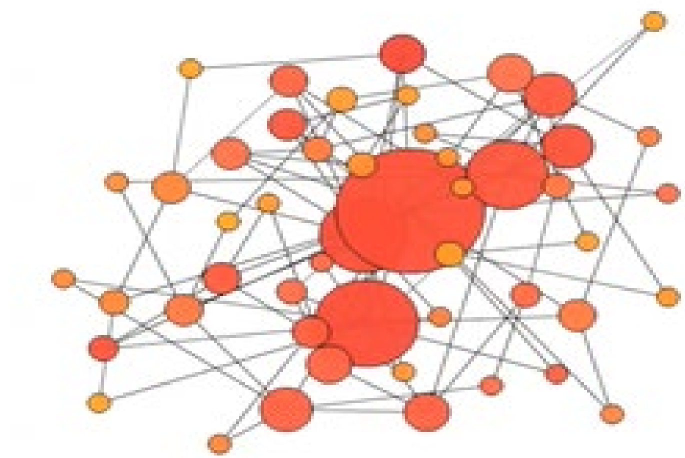
(i) $\gamma>4$: $S \approx f-f_c$ (similar to that of a random graph)

(i) $3>\gamma>4$: $S \approx (f-f_c)^{1/(\gamma-3)}$

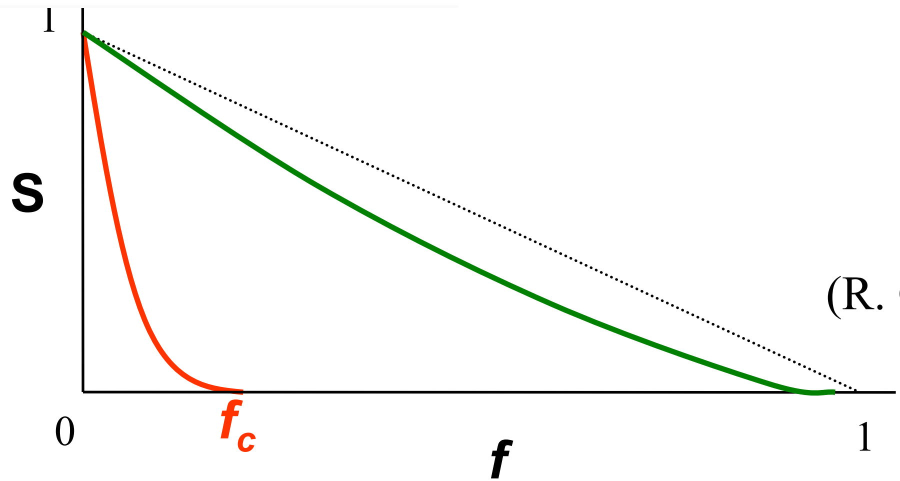
(i) $\gamma<3$: $f_c = 0$ and $S \approx f^{1+1/(3-\gamma)}$

R. Cohen, D. ben-Avraham, S. Havlin,
Percolation critical exponents in scale-free networks
Phys. Rev. E 66, 036113 (2002);
See also: Dorogovtsev S, Lectures on Complex Networks, Oxford, pg44

ACHILLES' HEEL OF SCALE-FREE NETWORKS



Attacks



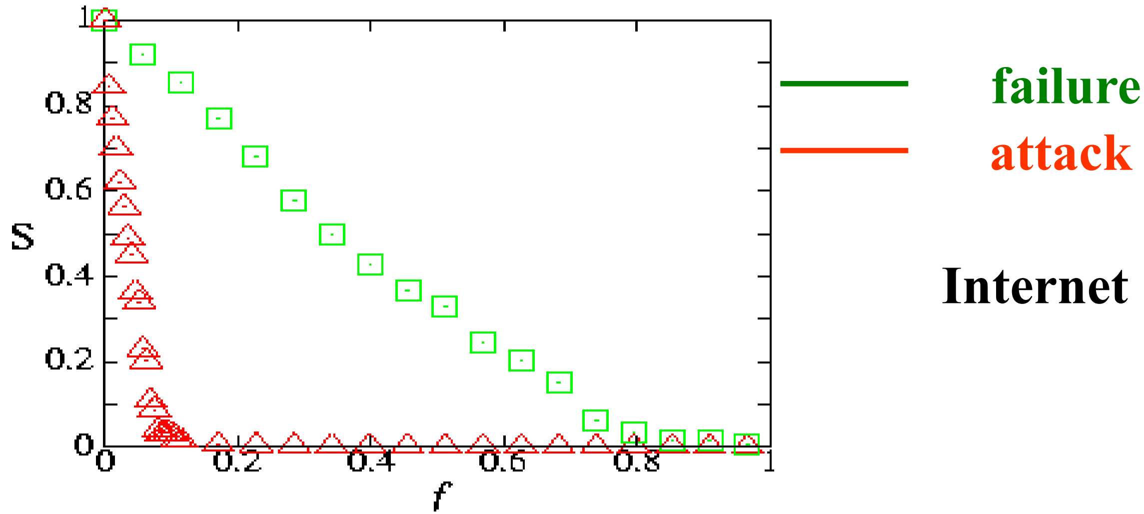
Failures

$\gamma \leq 3 : f_c = 1$

(R. Cohen et al PRL, 2000)

Albert, Jeong, Barabási, *Nature* **406** 378 (2000)

INTERNET'S ROBUSTNESS TO RANDOM FAILURES



R. Albert, H. Jeong, A.L. Barabasi, *Nature* **406** 378 (2000)

$$f_c = 1 - \frac{1}{\kappa - 1}$$

Internet: Router level map, N=228,263; γ=2.1±0.1; $\kappa=28$ → $f_c=0.962$

AS level map, N= 11,164; γ=2.1±0.1; $\kappa=264$ → $f_c=0.996$

Internet parameters: Pastor-Satorras & Vespignani, *Evolution and Structure of the Internet*: Table 4.1 & 4.4

ATTACK THRESHOLD FOR ARBITRARY P(K)

Attack problem: we remove a fraction *f* of the hubs.

At what threshold f_c will the network fall apart (no giant component)?

Hub removal changes

the maximum degree of the network [$K_{max} \rightarrow K'_{max} \leq K_{max}$)

the degree distribution [$P(k) \rightarrow P'(k')$]

A node with degree k will loose some links because some of its neighbors will vanish.

Claim: once we correct for the changes in K_{max} and P(k),we are back to the robustness problem.

That is, attack is nothing but a robusiness of the network with a new K_{max} and P(k).

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

ATTACK THRESHOLD FOR ARBITRARY P(K)

Attack problem: we remove a fraction *f* of the hubs.

the maximum degree of the network [$K_{max} \rightarrow K'_{max} \leq K_{max}$) `

If we remove an *f* fraction of hubs, the maximum degree changes:

$$\int_{K'_{max}}^{K_{max}} P(k)dk = f$$

$$\int_{K'_{max}}^{K_{max}} P(k)dk = (\gamma-1)K_{min}^{\gamma-1}\int_{K'_{max}}^{K_{max}} k^{-\gamma}dk = \frac{\gamma-1}{1-\gamma}K_{min}^{\gamma-1}(K_{max}^{1-\gamma} - K'^{1-\gamma}_{max})$$

As $K'_{max} \leq K_{max}$ we can ignore the K_{max} term

$$\left(\frac{K_{min}}{K'_{max}}\right)^{\gamma-1} = f \qquad K'_{max} = K_{min} f^{\frac{1}{1-\gamma}}$$

← The new maximum degree after removing f fraction of the hubs.

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

ATTACK THRESHOLD FOR ARBITRARY P(K)

Attack problem: we remove a fraction *f* of the hubs.

the degree distribution changes [P(k) → P'(k')]

A node with degree *k* will loose some links because some of its neighbors will vanish.

Let us calculate the fraction of links removed 'randomly' , f', as a consequence of we removing f fraction of hubs.

$$\int_0^{K_{max}} kP(k)dk$$

as $K'_{max} \leq K_{max}$

$$f' = \frac{\int_{K'_{max}}^{K_{max}} kP(k)dk}{<k>} = \frac{1}{<k>}(\gamma-1)K_{min}^{\gamma-1}\int_{K'_{max}}^{K_{max}} k^{1-\gamma}dk = \frac{1}{<k>}\frac{\gamma-1}{2-\gamma}K_{min}^{\gamma-1}(K_{max}^{2-\gamma} - K'^{2-\gamma}_{max}) = -\frac{1}{<k>}\frac{\gamma-1}{2-\gamma}K_{min}^{\gamma-1}K'^{2-\gamma}_{max}$$

$$f' = -\frac{1}{<k>}\frac{\gamma-1}{2-\gamma}K_{min}^{\gamma-1}K_{min}^{2-\gamma}f^{\frac{2-\gamma}{1-\gamma}} = -\frac{1}{<k>}\frac{\gamma-1}{2-\gamma}K_{min}f^{\frac{2-\gamma}{1-\gamma}}$$

$$K'_{max} = K_{min}f^{\frac{1}{\gamma-1}}$$

$$<k^m> = -\frac{(\gamma-1)}{(m-\gamma+1)}K_{min}^m$$

$$f' = f^{\frac{2-\gamma}{1-\gamma}}$$

For γ→2, f'→1, which means that even the removal of a tiny fraction of hubs will destroy the network. The reason is that for γ=2 hubs dominate the network

$$<k> = -\frac{(\gamma-1)}{(2-\gamma)}K_{min}$$

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

ATTACK THRESHOLD FOR ARBITRARY P(K)

Attack problem: we remove a fraction *f* of the hubs.

At what threshold f_c will the network fall apart (no giant component)?

Hub removal changes

the maximum degree of the network [$K_{max} \rightarrow K'_{max} \leq K_{max}$) $\quad K'_{max} = K_{min} f^{\frac{1}{1-\gamma}}$

the degree distribution [$P(k) \rightarrow P'(k')$]

A node with degree k will loose some links because some of its neighbors will vanish. $f' = f^{\frac{2-\gamma}{1-\gamma}}$

Claim: once we correct for the changes in K_{max} and P(k), we are back to the robustness problem.

That is, attack is nothing but a robustness of the network with a new K'_{max} and f'.

$$\boxed{f'} = 1 - \frac{1}{\kappa' - 1} \qquad \kappa' = \frac{\langle k'^2 \rangle}{\langle k' \rangle} = \frac{\langle k^2 \rangle}{(1-f_c)\langle k \rangle} = \frac{\kappa}{1-f_c}$$

$$\kappa = \left|\frac{2-\gamma}{3-\gamma}\right| \begin{cases} K_{min} & \gamma > 3 \\ \boxed{K_{max}^{3-\gamma}} K_{min}^{\gamma-2} & 3 > \gamma > 2 \\ K_{max} & 2 > \gamma > 1 \end{cases} \qquad f_c^{\frac{2-\gamma}{1-\gamma}} = 2 + \frac{2-\gamma}{3-\gamma} K_{min}\left(f_c^{\frac{3-\gamma}{1-\gamma}} - 1\right)$$

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

ATTACK THRESHOLD FOR ARBITRARY P(K)

Attack problem: we remove a fraction *f* of the hubs.

At what threshold f_c will the network fall apart (no giant component)?

$$f_c^{\frac{2-\gamma}{1-\gamma}} = 2 + \frac{2-\gamma}{3-\gamma} K_{\min}\left(f_c^{\frac{3-\gamma}{1-\gamma}} - 1 \right)$$

- f_c depends on γ; it reaches its max for γ<3
- f_c depends on K_{min} (m in the figure)
- Most important: f_c is tiny. Its maximum reaches only 6%, i.e. the removal of 6% of nodes can destroy the network in an attack mode.
- Internet: γ=2.1, so 4.7% is the threshold.

f_c

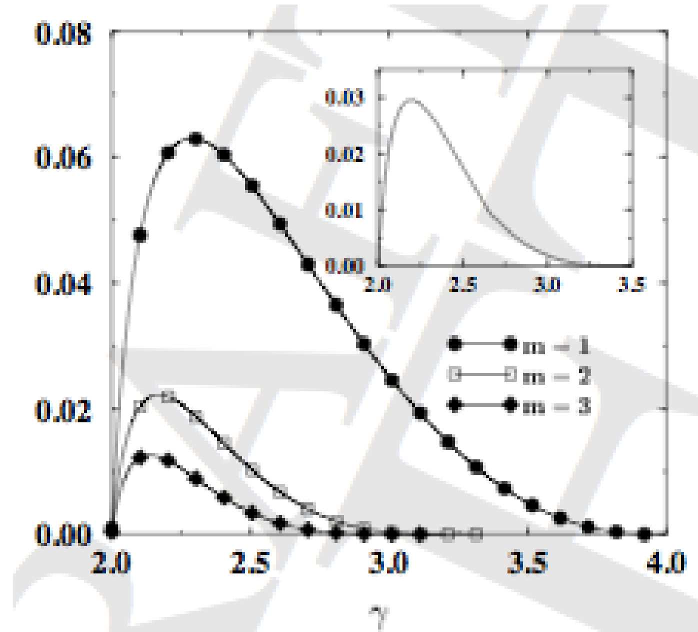


Figure: Pastor-Satorras & Vespignani, *Evolution and Structure of the Internet*: Fig 6.12

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

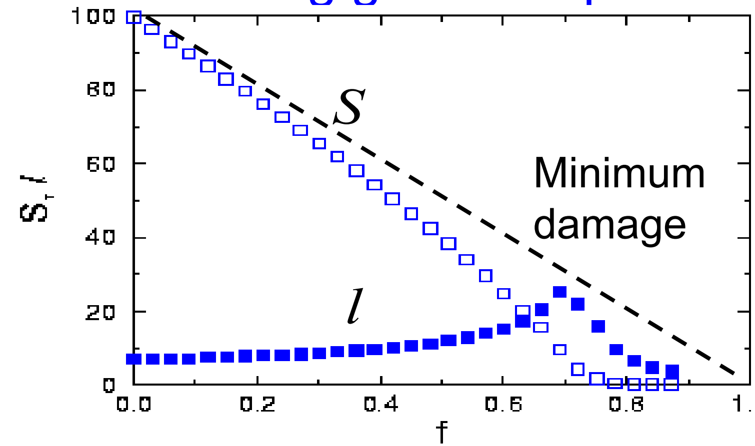
APPLICATION: ER RANDOM GRAPHS

Consider a random graph with connection probability *p* such that at least a giant connected component is present in the graph.

Find the critical fraction of removed nodes such that the giant connected component is destroyed.

$$f_c = 1 - \frac{1}{\frac{\langle k_0^2 \rangle}{\langle k_0 \rangle} - 1} = 1 - \frac{1}{pN} = 1 - \frac{1}{\langle k_0 \rangle}$$

S surviving giant component



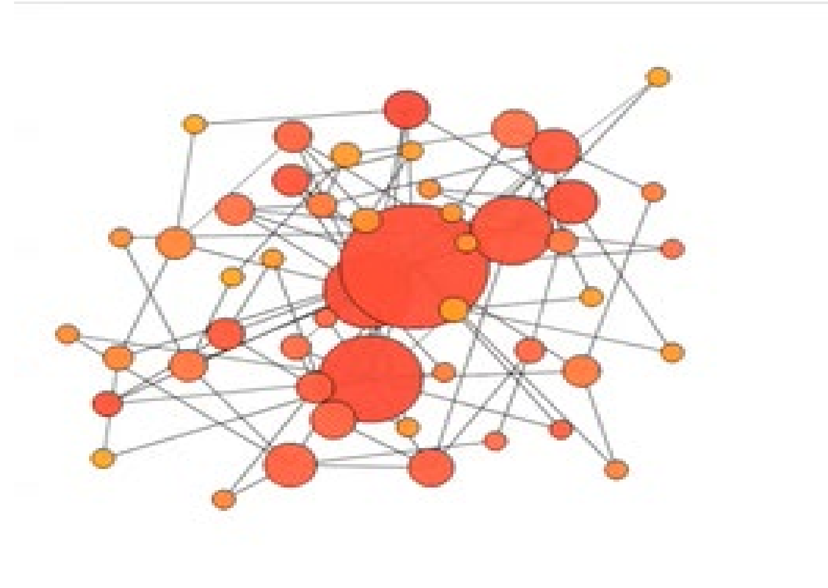
The higher the original average degree, the larger damage the network can survive.

Empty squares show *S*

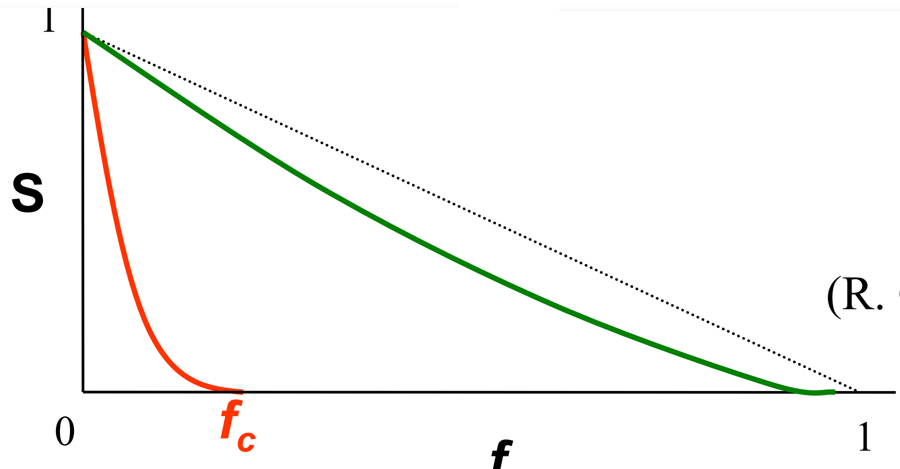
Filled squares *l* – avg. distance

Q: How do you explain the peak in the average distance?

SUMMARY: ACHILLES' HEEL OF SCALE-FREE NETWORKS



Attacks



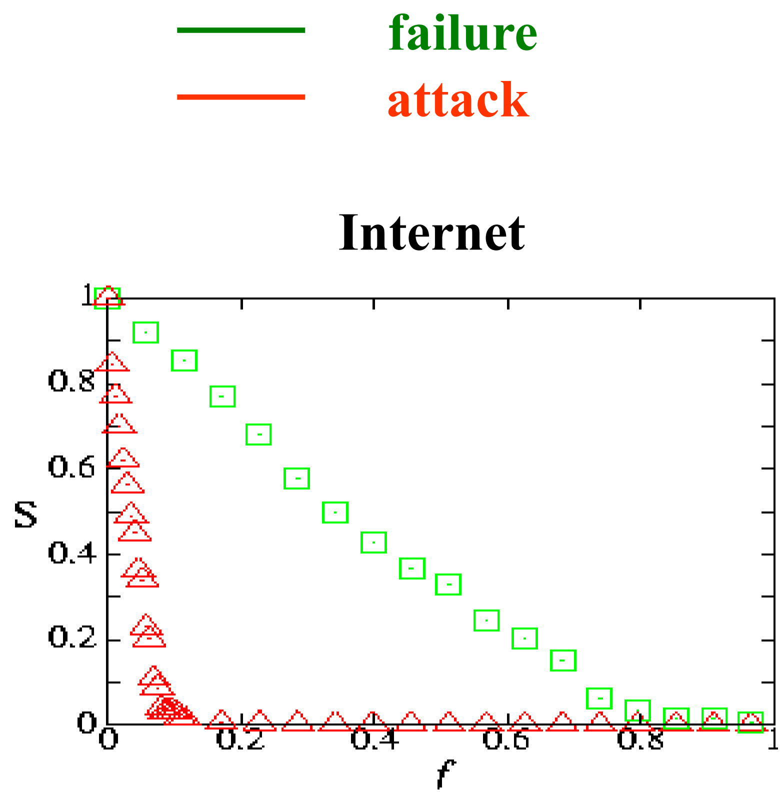
Failures

$\gamma \leq 3 : f_c = 1$

(R. Cohen et al PRL, 2000)

Albert, Jeong, Barabási, *Nature* **406** 378 (2000)

SUMMARY: ACHILLES' HEEL OF COMPLEX NETWORKS

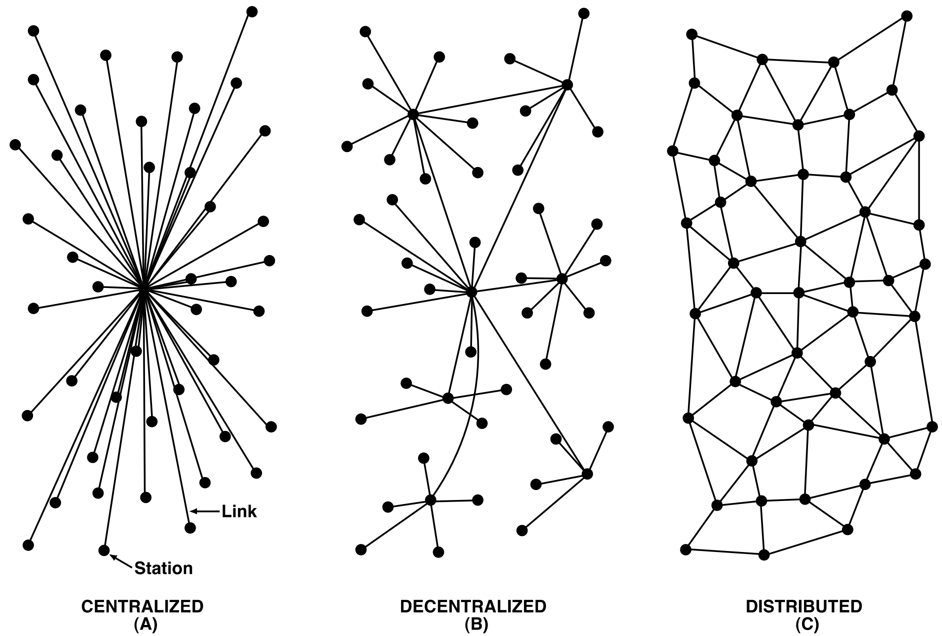


R. Albert, H. Jeong, A.L. Barabasi, *Nature* **406** 378 (2000)

HISTORICAL DETOUR: PAUL BARAN AND INTERNET

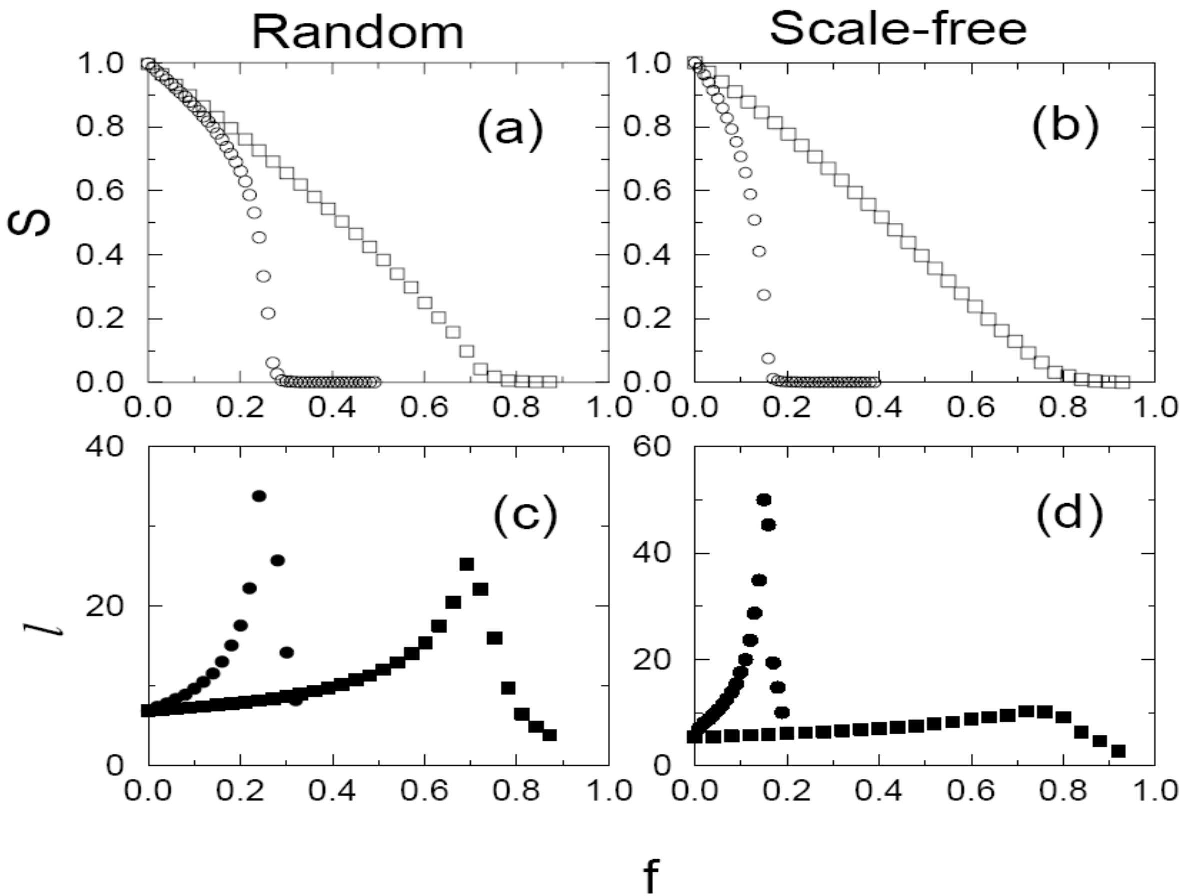
1958

A network of n-ary degree of connectivity has n links per node was simulated



The simulation revealed that networks where n ≥ 3 had a significant increase in resilience against even as much as 50% node loss. Baran's insight gained from the simulation was that redundancy was the key.

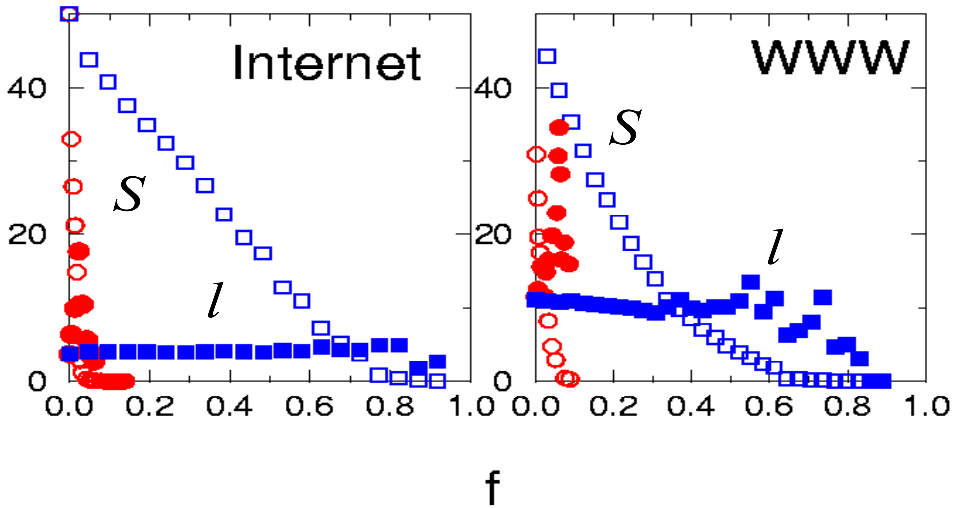
SCALE-FREE NETWORKS ARE MORE ERROR TOLERANT, BUT ALSO MORE VULNERABLE TO ATTACKS



- squares: random failure
- circles: targeted attack
- S surviving fraction of GC
- *l* average distance

Failures: little effect on the integrity of the network.
Attacks: fast breakdown

REAL SCALE-FREE NETWORKS SHOW THE SAME DUAL BEHAVIOR



- blue squares: random failure
- red circles: targeted attack
- open symbols: ***S*** *(size of surviving component)*
- filled symbols: ***l*** *(average distance)*

- break down if 5% of the nodes are eliminated selectively (always the highest degree node)
- resilient to the random failure of 50% of the nodes.

Similar results have been obtained for metabolic networks and food webs.

CASCADES

Potentially large events triggered by small initial shocks

- **Information cascades**

  social and economic systems

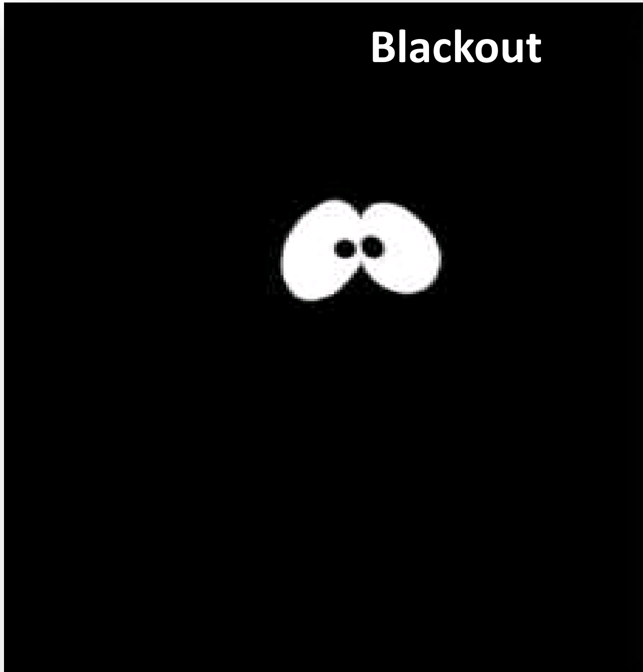
  diffusion of innovations

- **Cascading failures**

  infrastructural networks

  complex organizations

CASCADING FAILURES IN NATURE AND TECHNOLOGY



Flows of physical quantities

- congestions
- instabilities
- Overloads

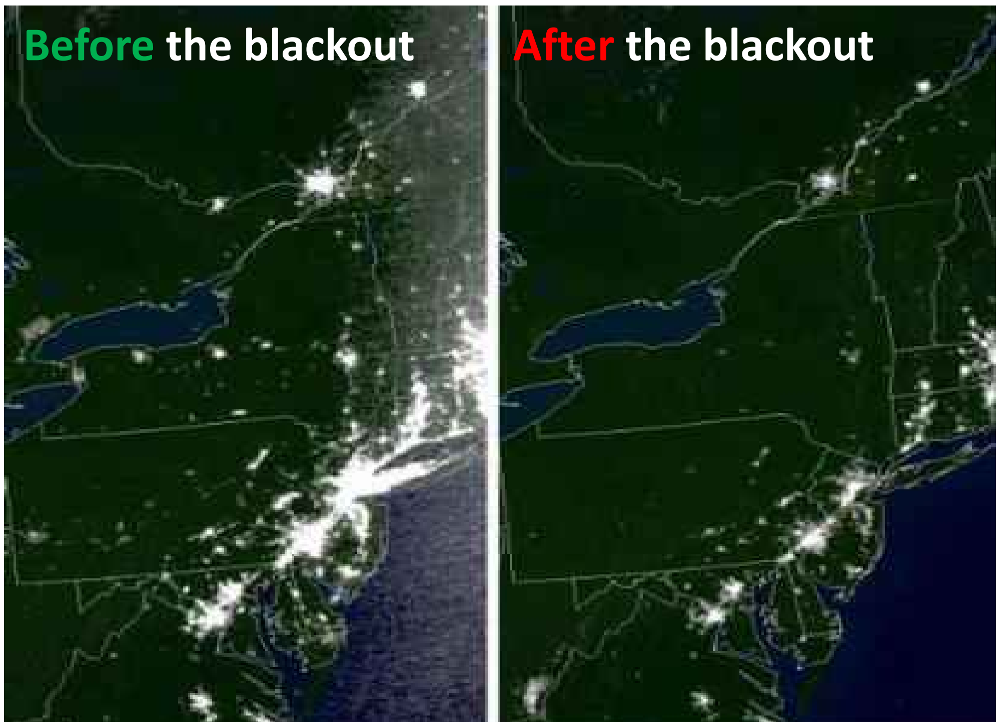
Cascades depend on

- Structure of the network
- Properties of the flow
- Properties of the net elements
- Breakdown mechanism

NORTHEAST BLACKOUT OF 2003

Origin

A 3,500 MW power surge (towards Ontario) affected the transmission grid at 4:10:39 p.m. EDT. (Aug-14-2003)



Consequences

More than 508 generating units at 265 power plants shut down during the outage. In the minutes before the event, the NYISO-managed power system was carrying 28,700 MW of load. At the height of the outage, the load had dropped to 5,716 MW, a loss of 80%.

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NUMBER 73
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$1.50 outside of
Metro Boston

The Boston Globe

MONDAY, MARCH 14, 2011

A NEW WEAK

TODAY: *Partly sunny and colder. Hi 37-42. Low 27-32.*
TOMORROW: *Mostly sunny, milde High 42-47. Low 32-37.*
HIGH TIDE: *6:42 a.m., 7:25 p.m.*
SUNRISE: *6:59* SUNSET: *6:49*
FULL REPORT: PAGE B13

Cascading disaster in Japan

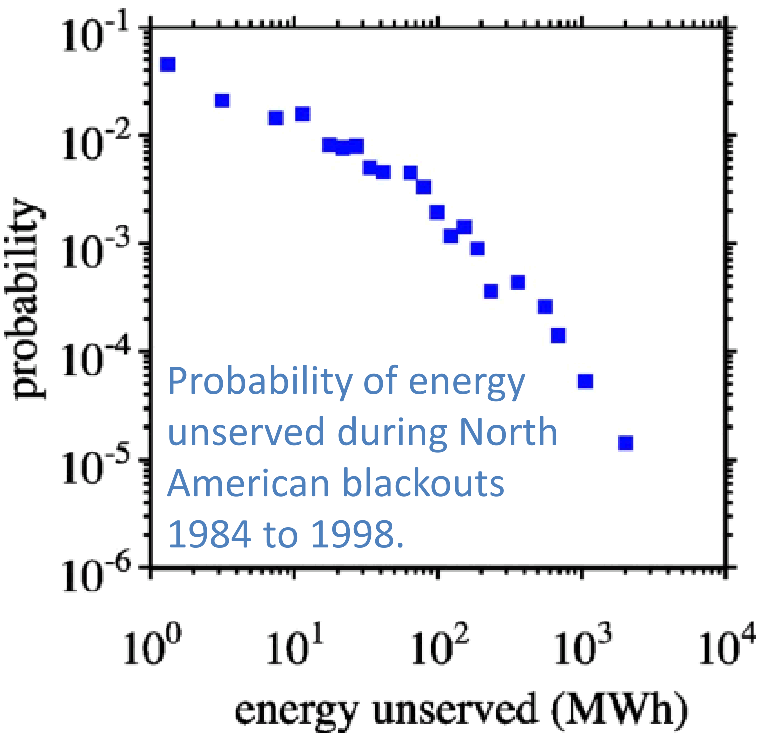
Blast shakes a second reactor death toll soar

By Martin Fackler
and Mark McDonald
NEW YORK TIMES

SENDAI, Japan — Japan reel from a rapidly unfolding disaster epic scale yesterday, pummeled by t death toll, destruction, and homele ness caused by the earthquake a tsunami and new hazards from da aged nuclear reactors. The prime m ister called it Japan's worst crisis sir World War II.

Japan's $5 trillion economy, t world's third largest, was threaten with severe disruptions and partial p ralysis as many industries shut dov temporarily. The armed forces and v unteers mobilized for the far more gent crisis of finding survivors, evac ating residents near the strick power plants and caring for the v tims of the record 8.9 magnitu quake that struck on Friday.

CASCADES SIZE DISTRIBUTION OF BLACKOUTS



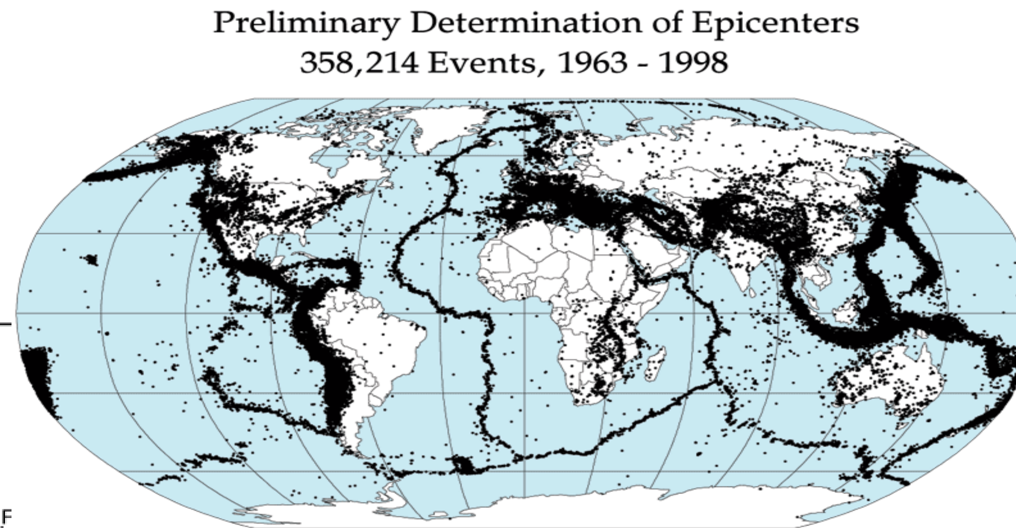
Unserved energy/power magnitude (*S*) distribution

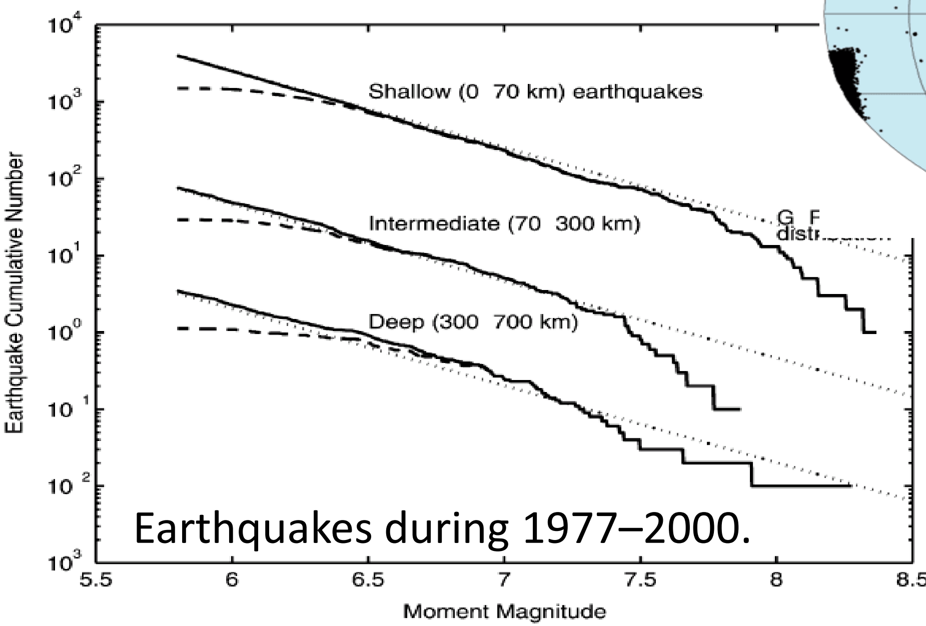
$$P(S) \sim S^{-\alpha},\ 1 < \alpha < 2$$

Source	Exponent	Quantity
North America	2.0	Power
Sweden	1.6	Energy
Norway	1.7	Power
New Zealand	1.6	Energy
China	1.8	Energy

I. Dobson, B. A. Carreras, V. E. Lynch, D. E. Newman, *CHAOS* 17, 026103 (2007)

CASCADES SIZE DISTRIBUTION OF EARTHQUAKES



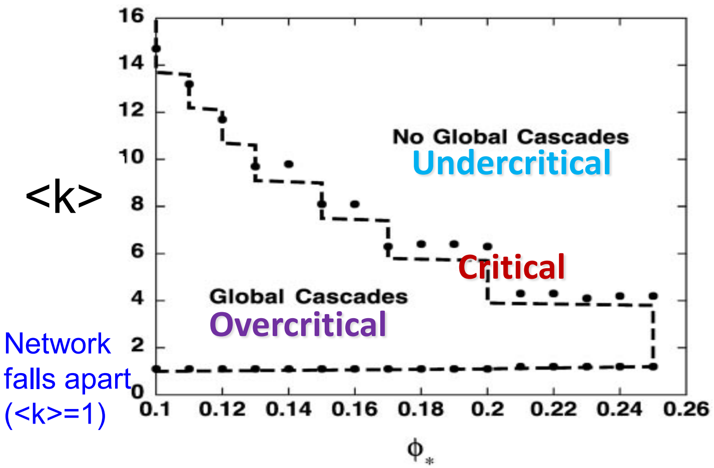


Earthquake size *S* distribution

$$P(S) \sim S^{-\alpha}, \alpha \approx 1.67$$

Y. Y. Kagan, *Phys. Earth Planet. Inter.* 135 (2–3), 173–209 (2003)

FAILURE PROPAGATION MODEL



Initial Setup

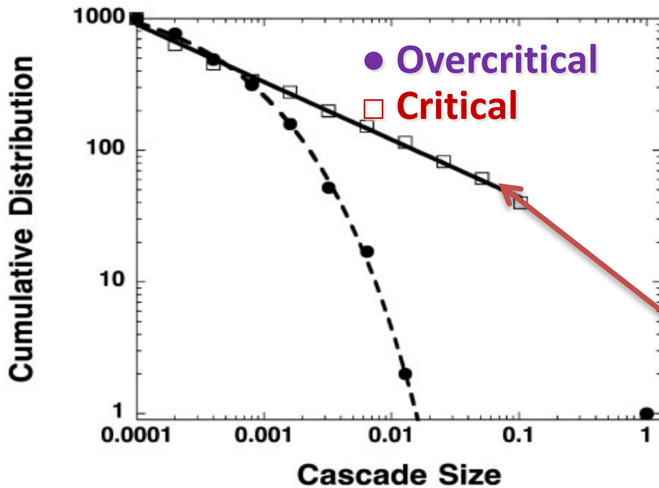
•Random graph with *N* nodes

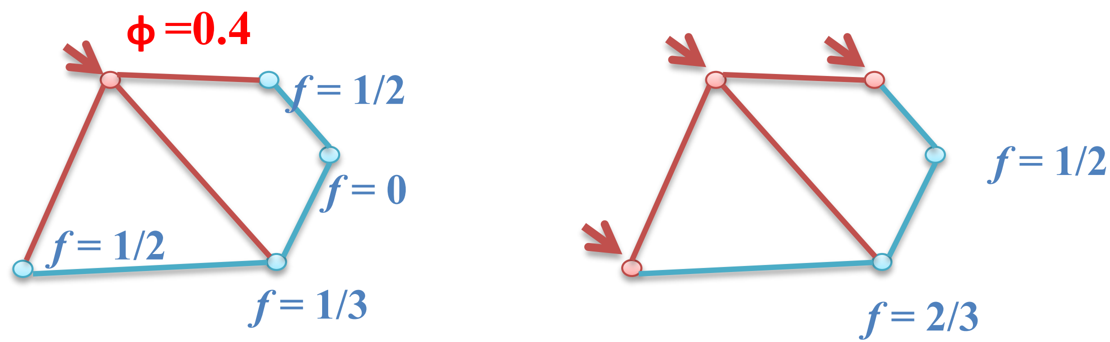
•Initially each node is functional.

Cascade

•Initiated by the failure of one node.

•f_i : fraction of failed neighbors of node *i*. Node *i* fails if f_i is greater than a global threshold ϕ.





Erdos-Renyi network

$$P(S) \sim S^{-3/2}$$

D. Watts, *PNAS* 99, 5766-5771 (2002)