Epidemic Modeling

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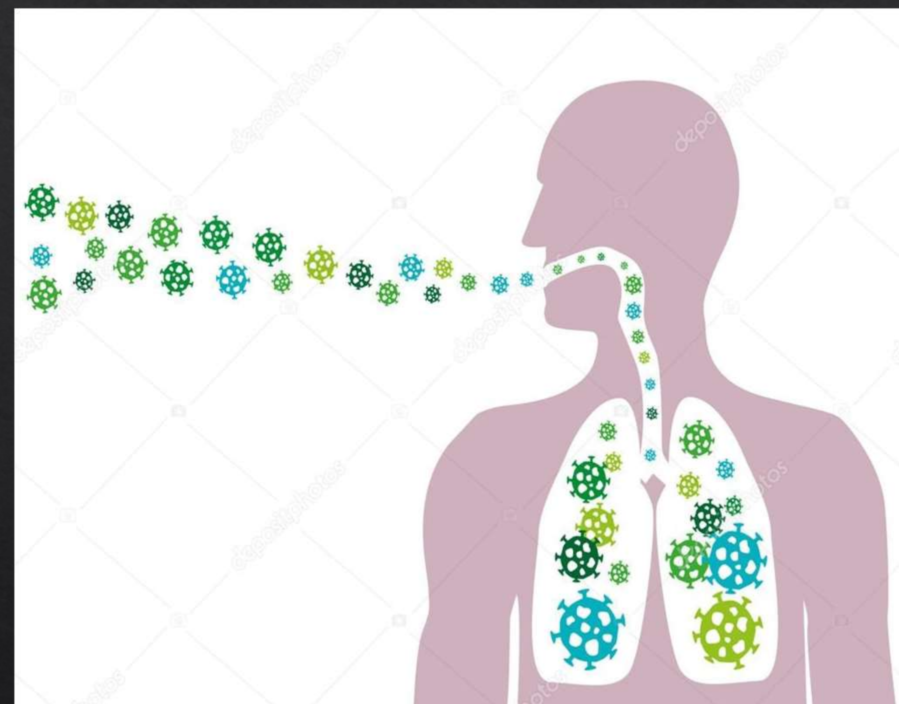
Outline

- Goal
- Setup and Assumptions
- Epidemic Models
  - Susceptible-Infected (SI) Model
  - Susceptible-Infected-Susceptible (SIS) Model
  - Susceptible-Infected-Recovered (SIR) Model
- Summary / Conclusion

Goal

Develop an analytical and numerical framework for modeling the spread of pathogens throughout a population



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Compartmentalization

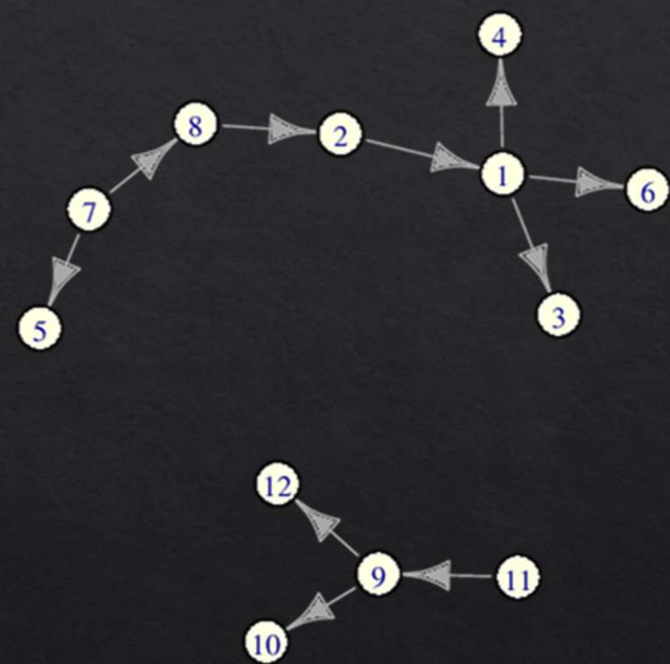
We group individuals within a population into one of three fluid categories, or states

Susceptible (S)	Infectious (I)	Recovered (R)
Individual is Healthy, and has not yet contracted the pathogen	Individual has contracted the pathogen and can transfer it to others.	Individual contracted the pathogen and recovered from it, thus can no longer transfer it to others

- Optional States:
  - Latent: Individual contracted pathogen, but is not yet contagious
  - Immune: Individual inherently unable to contract or transfer pathogen

Assumption: Homogenous Mixing

- At any point in time, a healthy individual has some probability of encountering an infected individual.
    - Saves us from having to first model individual-to-individual contact networks
    - Any individual can be infected by anyone else
- More assumptions
    - Population remains constant
    - If symptomatic, then contagious



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Susceptible-Infected (SI) Model

- Once infected, you’re infected forever
- Examples: Zombie apocalypse, HIV

Susceptible-Infected (SI) Model

- Variable Definitions
  - N – Population
  - <k> - Average number of contacts for each individual
  - β – Likelihood of infection upon contact with infected individual

  - S(t) – Count of susceptible individuals at time t
  - I(t) – Count of infected individuals at time t

  - $s(t) = S(t) / N = s$
  - $i(t) = I(t) / N = i$

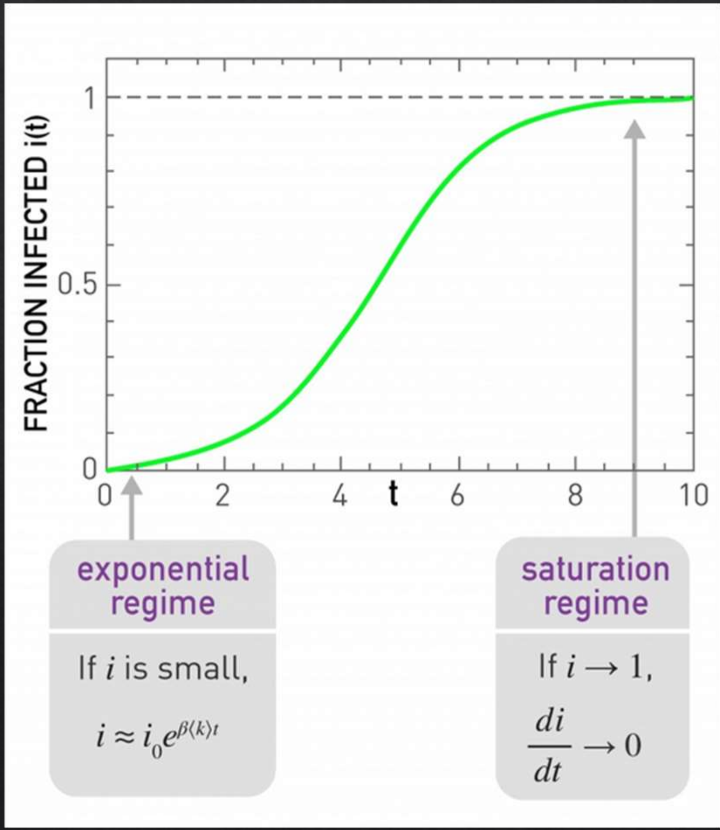
- Initial Conditions
  - $S(0) = N - 1$
  - $I(0) = 1$

Computing i for SI Model

- $\frac{di}{dt} = \beta < k > \frac{S(t)I(t)}{N}$
- Product $\beta < k >$ known as transmission rate
- For simplicity, solving of differential eqution is omitted
- Solution: $i = \frac{i_0 e^{\beta<k>t}}{1 - i_0 + i_0 e^{\beta<k>t}}$

SI Model Results

- Early on, infection rate increases exponentially
- Epidemic ends when everyone is infected
  - $I(t \to \infty) = N$
  - $S(t \to \infty) = 0$
- 36% (1/e) infected after $\tau = 1/\beta < k >$
  - Increasing either β or <k> increases initial infection rate
  - Characteristic Time



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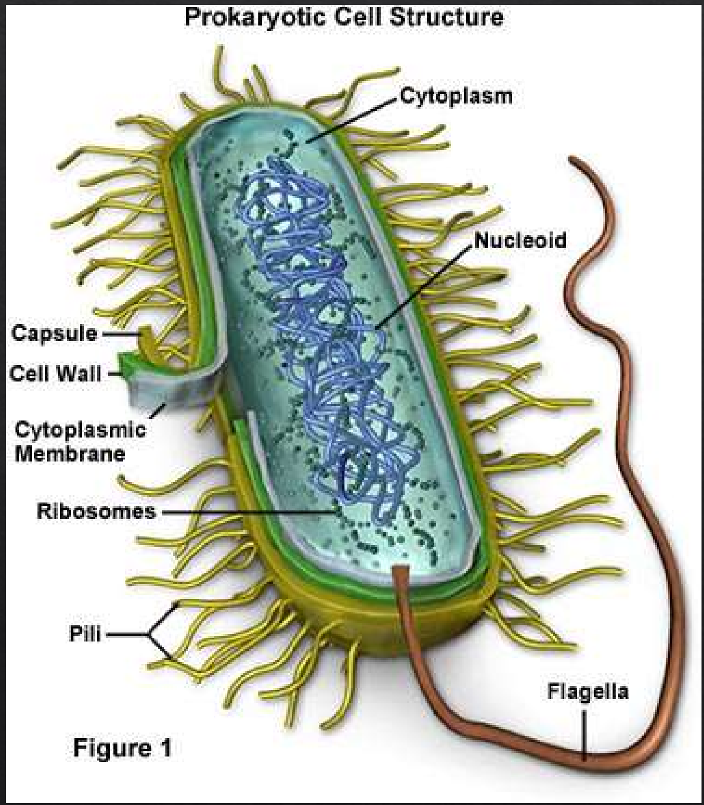
Susceptible-Infected-Susceptible (SIS) Model

- Infected individuals recover at some rate μ
- Once recovered, you can still contract the pathogen again.
- Example: Bacterial diseases



Computing i for SIS Model

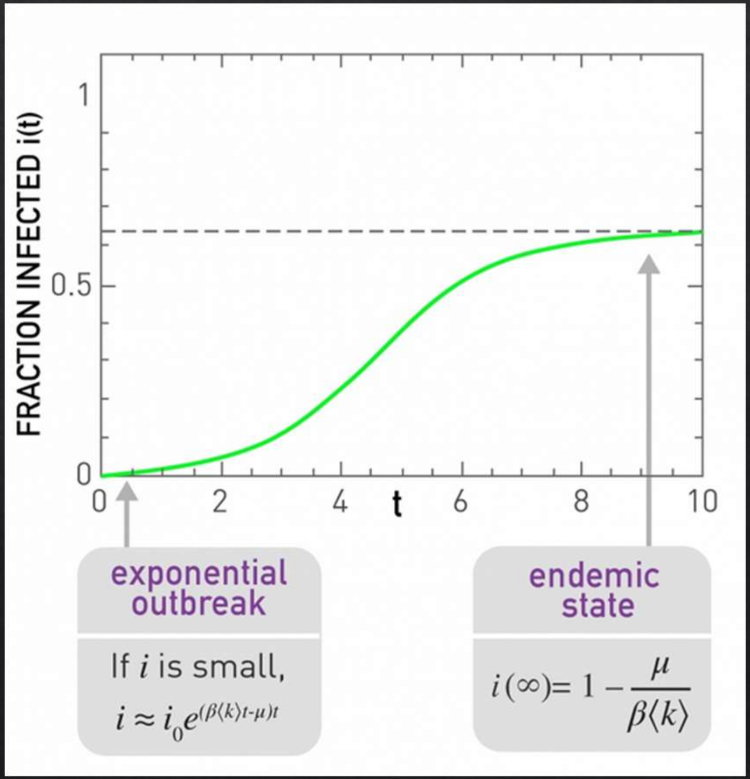
- $\frac{di}{dt} = \beta < k > \frac{S(t)I(t)}{N}$ - μi
- Solution:

$$i = \left(1 - \frac{\mu}{\beta\langle k\rangle}\right) \frac{Ce^{(\beta\langle k\rangle - \mu)t}}{1+Ce^{(\beta\langle k\rangle-\mu)t}} \qquad (10.7)$$

where the initial condition $i_0=i(t=0)$ gives $C=i_0/(1-i_0-\mu/\beta\langle k\rangle)$.

SIS Model Results

- Infection rate still increases exponentially early on.
- Can end up in one of two final states:
  - $\mu < (\beta < k >)$: Some fraction of the population will always be infected.
  - else: Pathogen dies and nobody is infected.



SIS – Basic Reproductive Number

- Number of new infections an infected individual causes under ideal circumstances.
- $R_0 = \frac{\beta <k>}{\mu}$
- If $R_0 < 1$, then epidemic will end in disease-free state
- If $R_0 >= 1$, then epidemic will end in endemic state

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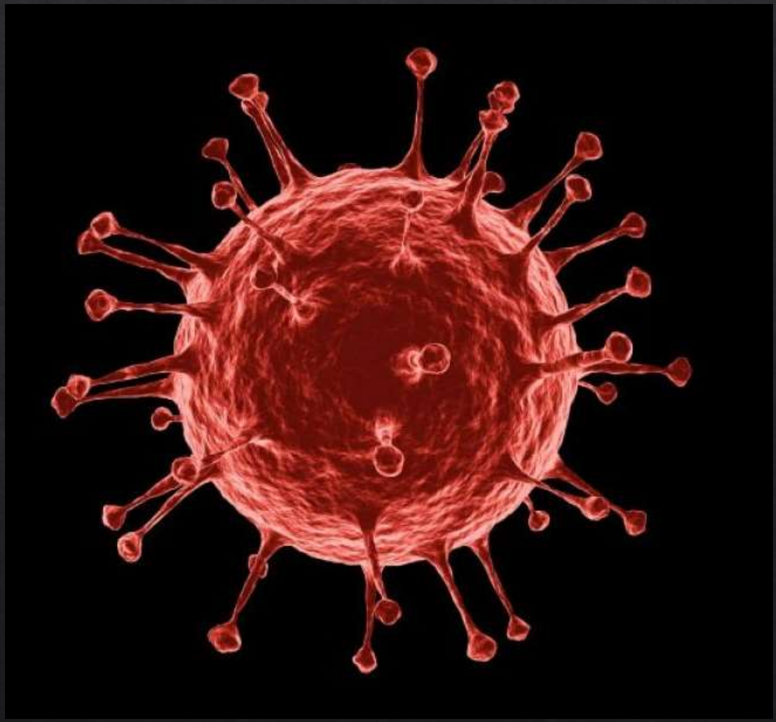
Susceptible-Infected-Recovered (SIR) Model

- Once an infected individual recovers , he/she is immune to the pathogen, thus is removed from the model's population.
- Example: Most viral infections

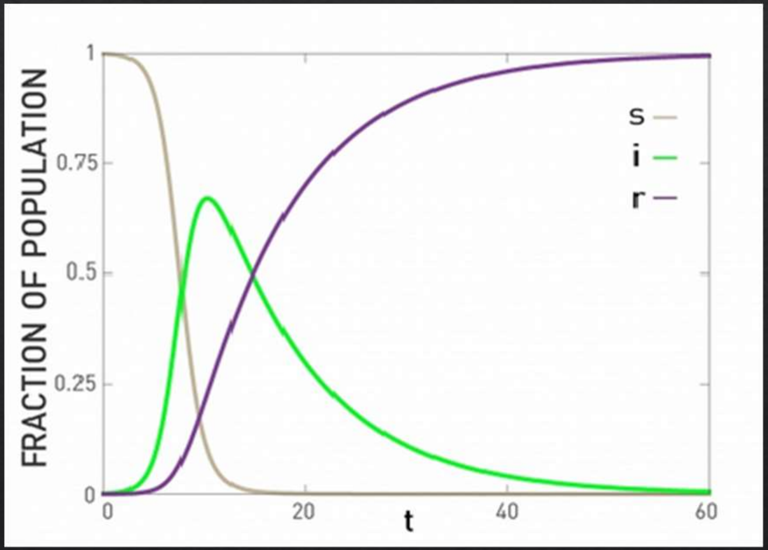


Susceptible-Infected-Recovered (SIR) Model

$$\frac{ds}{dt} = -\beta \langle k \rangle i \, [1 - r - i]$$

$$\frac{di}{dt} = -\mu i + \beta \langle k \rangle i \, [1 - r - i]$$

$$\frac{dr}{dt} = \mu i$$



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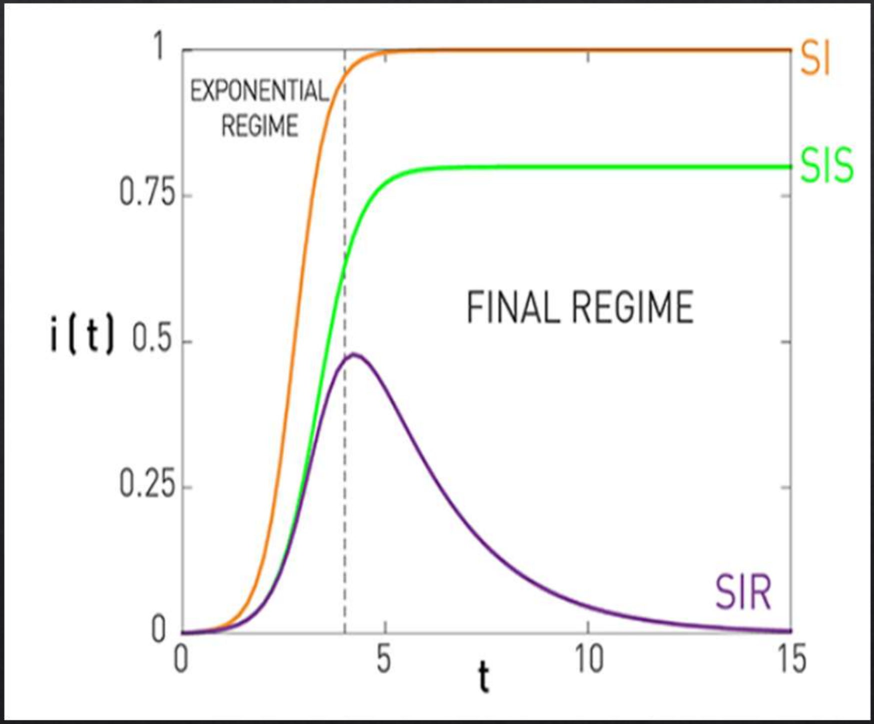
Summary

- All models show exponential similar exponential increase in infection rate early on in epidemic.
- For SI, epidemic ends with everyone infected.
- For SIS, epidemic either ends with nobody infected, or a fraction infected based on R_0
- For SIR, infection eventually dies out.



Questions?